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Full Length Research Paper

Many exact solutions for nonlinear dynamics of DNA model using the generalized Riccati equation mapping method

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The objective of this paper is to apply the generalized Riccati equation mapping method for constructing the exact traveling wave solutions of a nonlinear partial differential equation describing the dynamics of DNA modeling. This model consists of two long elastic homogeneous strands connected with each other by an elastic membrane. Hyperbolic and trigonometric function solutions of this model are obtained. Comparison between our new results and the well-known results are given.

Key words: Nonlinear dynamics of DNA model, generalized Riccati equation mapping method, exact traveling wave solutions, generalized Riccati equation.

INTRODUCTION

Seeking traveling wave solutions of nonlinear evolution equations is of important and significance in mathematical physics and becomes of the most exciting and extremely active areas of research investigation. These solutions play an important role in many phenomena in physics, such as fluid mechanics, hydrodynamics, optics, plasma physics and so on. Because of the increased concentration in the theory of solitary waves, a large variety of analytic and computation methods have been established in the analysis of the nonlinear models (Wang et al., 2008; Lu and Shi, 2010; Miura, 1978; Parkes et al., 2005; Hirota, 1971; Ablowitz and Clarkson, 1991; Tascan and Bekir, 2009; El-Wakil et al., 2007; Weiss et al., 1983; Zayed, 2011; Zayed and Gepreel, 2009; Zayed and Arnous, 2012; Zayed and Hoda Ibrahim, 2012; Kudryashov and Loguinova, 2008; Ryabov, 2010; Ryabov et al., 2011; Wang et al., 1996; Kudryashov, 2012). An attractive nonlinear model of the nonlinear science is deoxyribonucleic acid (DNA). The DNA molecule encodes the information that organisms need to live and reproduce themselves. The DNA structure has been studied during last decades. It consists of a pair of molecules organized as strands and joined by hydrogen as well as covalent bonds. The investigation of DNA dynamics has successfully predicted the appearance of important nonlinear structures. It has been shown that the nonlinearity is responsible for forming localized waves. These localized waves are interesting because they have the capability to transport energy without dissipation (Aguero et al., 2008; Gaeta, 1999; Gaeta et al., 1994; Yakushevich, 1987, 1989, 1998; Peyrard and Bishop, 1989). Recently, Kong et al. (2001) and Alka et al. (2011) studied the following nonlinear dynamics of DNA modeling:

$$\phi_{tt} - c_1^2 \phi_{xx} - A\phi^3 - B\phi^2 - C\phi = 0, \qquad (1)$$

where c_1^2, A, B and C are well-known constants which are related to the real physical quantities of DNA. These constants can be found in Alka et al. (2011) and can be derived in “the derivation of the nonlinear DNA modeling (1)” part of this work, while ϕ represents the out-of-phase motion.

The objective of this article is to apply the generalized Riccati equation mapping method to find many traveling wave solutions of the model (1), namely the hyperbolic and trigonometric function solutions. Alka et al. (2011)

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have investigated the nonlinear model (1) using the elliptic equation method and found the following solitary wave solutions:

$$\phi = \frac{-\sqrt{2}ah}{(2a^2-1)}\left[1 \pm \tanh\left(\sqrt{\frac{2a^2\mu l_0}{\rho\sigma h(2a^2-1)}}\xi\right)\right], \qquad (2)$$

where the plus or minus sign stands for anti-kinks or kinks respectively, while ξ is given by

$$\xi = (x - \sqrt{2}c_1 t)/c_1 \quad \text{and} \quad a^2 > \frac{1}{2}.$$

The rest of this article is organized as follows: First is a "derivation of the nonlinear DNA modeling (1)". This is followed by a "description of the generalized Riccati equation mapping method". Next, "applications" of this method to find the exact traveling wave solutions of the model (1) are presented. The findings are then discussed and conclusions are given.

