

Full Length Research Paper

Nonlinear dynamics of water ecosystem of three gorges reservoir area

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Accepted 29 August, 2011

A new nonlinear dynamics model of alga-aquatic animal is proposed based on the water ecosystem of Three Gorges Reservoir Area. The stability of the equilibria and Hopf bifurcation of the delay are studied. The results of the analysis show that with the increasing of the delay, the stability switch of the positive equilibrium and the asymptotically stable positive periodic solution resulting from Hopf bifurcation will occur. Thus, the time lag is one of the reasons that water quality is fluctuating in the water ecosystem of Three Gorges Reservoir Area, which may provide a theoretical basis for the oscillation of water quality.

Key words: Hopf bifurcation, delay, stability switch, water ecosystem.

INTRODUCTION

Environmental problem is an important challenge of the world during the 21st century and the major research directions include the project of water environment, the control of non-point source pollution and the project of water pollution, and so on (Ai et al., 2002). Ecosystem health and ecological risk assessment have become a new research topic and a bridge to connecting geoscience, environmental science, ecology, economics and social science (Chen et al., 2006; Wu and Niu, 2006). Since the 1990s, there is an intensified research on marine ecosystem and one of the important directions is the ocean ecosystem dynamics model (Feng et al., 2007; Liu et al., 2003), which has been applied in the research on complexity of freshwater ecosystem widely (Shen and Cai, 2003).

This paper considers a simple aquatic ecosystem based on Three Gorges Reservoir Area including the species of algae (for example, diatom) and aquatic animals (for example, fish). Algae species breed massive mainly due to environmental pollution and aquatic animals purify water quality because of preying algae (Kuang et al., 2000). So the simple aquatic ecosystem can simulate the water quality of Three Gorges Reservoir Area. On this basis, in order to explain the oscillation

of water quality of Three Gorges Reservoir Area, we proposed a new nonlinear dynamics model of the alga-aquatic animal ecosystem, and analyzed the dynamical behavior of the model based on the nonlinear dynamics theory.

MATERIALS AND METHODS

Model description

Considering the predator-prey relationship between the species of algae and aquatic animals, we proposed the following aquatic ecosystem dynamics model:

$$\begin{cases} x'(t) = (r - dx(t))x(t) - \beta x(t)y(t), \\ y'(t) = \beta kx(t - \tau)y(t - \tau) - uy(t). \end{cases} \quad (1)$$

where $x(t), y(t)$ is the number of alga and aquatic animals at time t , respectively, r, d represents the intrinsic growth rate and the density constraint coefficient of alga, respectively, r/d is the environmental carrying capacity of alga, β represents the predation coefficient of aquatic animals, k is the feed conversion efficiency ratio of aquatic animals, u expresses the sum of natural mortality rate and the fishing intensity of aquatic animals, τ is the delay time between aquatic preying alga and translating to effective biomass. All parameters are positive constants due to the biological

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significance. Note that $C^+([- \tau, 0], R_+^2)$ is the space of continuous functions mapping the interval $[- \tau, 0]$ into R_+^2 , where $R_+^2 = \{(x, y) : x \geq 0, y \geq 0\}$. The initial conditions for system (1) take the form:

$$\begin{aligned} x(\theta) &= \varphi_1(\theta), y(\theta) = \varphi_2(\theta), \\ (\varphi_1, \varphi_2) &\in C^+([- \tau, 0], R_+^2), \\ \varphi_i(0) &> 0, i = 1, 2. \end{aligned}$$

Dynamic analysis

We first present the preliminary results for system (1) as follows:

Theorem 1

Under the aforementioned initial conditions, all solutions of system (1) are nonnegative, and system (1) is dissipative.

This theorem is clear to be seen, thus, the process of proving is omitted.

From the work of van den Driessche et al. (2002), the basic reproductive number of aquatic animals is given by $R_0 = \beta kr / du$. It is easy to see that the extinction equilibrium $E_0 = (0, 0)$ and the elimination equilibrium of aquatic $E_1 = (r/d, 0)$ always exist. If $R_0 < 1$, system (1) has no positive equilibrium; if $R_0 > 1$, system (1) has a unique positive equilibrium $E_2 = (x^*, y^*)$, where $x^* = u / \beta k$, $y^* = (\beta kr - du) / \beta^2 k$.

When the delay time is ignored, system (1) is changed to the following ordinary differential equations:

$$\begin{cases} x'(t) = (r - dx(t) - \beta y(t))x(t), \\ y'(t) = \beta ky(t)x(t) - uy(t). \end{cases} \quad (2)$$

For system (2), taking a Dulac function $D = 1/xy$, we can easily obtain the following results by Dulac criterion and Poincaré-Bendixson Theorem (Zhang and Feng, 2000).

Theorem 2

For system (2), the following conclusions are always valid.

- i) Boundary equilibrium E_0 is always unstable.
- ii) If $R_0 < 1$, boundary equilibrium E_1 is globally asymptotically stable.
- iii) If $R_0 > 1$, positive equilibrium E_2 is globally asymptotically stable.

Now we discuss the dynamics of system (1) when $\tau > 0$. First, we have:

Theorem 3

For system (1), the following conclusions are always valid:

- i) Boundary equilibrium E_0 is always unstable;
- ii) Boundary equilibrium E_1 is globally asymptotically stable if $R_0 < 1$ and is unstable if $R_0 > 1$.

Proof 1

The linearization of Equation 1 at E_0 and E_1 are:

$$\begin{cases} x'(t) = rx(t), \\ y'(t) = -uy(t). \end{cases}$$

And

$$\begin{cases} x'(t) = -rx(t) - \frac{\beta r}{d} y(t), \\ y'(t) = \frac{\beta kr}{d} y(t - \tau) - uy(t). \end{cases}$$

Thus, boundary equilibrium E_0 is always unstable and the characteristic equation at equilibrium E_1 is given by:

$$H_{E_1}(\lambda) = (\lambda + r)(\lambda + u - \frac{\beta kr}{d} e^{-\lambda \tau}) = 0. \quad (3)$$

Suppose that $\lambda = a + bi, a > 0$, is a root characteristic of Equation 3, if $R_0 < 1$. Substituting it into Equation 3, we have:

$$\cos(b\tau) = \frac{da + du}{\beta kr} e^{a\tau} > 1,$$

Which leads to a contradiction. Thus, boundary equilibrium E_1 is locally asymptotically stable if $R_0 < 1$. When $R_0 > 1$, since there must be an intersection point in the first quadrant for $f(\lambda) = \lambda + u$ and $g(\lambda) = (\beta kr/d)e^{-\lambda \tau}$, boundary equilibrium E_1 is unstable if $R_0 > 1$.

Now, we prove E_1 is globally asymptotically stable if $R_0 < 1$. Note that it is locally asymptotically stable. We only need to prove $\lim_{t \rightarrow +\infty} x(t) = r/d$, $\lim_{t \rightarrow +\infty} y(t) = 0$. Using Theorem 1 and the second equation of Equation 1, we have:

$$y'(t) \leq \frac{\beta k(r + \varepsilon)}{d} y(t - \tau) - uy(t)$$

Here $0 < \varepsilon < (du - \beta kr) / \beta k$ is a constant. Thus, by the basic theory of stability of functional differential equations (Zheng, 1994) and the comparison theorem, we have $\lim_{t \rightarrow +\infty} y(t) = 0$. In order to prove $\lim_{t \rightarrow +\infty} x(t) = r/d$, we split this proof into two cases.

i) Suppose that $x(t)$ is ultimately monotone. Since $x(t)$ is bounded, there exists a positive constant \bar{x} such that $\lim_{t \rightarrow +\infty} x(t) = \bar{x}$. By the fluctuation lemma (Wolkowicz et.al., 1997), we have $\lim_{t \rightarrow +\infty} x'(t) = 0$. Note that $\lim_{t \rightarrow +\infty} y(t) = 0$. Based on the first equation of Equation 1, we have $0 = (r - d\bar{x})\bar{x}$, which means that $\bar{x} = r/d$.

(ii) Suppose that $x(t)$ is always oscillatory. Let $\liminf_{t \rightarrow +\infty} x(t) = x_\infty$ and $\limsup_{t \rightarrow +\infty} x(t) = x^\infty$. By the fluctuation lemma (Wolkowicz et.al., 1997), we know that there exists a sequence $\{t_m\} \uparrow \infty$ such that

$$\lim_{m \rightarrow +\infty} x(t_m) = x^\infty, x'(t_m) = 0.$$

Based on the first equation of Equation 1, we have:

$$0 = x(t_m)(r - dx(t_m)) - \beta y(t_m).$$

Letting $m \rightarrow +\infty$, we deduce that $0 = x^\infty(r - dx^\infty)$, which means that $x^\infty = r/d$. Similarly, we can prove $x_\infty = r/d$. Thus, according to (i) and (ii), we know that $\lim_{t \rightarrow +\infty} x(t) = r/d$ is always valid. This completes the proof.