THE DERIVATION OF THE NONLINEAR DNA MODELING (1)

Here we derive the nonlinear DNA model (1) as follows: With reference to Kong et al. (2001) and Alka et al. (2011), the DNA molecule is supposed to consist of two long elastic strands which represents two polynucleotide chains of the DNA molecule, connected to each other by an elastic membrane representing the hydrogen bonds between the pairs of bases in the two chains. This model includes four degrees of freedom u_1, υ_1 and u_2, υ_2 for the two strands respectively. The u_1 and u_2 represent respectively the longitudinal displacements of the top strand and the bottom strand, while υ_1 and υ_2 denote respectively the transverse displacements of the top strand and the bottom strand. The Hamiltonian of such a model has the form:

$$H = T + V_1 + V_2,$$

where the kinetic energy of the elastic strands

$$T = \int \frac{1}{2}\rho\sigma\left[\left(\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial u_2}{\partial t}\right)^2 + \left(\frac{\partial v_1}{\partial t}\right)^2 + \left(\frac{\partial v_1}{\partial t}\right)^2\right]dx,$$

the potential energy of the elastic strands

$$V_1 = \int \frac{1}{2}Y\sigma\left[\left(\frac{\partial u_1}{\partial x}\right)^2 + \left(\frac{\partial u_2}{\partial x}\right)^2\right]dx + \int \frac{1}{2}F\sigma\left[\left(\frac{\partial v_1}{\partial x}\right)^2 + \left(\frac{\partial v_2}{\partial x}\right)^2\right]dx,$$

and the potential energy of the elastic membrane

$$V_2 = \int \frac{1}{2}\mu\left[\Delta l(x)\right]^2 dx.$$

Here ρ, σ, Y and F are respectively the mass density, the area of transverse cross-section, the Young's modulus and the tension density of each strand; μ is the rigidity of the elastic membrane and $\Delta l(x)$ is the stretching of the elastic membrane at x due to longitudinal vibrations and is given by

$$\Delta l = \sqrt{(h + v_1 + v_2)^2 + (u_2 - u_1)^2} - l_0,$$

where h is the distance between the two strands, l_0 is the height of the membrane in the equilibrium position. To obtain the equation of motion let us introduce the new variables: for in-phase motion

$$u_+ = \frac{u_1 + u_2}{\sqrt{2}} \quad \text{and} \quad v_+ = \frac{v_1 + v_2}{\sqrt{2}};$$

for out-phase motion

$$u_- = \frac{u_2 - u_1}{\sqrt{2}} \quad \text{and} \quad v_- = \frac{v_2 - v_1}{\sqrt{2}}.$$

Assuming $|u_1 - u_2| << h$, $|v_1 - v_2| << h$ and neglecting the higher order term, we get

$$\frac{\Delta l}{l_0 + \Delta l} = 1 - \frac{l_0}{h} + \frac{l_0}{h^2}(v_1 - v_2) - \frac{l_0}{h^3}(v_1 - v_2)^2 + \frac{l_0}{2h^3}(u_2 - u_1)^2$$

As out-of-phase motion stretches the hydrogen bond (Peyrard and Bishop, 1989) we will consider only out-of-phase motion for which the equation of motion can be written as

$$\frac{\partial^2 u_-}{\partial t^2} - c_1^2\frac{\partial^2 u_-}{\partial x^2} = \lambda_1 u_- + \gamma_1 u_- v_- + \mu_1 u_-^3 + \beta_1 u_- v_-^2,$$
$$\frac{\partial^2 v_-}{\partial t^2} - c_2^2\frac{\partial^2 v_-}{\partial x^2} = \lambda_2 v_- + \gamma_2 u_-^2 + \mu_2 u_-^2 v_- + \beta_2 v_-^3 + c_0, \qquad (3)$$

Where

$$c_1^2 = \frac{Y}{\rho}, \quad c_2^2 = \frac{F}{\rho}, \quad \lambda_1 = \frac{-2\mu}{\rho\sigma h}(h - l_0), \quad \lambda_2 = \frac{-2\mu}{\rho\sigma},$$

$$\gamma_1 = 2\gamma_2 = \frac{2\sqrt{2}\mu l_0}{\rho\sigma h^2}, \quad \mu_1 = \mu_2 = \frac{-2\mu l_0}{\rho\sigma h^3}, \quad \beta_1 = \beta_2 = \frac{4\mu l_0}{\rho\sigma h^3},$$