Next, we analyze the dynamics of system (1) when $R_0 > 1$ and obtain the following result of uniform persistence based on the persistence theory for infinite dimensional systems. Let X be a complete metric space. Suppose that $X^0 \subset X, X_0 \subset X, X^0 \cap X_0 = \Phi$. Assume that $T(t)$ is a C_0 -semigroup of X satisfying:

$$\begin{cases} T(t) : X^0 \rightarrow X^0, \\ T(t) : X_0 \rightarrow X_0. \end{cases} \tag{4}$$

Then the following lemma is the persistence theory for infinite dimensional systems in Wang and Chen, (1997).

Lemma 1

Suppose that $T(t)$ satisfies Equation 4 and we have the following:

- i) there is a $t_0 \geq 0$ such that $T(t)$ is compact for $t > t_0$;
- ii) $T(t)$ is point dissipative in X ;

iii) $\tilde{A}_0 = \bigcup_{x \in A_0} \omega(x)$ is isolated and has an acyclic covering \tilde{M} ,

where $\tilde{M} = \{M_1, M_2, \dots, M_n\}$;

iv) $W^s(M_i) \cap X^0 = \Phi, i = 1, 2, \dots, n$.

Then X^0 is a uniform repeller with respect to X_0 , that is, there is an $\varepsilon > 0$ such that for any $x \in X^0$, $\liminf_{t \rightarrow +\infty} d(T(t)x, X_0) \geq \varepsilon$, where d is the distance of $T(t)x$ from X_0 .

Theorem 4

If $R_0 > 1$, then system (1) is uniformly persistent.

Proof 2

Let:

$$C_1 = \{(\varphi_1, \varphi_2) \in C([- \tau, 0], R_+^2) : \varphi_1(\theta) \equiv 0, \theta \in [- \tau, 0]\}$$

$$C_2 = \{(\varphi_1, \varphi_2) \in C([- \tau, 0], R_+^2) : \varphi_2(\theta) \equiv 0, \theta \in [- \tau, 0]\}$$

If $X_0 = C_1 \cup C_2, X^0 = \text{int } C([- \tau, 0], R_+^2)$, it is easy to show that X_0, X^0 are positive invariant set, so Equation 4 is satisfied. Since $T(t)$ is dissipative in $X = X_0 \cup X^0$ by Theorem 1, conditions (i) and (ii) of Lemma 1 are satisfied. Note that the system (1) only has two boundary equilibria E_0, E_1 , and E_1 is a global attractor of C_2 . The condition (iii) of Lemma 1 is satisfied. Next, we show that condition (iv) of Lemma 1 is also satisfied.

It is easy to prove $\omega^s(E_0) \cap X^0 = \Phi$. Now we begin to prove $\omega^s(E_1) \cap X^0 = \Phi$. Assume that it is not satisfied. Then there is a positive solution $(\tilde{x}(t), \tilde{y}(t))$ such that $(\tilde{x}(t), \tilde{y}(t)) \rightarrow (r/d, 0)$ as $t \rightarrow +\infty$. Since $R_0 > 1$, we can choose a sufficiently small $0 < \varepsilon < (\beta kr - du) / \beta k$ such that $\beta k(r - \varepsilon) / d > u$. Suppose that there is a sufficiently large T such that $(r - \varepsilon) / d < \tilde{x}(t)$ as $t \geq T - \tau$. Then the following is true when $t > T$:

$$\tilde{y}'(t) = \beta k \tilde{x}(t - \tau) \tilde{y}(t - \tau) - u \tilde{y}(t) > \frac{\beta k(r - \varepsilon)}{d} \tilde{y}(t - \tau) - u \tilde{y}(t),$$

For linear equations:

$$\tilde{y}'(t) = \frac{\beta k(r - \varepsilon)}{d} \tilde{y}(t - \tau) - u\tilde{y}(t),$$

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It is clear that the zero solution is unstable when $\beta k(r - \varepsilon) / d > u$.

Thus $\lim_{t \rightarrow +\infty} \tilde{y}(t) \neq 0$, which is a contradiction. So the condition (iv) of Lemma 1 is satisfied. Therefore, all positive solutions of system (1) are uniform repeller with respect to X_0 , that is, there is $\delta > 0$ such that for any solution $(x(t), y(t))$ of system (1) with initial conditions, $\liminf_{t \rightarrow +\infty} x(t) \geq \delta, \liminf_{t \rightarrow +\infty} y(t) \geq \delta$. The proof is completed.

Now, we focus on the stability of the unique positive equilibrium of system (1) when $R_0 > 1$. The linearization of Equation 1 at the positive equilibrium E_2 is:

$$\begin{cases} x'(t) = -dx^*x(t) - \beta x^*y(t), \\ y'(t) = \beta ky^*x(t - \tau) + \beta kx^*y(t - \tau) - uy(t). \end{cases}$$

Its characteristic equation is:

$$\begin{aligned} H_{E_2}(\lambda) &= \lambda^2 + A_1\lambda + A_2 + A_3\lambda e^{-\lambda\tau} + A_4e^{-\lambda\tau} \\ &= Q_1(\lambda) + Q_2(\lambda)e^{-\lambda\tau} = 0, \end{aligned} \quad (5)$$

Where:

$$A_1 = u + du / \beta k, A_2 = du^2 / \beta k, A_3 = -u,$$

$$A_4 = (\beta kr - 2du)u / \beta k, Q_1(\lambda) = \lambda^2 + A_1\lambda + A_2,$$

$$Q_2(\lambda) = A_3\lambda + A_4.$$

Calculating characteristic equation (5), we have:

$$\begin{aligned} F(b) &= |Q_1(ib)|^2 - |Q_2(ib)|^2 \\ &= b^4 + \frac{d^2u^2}{\beta^2k^2}b^2 + \frac{(\beta kr - du)(3du - \beta kr)u^2}{\beta^2k^2} \end{aligned}$$

So $F(b) = 0$ has not positive root when $1 < R_0 < 3$;

$F(b) = 0$ has a unique positive root when $R_0 > 3$. Therefore, using Theorem 2 and Lemma 1 in Wang et al. (2007), we have Theorem 5.

Theorem 5

For system (1), if $1 < R_0 < 3$, then the positive equilibrium E_2 is asymptotically stable for all $\tau \geq 0$; if $R_0 > 3$, then stability switches will occur, i.e., there exists a positive number τ^*

such that E_2 is unstable for all $\tau > \tau^*$ and there are at most a finite number of stability switches occur when τ changes from 0 to τ^* .

Furthermore, we determine τ^* and analyze the dynamical behavior of system (1) near τ^* . Based on the Hopf bifurcation theorem in Kuang (1993), we have:

Theorem 6

Suppose that $R_0 > 3$. If:

$$\tau > \tau_0 = \frac{1}{b} \arccos \frac{A_4(b^2 - A_2) - A_3A_1b^2}{A_4^2 + A_3^2b^2},$$

And $\tau - \tau_0$ is sufficiently small, then there is asymptotic stability positive periodic solution near the positive equilibrium E_2 , which is induced by Hopf bifurcation, where $\lambda = \pm ib$, $b > 0$ is a pair of pure imaginary roots of characteristic equation (5).

Proof 3

Suppose that $\lambda = \pm ib, b > 0$ is a pair of pure imaginary roots of characteristic equation (5). Substituting it into Equation 5 and separating the real and imaginary parts, we have:

$$\begin{cases} A_4 \cos(b\tau) + A_3b \sin(b\tau) = b^2 - A_2, \\ A_3b \cos(b\tau) - A_4 \sin(b\tau) = -bA_1. \end{cases} \quad (6)$$

From Equation 6, we obtain $F(b) = 0$. So $F(b) = 0$ has a unique positive root when $R_0 > 3$. Taking λ is a function of delay τ and derivating on both sides of Equation 5, we have:

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{2\lambda + A_1}{A_3\lambda^2 + A_4\lambda} e^{\tau\lambda} + \frac{A_3}{A_3\lambda^2 + A_4\lambda} - \frac{\tau}{\lambda}.$$

Thus,

$$\begin{aligned} \text{sign} \left\{ \frac{d(\text{Re } \lambda)}{d\tau} \right\}_{\lambda=ib} &= \text{sign} \left\{ \text{Re} \left(\frac{d\lambda}{d\tau} \right)^{-1} \right\}_{\lambda=ib} \\ &= \text{sign} \left\{ \text{Re} \left(\frac{2\lambda + A_1}{A_3\lambda^2 + A_4\lambda} e^{\tau\lambda} + \frac{A_3}{A_3\lambda^2 + A_4\lambda} - \frac{\tau}{\lambda} \right) \right\}_{\lambda=ib} \\ &= \text{sign} \left\{ \frac{(2b^2 + A_1^2 - A_3^2 - 2A_2)b^2}{(A_3^2b^2 + A_4^2)b^2} \right\} \\ &= \text{sign} \{ 2b^2 + A_1^2 - A_3^2 - 2A_2 \} \\ &= \text{sign} \left\{ 2b^2 + \left(\frac{du}{\beta k} \right)^2 \right\} > 0, \end{aligned}$$

And the transversality condition of Hopf bifurcation is satisfied, that is, the pure imaginary characteristic roots will cross the imaginary

axis from left to right when τ increase. Furthermore, from Equation 6, we can obtain that:

Table 1. Parameters and its range.