$$c_0 = \frac{\sqrt{2}\mu(h - l_0)}{\rho\sigma}.$$

By introducing the transformation $v_{\_} = au_{\_} + b,$ (where a and b are some constant), putting $u_{\_} = \phi,\ F = Y$ and $b = \frac{h}{\sqrt{2}}$, then the derivation of the model (1) can be found from the system (3). The constants c_1^2, A, B and C of the model (1) can be derived from the above quantities of the Equation (3) which are related to the real physical quantities given by the formulas

$$c_1^2 = \frac{Y}{\rho}, \quad A = \left(\frac{-2\alpha}{h^3} + \frac{4a^2\alpha}{h^3}\right), \quad B = \frac{6\sqrt{2}a\alpha}{h^2},$$

$$C = \left(\frac{-2\alpha}{l_0} + \frac{6\alpha}{h}\right), \quad \alpha = \frac{\mu l_0}{\rho\sigma}.$$

DESCRIPTION OF THE GENERALIZED RICCATI EQUATION MAPPING METHOD

Suppose we have a nonlinear evolution equation in the form

$$F(\phi, \phi_t, \phi_{tt}, \phi_x, \phi_{xx}, \phi_{tt}, \ldots) = 0, \tag{4}$$

Where F is a polynomial in $\phi(x,t)$ and its partial derivatives in which the highest order derivatives and the nonlinear terms are involved. In the following, we give the main steps of this method (Zhu, 2008; Li and Zhang, 2010; Ding et al., 2005).

Step 1. We use the wave transformation

$$\phi(x,t) = \phi(\xi), \quad \xi = kx + \omega t, \tag{5}$$

where k and ω are constants, to reduce Equation (4) into the following ODE:

$$P(\phi, \phi', \phi'', \ldots) = 0, \tag{6}$$

where P is a polynomial in $\phi(\xi)$ and its total derivatives, while $' = \frac{d}{d\xi}$.

Step 2. We suppose that Equation (6) has the formal solution:

$$\phi(\xi) = \sum_{n=0}^{N} a_n Q^n(\xi), \tag{7}$$

where a_n are constants to be determined later, such that $a_N \neq 0,$ while $Q(\xi)$ satisfies the generalized Riccati differential equation

$$Q'(\xi) = r + pQ(\xi) + qQ^2(\xi), \tag{8}$$

where r, p and q are real constants such that $q \neq 0.$

Step3. We determine the positive integer N in Equation (7) by balancing the homogeneous balance between the highest order derivatives and the nonlinear term in Equation (6).

Step 4. We substitute (7) along with Equation (8) into Equation (6) and vanish all the coefficients of the powers of $Q(\xi)$ to yield a system of algebraic equations which can be solved using computer programs such that Maple or Mathematica to find the values of a_n $(n = 0,1,2,\ldots,N)$ as well as k and ω.

Step 5. We substitute these values together with the well-known solutions of Equation (8) into (7) to obtain the explicit traveling wave solutions of Equation (4).

Step 6. It is well-known (Zhu, 2008; Li and Zhang, 2010; Ding et al., 2005) that the generalized Riccati differential Equation (8) has the following many solutions:

Type 1: When $\Delta = p^2 - 4qr > 0,$ and $pq \neq 0$ (or $qr \neq 0$) we have the solutions

$$Q_1(\xi) = \frac{-1}{2q}[p + \sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)],$$

$$Q_2(\xi) = \frac{-1}{2q}[p + \sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)],$$

$$Q_3(\xi) = \frac{-1}{2q}\left\{p + \sqrt{\Delta}[\tanh(\sqrt{\Delta}\xi) \pm i\ \mathrm{sec}h(\sqrt{\Delta}\xi)]\right\},$$