Parameters	Meaning	Range
r	The Intrinsic growth rate of alga	0.5 h^{-1}
d	The density constraint coefficient of alga	0.05 mg/L
β	The predation coefficient of aquatic animals	0.04 h^{-1}
k	The feed conversion efficiency ratio of aquatic animals	0.6
u	The sum of natural mortality rate and the fishing intensity of aquatic animals	$0.02 - 0.5 \text{ h}^{-1}$
τ	The delay time between aquatic preying alga and translating to effective biomass	$0.1 - 12 \text{ h}$

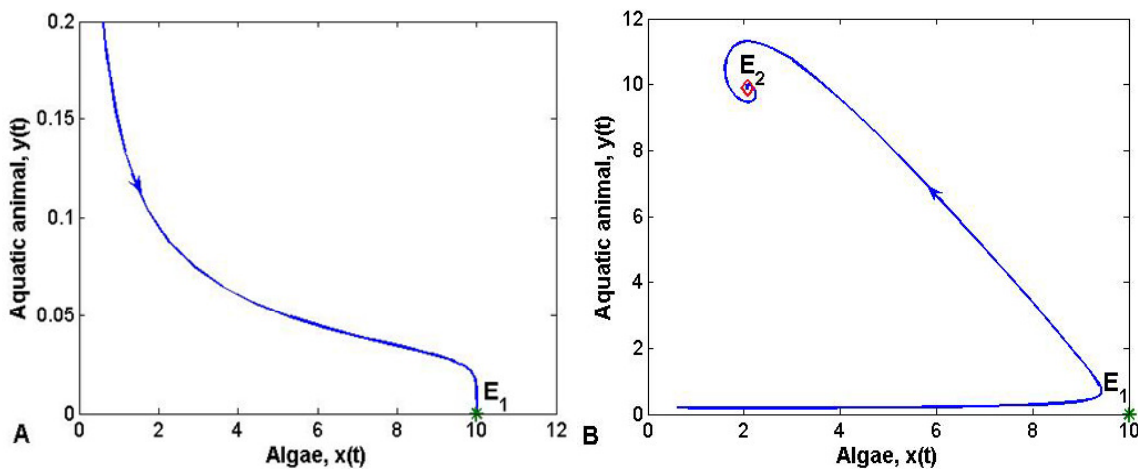


Figure 1. Phase diagram of system (1) when $\tau = 0$. A) $u = 0.3$ and B) $u = 0.05$.

Note that $R_0 = 0.8 < 1$ in (A) and the extinct equilibrium of aquatic animal $E_1 = (10, 0)$ is globally asymptotically stable; $R_0 = 4.8 > 1$ in (B) and the survival equilibrium of aquatic animal $E_2 = (2.0833, 9.8958)$ is globally asymptotically stable. Here the initial condition is $(x_0, y_0) = (0.6, 0.2)$.

$$\cos(b\tau) = \frac{A_4(b^2 - A_2) - A_3A_1b^2}{A_4^2 + A_3^2b^2}.$$

Hence,

$$\tau_n = \frac{1}{b} \arccos(b\tau) \frac{A_4(b^2 - A_2) - A_3A_1b^2}{A_4^2 + A_3^2b^2} + \frac{2n\pi}{b}, n = 0, \pm 1, \pm 2, \dots$$

Summarizing the aforementioned analysis, when $R_0 > 3$, we can obtain that the positive equilibrium E_2 is stable if $\tau = 0$. When τ increases to τ_0 , characteristic equation (5) will have a pair of pure imaginary eigenvalues. There is an eigenvalue with positive real part if τ continues to increase and the positive equilibrium

E_2 becomes unstable. Because $F(b) = 0$ only has one positive real root, the root of Equation will cross the imaginary axis and will not go back to the complex plane left plane with the increasing of τ . Thus, the positive equilibrium E_2 will always be unstable and $\tau^* = \tau_0$. According to Theorem 4 and the Hopf bifurcation theorem, τ_0 is the Hopf bifurcation values of system (1). When $\tau > \tau_0$ ($\tau - \tau_0$ is sufficiently small), system (1) has an asymptotically stable periodic solution near the positive equilibrium point E_2 and its period is $2\pi/b$.