$i = \sqrt{-1}$

$$Q_4(\xi) = \frac{-1}{2q}\left\{p + \sqrt{\Delta}[\coth(\sqrt{\Delta}\xi) \pm cs\,\mathrm{c}h(\sqrt{\Delta}\xi)]\right\},$$

$$Q_5(\xi) = \frac{-1}{4q}\left\{2p + \sqrt{\Delta}[\tanh(\frac{\sqrt{\Delta}}{4}\xi) \pm \cot h(\frac{\sqrt{\Delta}}{4}\xi)]\right\},$$

$$Q_6(\xi)=\frac{1}{2q}\left\{-p+\frac{\sqrt{(E^2+F^2)\Delta}-E\sqrt{\Delta}cosh\left(\sqrt{\Delta}\xi\right)}{Esinh\left(\sqrt{\Delta}\xi\right)+F}\right\},$$

$$Q_7(\xi)=\frac{1}{2q}\left\{-p-\frac{\sqrt{(F^2-E^2)\Delta}+E\sqrt{\Delta}cosh\left(\sqrt{\Delta}\xi\right)}{Esinh\left(\sqrt{\Delta}\xi\right)+F}\right\}, \qquad (9)$$

where E and F are nonzero constants, such that $F^2-E^2>0$,

$$Q_8(\xi)=\frac{2rcosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}{\sqrt{\Delta}sinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)-pcosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)},$$

$$Q_9(\xi)=\frac{-2rsinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}{psinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)-\sqrt{\Delta}cosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)},$$

$$Q_{10}(\xi)=\frac{2rcosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}{\sqrt{\Delta}sinh\left(\sqrt{\Delta}\xi\right)-pcosh\left(\sqrt{\Delta}\xi\right)\pm i\sqrt{\Delta}},$$

$$Q_{11}(\xi)=\frac{2rsinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}{-psinh\left(\sqrt{\Delta}\xi\right)+\sqrt{\Delta}cosh\left(\sqrt{\Delta}\xi\right)\pm\sqrt{\Delta}},$$

$$Q_{12}(\xi)=\frac{4rsinh\left(\frac{\sqrt{\Delta}\xi}{4}\right)cosh\left(\frac{\sqrt{\Delta}\xi}{4}\right)}{-2psinh\left(\frac{\sqrt{\Delta}\xi}{4}\right)cosh\left(\frac{\sqrt{\Delta}\xi}{4}\right)+2\sqrt{\Delta}cosh^2\left(\frac{\sqrt{\Delta}\xi}{4}\right)-\sqrt{\Delta}},$$

Type 2: When $\Delta=p^2-4qr<0,$ and $pq\neq 0$ (or $qr\neq 0$) we have the solutions

$$Q_{13}(\xi)=\frac{-1}{2q}\left\{p-\sqrt{-\Delta}\tan\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\right\},$$

$$Q_{14}(\xi)=\frac{-1}{2q}\left\{p+\sqrt{-\Delta}\cot\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\right\},$$

$$Q_{15}(\xi)=\frac{-1}{2q}\left\{p-\sqrt{-\Delta}\left[\tan\left(\sqrt{-\Delta}\xi\right)\pm\sec\left(\sqrt{-\Delta}\xi\right)\right]\right\},$$

$$Q_{16}(\xi)=\frac{-1}{2q}\left\{p+\sqrt{-\Delta}\left[\cot\left(\sqrt{-\Delta}\xi\right)\pm\csc\left(\sqrt{-\Delta}\xi\right)\right]\right\},$$

$$Q_{17}(\xi)=-\frac{1}{4q}\left\{2p-\sqrt{-\Delta}\left[\tan\left(\frac{\sqrt{-\Delta}\xi}{4}\right)-\cot\left(\frac{\sqrt{-\Delta}\xi}{4}\right)\right]\right\},$$

$$Q_{18}(\xi)=\frac{1}{2q}\left\{-p+\frac{\pm\sqrt{-(E^2-F^2)\Delta}-E\sqrt{-\Delta}cos\left(\sqrt{-\Delta}\xi\right)}{Esin\left(\sqrt{-\Delta}\xi\right)+F}\right\},$$

$$Q_{19}(\xi)=\frac{1}{2q}\left\{-p-\frac{\pm\sqrt{-(E^2-F^2)\Delta}+E\sqrt{-\Delta}cos\left(\sqrt{-\Delta}\xi\right)}{Esin\left(\sqrt{-\Delta}\xi\right)+F}\right\},$$

where E and F are nonzero constants, such that $E^2-F^2>0$,

$$Q_{20}(\xi)=\frac{-2rcos\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}{\sqrt{-\Delta}sin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)+pcos\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}, \qquad (10)$$

$$Q_{21}(\xi)=\frac{2rsin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}{-psin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)+\sqrt{-\Delta}cos\left(\frac{\sqrt{-\Delta}\xi}{2}\right)},$$

$$Q_{22}(\xi)=\frac{-2rcos\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}{\sqrt{-\Delta}sin\left(\sqrt{-\Delta}\xi\right)+pcos\left(\sqrt{-\Delta}\xi\right)\pm\sqrt{-\Delta}},$$

$$Q_{23}(\xi)=\frac{2rsin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}{-psin\left(\sqrt{-\Delta}\xi\right)+\sqrt{-\Delta}cos\left(\sqrt{-\Delta}\xi\right)\pm\sqrt{-\Delta}},$$

$$Q_{24}(\xi)=\frac{4rsin\left(\frac{\sqrt{-\Delta}\xi}{4}\right)cos\left(\frac{\sqrt{-\Delta}\xi}{4}\right)}{-2psin\left(\frac{\sqrt{-\Delta}\xi}{4}\right)cos\left(\frac{\sqrt{-\Delta}\xi}{4}\right)+2\sqrt{-\Delta}cos^2\left(\frac{\sqrt{-\Delta}\xi}{4}\right)-\sqrt{-\Delta}},$$

APPLICATIONS

In this part of the work, we will apply the generalized Riccati equation mapping method described in "description of the generalized Riccati equation mapping method" part of this work, to find many exact solutions of the nonlinear dynamics of DNA model (1). To this end, we use the wave transformation (5) to reduce Equation (1) to the following ODE:

$$(\omega^2 - k^2c_1^2)\phi'' - A\phi^3 - B\phi^2 - C\phi = 0, \qquad (11)$$

where $\omega^2 - k^2c_1^2 \neq 0$. Balancing ϕ'' with ϕ^3 yields $N = 1$. Thus, the formal solution of Equation (11) has the form:

$$\phi(\xi) = a_0 + a_1 Q(\xi), \qquad (12)$$

where a_0 and a_1 are constants to be determined later. Substituting (12) along with Equation (8) into (11) and setting all the coefficients of the powers of $Q(\xi)$ to be zero yields a system of the following algebraic equations:

$$2(\omega^2 - k^2c_1^2)a_1q^2 - Aa_1^3 = 0, \qquad (13)$$

$$3a_1pq(\omega^2 - k^2c_1^2) - 3a_0a_1^2A - Ba_1^2 = 0, \qquad (14)$$

$$(a_1p^2 + 2a_1qr)(\omega^2 - k^2c_1^2) - 3a_0^2a_1A - 2a_0a_1B - a_1C = 0, \qquad (15)$$

$$a_1pr(\omega^2 - k^2c_1^2) - Aa_0^3 - Ba_0^2 - Ca_0 = 0. \qquad (16)$$

Solving the algebraic Equations (13)-(16) using the Maple or Mathematica yields the following results:

$$a_1 = a_0 z_{1,2},$$

$$A = \frac{-2rq^2(\omega^2 - k^2c_1^2)}{a_0^2(q - pz_{1,2})},$$

$$B = \frac{3rq(2q - pz_{1,2})(\omega^2 - k^2c_1^2)}{a_0(q - pz_{1,2})},$$

$$C = (\omega^2 - k^2c_1^2)(p^2 - 4qr),$$

$$z_{1,2} = \frac{1}{2r}(p \pm \sqrt{p^2 - 4qr}), \qquad (17)$$

where $a_0 \neq 0$ and $r \neq 0$. Now the exact traveling wave solutions of Equation (1) have the following forms:

Type 1: When $\Delta = p^2 - 4qr > 0$ and $qr \neq 0$ we have the hyperbolic solutions:

$$\phi_1(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{4qr}\left[p + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right]\right\},$$

$$\phi_2(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{4qr}\left[p + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}\xi}{2}\right)\right]\right\},$$

$$\phi_3(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{4qr}\left(p + \sqrt{\Delta}[\tanh[\sqrt{\Delta}\xi] \pm i\,sech[\sqrt{\Delta}\xi]]\right)\right\},$$

$$\phi_4(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{4qr}\left(p + \sqrt{\Delta}[coth[\sqrt{\Delta}\xi] \pm csch[\sqrt{\Delta}\xi]]\right)\right\},$$

$$\phi_5(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{8qr}\left(2p + \sqrt{\Delta}\left[tanh\left(\frac{\sqrt{\Delta}\xi}{4}\right) \pm coth\left(\frac{\sqrt{\Delta}\xi}{4}\right)\right]\right)\right\},$$

$$\phi_6(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{4qr}\left[p - \frac{\sqrt{(E^2 + F^2)\Delta} - E\sqrt{\Delta}\cosh\left(\sqrt{\Delta}\xi\right)}{F + E\sinh\left(\sqrt{\Delta}\xi\right)}\right]\right\},$$

$$\phi_7(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{4qr}\left[p + \frac{\sqrt{(F^2 - E^2)\Delta} + E\sqrt{\Delta}\cosh\left(\sqrt{\Delta}\xi\right)}{F + E\sinh\left(\sqrt{\Delta}\xi\right)}\right]\right\},$$

where E and F are nonzero constants, such that $F^2 - E^2 > 0$,

$$\phi_8(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{p - \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}\right\},$$

$$\phi_9(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{\Delta})}{p - \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}\xi}{2}\right)}\right\},$$

$$\phi_{10}(x,t) = a_0\left\{1 + \frac{(p \pm \sqrt{\Delta})cosh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}{\sqrt{\Delta}sinh\left(\sqrt{\Delta}\xi\right) - pcosh\left(\sqrt{\Delta}\xi\right) \pm i\sqrt{\Delta}}\right\},$$

$$\phi_{11}(x,t) = a_0\left\{1 + \frac{(p \pm \sqrt{\Delta})sinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}{\sqrt{\Delta}cosh\left(\sqrt{\Delta}\xi\right) \pm \sqrt{\Delta} - psinh\left(\sqrt{\Delta}\xi\right)}\right\},$$

$$\phi_{12}(x,t) = a_0\left\{1 + \frac{(p \pm \sqrt{-\Delta})sinh\left(\frac{\sqrt{\Delta}\xi}{2}\right)}{2\sqrt{\Delta}cosh^2\left(\frac{\sqrt{\Delta}\xi}{4}\right) - psinh[\frac{\sqrt{\Delta}\xi}{2}] - \sqrt{\Delta}}\right\}. \qquad (18)$$

Type 2: When $\Delta = p^2 - 4qr < 0$ and $qr \neq 0$ we have the trigonometric periodic solutions:

$$\phi_{13}(x,t) = a_0\left\{1 - \frac{(p \pm \sqrt{-\Delta})}{4qr}\left[p - \sqrt{-\Delta}\tan\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\right]\right\},$$

$$\phi_{14}(x,t)=a_0\left\{1-\frac{\left(p\pm\sqrt{-\Delta}\right)}{4qr}\left[p+\sqrt{-\Delta}\cot\left(\frac{\sqrt{-\Delta}\xi}{2}\right)\right]\right\},$$

$$\phi_{15}(x,t)=a_0\left\{1-\frac{(p\pm\sqrt{-\Delta})}{4qr}\left(p-\sqrt{-\Delta}\left[\tan\left(\sqrt{-\Delta}\xi\right)\pm\sec\left(\sqrt{-\Delta}\xi\right)\right]\right)\right\},$$

$$\phi_{16}(x,t)=a_0\left\{1-\frac{(p\pm\sqrt{-\Delta})}{4qr}\left(p+\sqrt{-\Delta}\left[cot\left(\sqrt{-\Delta}\xi\right)\pm csc\left(\sqrt{-\Delta}\xi\right)\right]\right)\right\},$$

$$\phi_{17}(x,t)=a_0\left\{1-\frac{(p\pm\sqrt{-\Delta})}{8qr}\left(2p+\sqrt{-\Delta}\left[\cot\left(\frac{\sqrt{-\Delta}\xi}{4}\right)-\tan\left(\frac{\sqrt{-\Delta}\xi}{4}\right)\right]\right)\right\},$$