RESULTS

Parameters

The range of parameters and its meaning are shown in
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Table 1. Note that these parameter values are within the similar ranges as those ones employed by Feng et al.

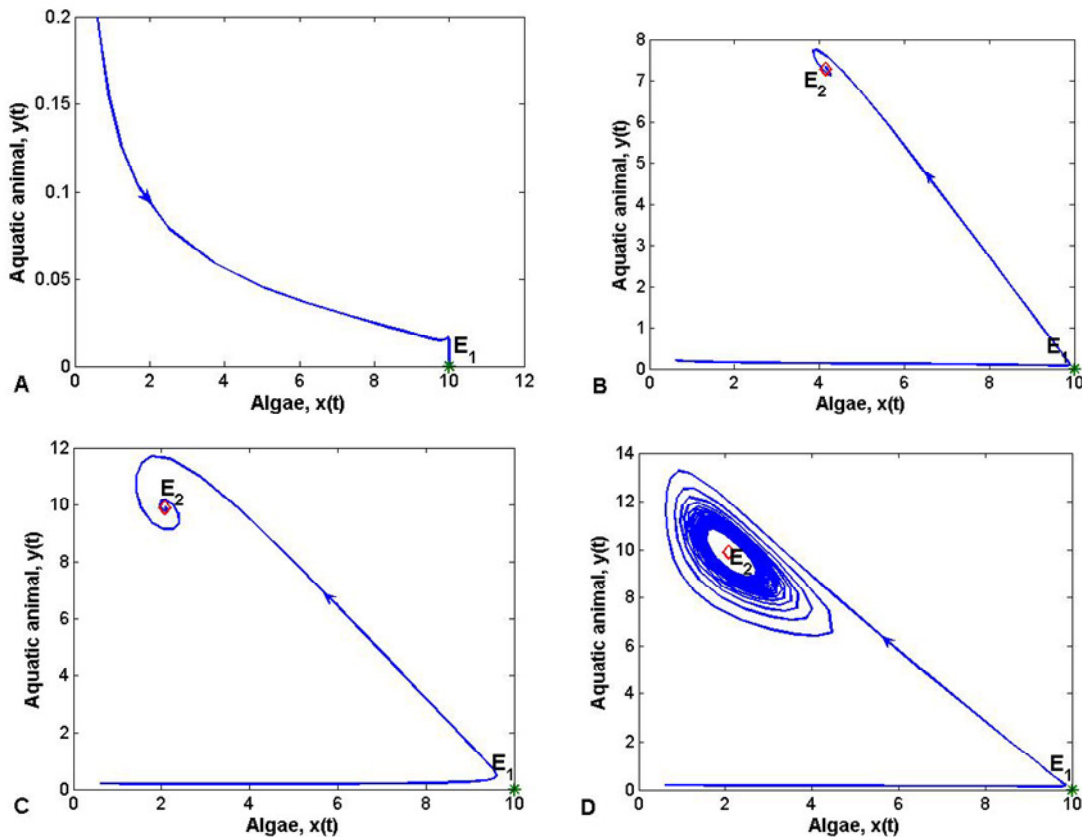


Figure 2. Phase diagram of system (1) when $\tau \neq 0$. A) $u = 0.3, \tau = 10.5$; B) $u = 0.1, \tau = 10.5$; C) $u = 0.05, \tau = 1.5$ and D) $u = 0.05, \tau = 10.5$.

Note that $R_0 = 0.8 < 1$ in (A) and the extinct equilibrium of aquatic animal $E_1 = (10, 0)$ is globally asymptotically stable; $R_0 = 2.4 > 1$ in (B) and the survival equilibrium of aquatic animal $E_2 = (4.1667, 7.2917)$ is asymptotically stable; $R_0 = 4.8 > 3$ in (C) and the survival equilibrium of aquatic animal $E_2 = (2.0833, 9.8958)$ is asymptotically stable; $R_0 = 4.8 > 3$ in (D), the survival equilibrium of aquatic animal is unstable, Hopf bifurcation occurs and a stable periodic solution bifurcates from E_2 . Here the initial condition is $(\varphi_1(\theta), \varphi_2(\theta)) = (0.6, 0.2), \theta \in [-\tau, 0]$.