$$\phi_{18}(x,t)=a_0\left\{1+\frac{(p\pm\sqrt{-\Delta})}{4qr}\left[-p+\frac{\pm\sqrt{-(E^2-F^2)\Delta}-E\sqrt{-\Delta}cos\left(\sqrt{-\Delta}\xi\right)}{F+Esin\left(\sqrt{-\Delta}\xi\right)}\right]\right\},$$

$$\phi_{19}(x,t)=a_0\left\{1-\frac{(p\pm\sqrt{-\Delta})}{4qr}\left[p+\frac{\pm\sqrt{-(E^2-F^2)\Delta}+E\sqrt{-\Delta}cos\left(\sqrt{-\Delta}\xi\right)}{F+Esin\left(\sqrt{-\Delta}\xi\right)}\right]\right\},$$

where E and F are nonzero constants, such that $E^2-F^2>0$,

$$\phi_{20}(x,t)=a_0\left\{1-\frac{p\pm\sqrt{-\Delta}}{p+\sqrt{-\Delta}tan\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}\right\},$$

$$\phi_{21}(x,t)=a_0\left\{1-\frac{p\pm\sqrt{-\Delta}}{p-\sqrt{-\Delta}\cot\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}\right\},$$

$$\phi_{22}(x,t)=a_0\left\{1-\frac{(p\pm\sqrt{-\Delta})cos\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}{pcos\left(\sqrt{-\Delta}\xi\right)+\sqrt{-\Delta}sin\left(\sqrt{-\Delta}\xi\right)\pm\sqrt{-\Delta}}\right\},$$

$$\phi_{23}(x,t)=a_0\left\{1+\frac{(p\pm\sqrt{-\Delta})sin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}{-psin\left(\sqrt{-\Delta}\xi\right)+\sqrt{-\Delta}cos\left(\sqrt{-\Delta}\xi\right)\pm\sqrt{-\Delta}}\right\},$$

$$\phi_{24}(x,t)=a_0\left\{1-\frac{(p\pm\sqrt{-\Delta})sin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)}{psin\left(\frac{\sqrt{-\Delta}\xi}{2}\right)-2\sqrt{-\Delta}cos^2\left(\frac{\sqrt{-\Delta}\xi}{4}\right)+\sqrt{-\Delta}}\right\}. \quad (19)$$

where a_0 and ξ can be found using (17) to take the forms:

$$a_0=\frac{-2\sqrt{2}qah}{(2a^2-1)(2q-pz_{1,2})},\ a^2>\frac{1}{2} \text{ and } \xi=kx\pm\left(\sqrt{k^2c_1^2+\frac{C}{\Delta}}\right)t. \quad (20)$$

Remark. All solutions of this article have been checked with Maple by putting them back into the original Equation (1).

DISCUSSION

On comparing our first result $\phi_1(x,t)$ obtained using the generalized Riccati equation mapping method with the well-known result (2) obtained in Alka et al. (2011) using the elliptic equation method, we deduce that the result (2) follows from our result $\phi_1(x,t)$ by setting $\frac{2q}{2q-pz_{1,2}}=1.$ Then we get $p=0$ since $z_{1,2}\neq 0.$ In this case the solution $\phi_1(x,t)$ reduces to the form

$$\phi_1=\frac{-\sqrt{2}ah}{(2a^2-1)}\left\{1\pm\tanh\left(\sqrt{\frac{\mu(3l_0-h)}{2\rho\sigma h}}\xi\right)\right\}, \quad (21)$$

If we set $a^2=\frac{3l_0-h}{2(l_0-h)},$ the formula (21) reduces to the form (2). This shows that the well-known solution (2) is a special case of our first solution $\phi_1(x,t).$ Thus, on applying the generalized Riccati equation mapping method used in this article with the help of the formal solution (7) along with the generalized Riccati Equation (8), we obtain the exact traveling wave solutions (18) and (19) of the nonlinear dynamics of DNA model (1) which look new and represent the solitary wave solutions of the forms (18) and the periodic solutions of the forms (19).

CONCLUSIONS

From these discussions we conclude that the generalized Riccati equation mapping method is more effective and gives more exact solutions than the elliptic equation method. Finally, we deduce that the proposed method in this article can be applied to many other nonlinear evolution equations in mathematical physics.

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