(2007) for their plankton ecosystem. Because the exploitation of aquatic animals (for example fish) is a regular direct interference on the ecosystems, in order to analyze its effect on dynamical characteristics, we focus on the variety of parameters u and τ .

Numerical simulations

First, we fix the parameter $\tau = 0$. Let $u = 0.3$ and $u = 0.05$, respectively. The phase diagram of system (1)

has been obtained (Figure 1). Based on Figure 1A, we can obtain that the basic reproductive number of aquatic animal $R_0 < 1$ implies that the extinct equilibrium of aquatic animal E_1 is globally asymptotically stable, whereas Figure 1B shows that $R_0 > 1$ imply that the survival equilibrium of aquatic animal E_2 is globally asymptotically stable. These conclusions are consistent with the results of Theorem 2. Since R_0 is inversely related to u , we can conclude that a small fishing

intensity of aquatic animals will be benefit for aquatic's consumption for alga in the water ecosystems. Therefore, the proper reduce of fishing intensity can relieve the water pollution and improve the water quality.

Figure 2 shows that the dynamical behaviour of system(1) when $\tau \neq 0$. If $R_0 < 1$, the extinct equilibrium

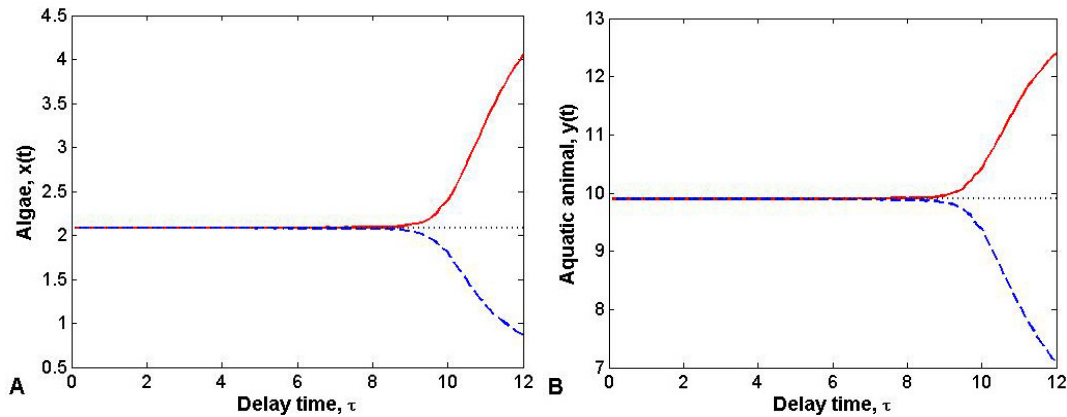


Figure 3. The global bifurcation diagram when $u = 0.05$. With model (1) integrated over $[0, 2000]$, the maximum value (solid line), the minimum value (dash line) of the last 150 iterations and the value of survival equilibrium of aquatic animal (dot line) are plotted.

of aquatic animal E_1 is always globally asymptotically stable although the delay τ is sufficient large (Figure 2A); if $1 < R_0 < 3$, the survival equilibrium of aquatic animal E_2 is always globally asymptotically stable for any delay τ (Figure 2B); if $R_0 > 3$, small delay τ implies that the survival equilibrium of aquatic animal E_2 is globally asymptotically stable (Figure 2C). However, when the delay time become large, the survival equilibrium of aquatic animal is unstable, Hopf bifurcation occurs and a stable periodic solution bifurcates from E_2 (Figure 2D). These conclusions are consistent with the conclusions of Theorems 3, 5 and 6. Furthermore, fix $u = 0.05$ and τ changes from 0.1 to 12, the global bifurcation diagram is given in Figure 3.

From Figure 3, we can find that the process of survival equilibrium of aquatic animal E_2 change from stable to unstable with the increase of delay τ , and the period of the periodic solution is gradually increase.

DISCUSSION

Currently, water quality deterioration has become a worldwide public problem and mathematical model has been used to study the oscillation of water quality. Feng et al. (2007) and Wang et al. (2009) explored the conditions of the oscillation of water quality based on the nonlinear dynamics model of planktonic ecosystem without delay time. In the present paper, we proposed a

new nonlinear dynamics model of alga-aquatic animal based on the water ecosystem of Three Gorges Reservoir Area, which contains the delay time between aquatic preying alga and translating to effective biomass. The theoretical and simulated results indicate that the delay time may lead to the oscillation of water quality under proper conditions.

According to measuring data in Three Gorges Reservoir Area, Liu et al. (2005) studied algal growth mechanism and found some strong interactions and inherence rules when nitrogen (N) and phosphorus (P) concentration absorbed by algae body varied with the different N and P concentration. However, for simplicity, model (1) does not directly contain the essential nutrients (such as N and P) for the growth of alga. Furthermore, the parameter values in our simulations are artificial. To conquer the above-mentioned limitations, using the idea of Cabrera (2011), we will collect the real monitoring data of water quality of Three Gorges Reservoir Area and discuss these cases in mathematics to understand these topics in future work.

CONCLUSIONS

Note that the survival equilibrium of aquatic animal becomes unstable only when the delay time is sufficient large and the basic reproductive number of aquatic animal is greater than three. If there is a large delay time between aquatic preying alga and translating to effective biomass, the mild fishing intensity may cause the fluctuation of the water quality due to the inversely-proportional relationship between the basic reproductive

number and the fishing intensity of aquatic animals. However, excessive fishing intensity will induce the aquatic animal tends to extinction, that is, the extinct equilibrium of aquatic animal is globally asymptotically stable, which acts inimical to water purification. Therefore, based on the theoretical analysis and numerical simulations, we can conclude that the delay time and fishing intensity together may induce the

oscillation of water quality. Thus, in order to keep the good ecology function of the aquatic ecosystem of Three Gorges Reservoir Area, the administrative office will careful to work out measures for fishing intensity.

ACKNOWLEDGMENTS

This work was supported by the Major Special projects of national science and technology (No. 2009ZX07528-003-09) and the Natural Science Foundation Project of CQ CSTC (No. 2010BB2093).

REFERENCES

- Ai XS, Li WH (2002). Research frontiers in some aspects of water sciences. *J. Hydraulic Eng.*, 33(7): 125-128.
- Cabrera FMI (2011). Deterministic approach to the study of the interaction predator-prey in a chemostat with predator mutual interference. Implications for the paradox of enrichment. *Ecol. Model.*, 222(3): 598-605.
- Chen H, Liu JS, Cao Y, Li SC, Ou-Yang H (2006). Progresses of ecological risk assessment. *Acta Ecol. Sin.*, 26(5): 1558-1566.
- Feng JF, Li HM, Wang HL (2007). Research on the nonlinear dynamics of the plankton ecosystem. *Ocean Tech.*, 26(3): 67-69.
- Kuang QJ, Tan YY, Wan DB, Zhang JY (2000). On the phytoplankton in the middle and lower reaches of the Hanjiang River and the prevention of water-blooms. *Resour. Environ. Yangtze Basin*, 9(1): 63-70.
- Kuang Y (1993). *Delay Differential Equations with Applications in Population Dynamics*. Academic Press, pp. 58-62, 74-81.
- Liu GM, Sun S, Wang H (2003). Review on the marine ecosystem dynamics model. *Adv. Earth Sci.*, 18(3): 427-432.
- Liu XA, Zhan M, MA YE (2005). Relationship between algal growth and nutritious materials absorbability in the Three -Gorges Valley. *Environ. Sci.*, 26(4): 95-99
- Shen YF, Cai QH (2003). Complexity in freshwater ecosystems. *J. Grad. School Chin. Acad. Sci.*, 20(2): 131-138.
- van den Driessche P, Watmough J (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Math. Biosci.*, 180(1): 29-48.
- Wang HL, Shen YH, Ge G (2009). Nonlinear dynamical research of the steady of marine planktonic ecosystem's model. *Mar. Sci. Bull.*, 28(5): 97-101.
- Wang K, Wang W, Pang H, Liu X (2007). Complex dynamic behavior in a viral model with delayed immune response. *Physica D*, 226(2): 197-208.
- Wang W, Chen L (1997). A predator-prey system with stage-structure for predator. *Comput. Math. Appl.*, 33(8): 83-91.
- Wolkowicz GSK, Xia H (1997). Global asymptotic behavior of a Chemostat model with discrete delays. *SIAM J Appl. Math.*, 57(4): 1019-1043.
- Wu CH, Niu WH (2006). Study on the health of river ecosystem. *Yellow River*, 28(2): 10-12.
- Zhang JY, FengBY (2000). *Geometry theory and branch problem on ordinary differential equation*. Beijing: Peking university press, pp. 88, 126-131.
- Zheng ZX (1994). *Theory of functional differential equations*. Hefei: Anhui education press, pp. 236-246.