

Full Length Research Paper

Control chaos for Lorenz system based on tridiagonal matrix stability theory

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Accepted 17 June, 2013

The Lorenz system is chaos when parameters vary in certain scope. In order to control chaotic Lorenz system, a new controller is proposed based on tridiagonal matrix stability theory. The proposed controller is simple and easy to be implemented. Simulation results show the effectiveness of the controller.

Key words: control chaos, Lorenz system, tridiagonal matrix stability theory.

INTRODUCTION

Since 1963, Lorenz has found the first attractor in which chaos theory had been developed. Control chaos is an important technology in the field of nonlinear theory. In 1990; Ott et al. (1990) presented the OGY method to control chaos. After their pioneering work, chaotic control has become a focus in nonlinear problems and a lot of work has been done in the field (Chen and Dong, 1998; Wang et al., 2001; Guan et al., 2002). Nowadays, many methods have been proposed to control chaos (Wang, 2003). Generally speaking, there are two control ways: feedback control and non feedback control. Feedback methods (Jang et al., 2002; Wang et al., 2000; Zheng, 2006; Gong, 2005; Wang and Wu, 2006) are used to stabilize the unstable periodic orbit of chaotic systems by feeding back their states. Non feedback methods (Wang and Zhao, 2005; Chen and Wang, 2007) are adopted to suppress chaotic behaviors by applying periodic perturbations to some parameters or variables. However, OGY method is a basic methodology for controlling chaos. In OGY method, finding an adjustable parameter is not often simple. Control chaos via the time-delay feedback control (TDFC) is also an effective method. However, it encounter with some problems as the control objective must be the equilibrium or the unstable periodic (UPO), moreover determining the time delay for TDFC method is difficult. At the same time, complex chaotic behaviors in Lorenz system were detected and reported

due to Lorenz system which is applied in secure communication and cryptography. When parameters of Lorenz system fall into a certain area, the Lorenz system is experiencing chaotic behavior. How to control chaos in Lorenz system is an important task for scholars.

In brief, researchers have done some important works to control chaos in the Lorenz system; some methods are applied to control chaos in the Lorenz system (Wu et al., 2007; Chen and Lü, 2003). All the existed methods can work well theoretically. However, there are some issues in most of these mentioned works such as; a lot of them require a great amount of computation, or calculation process is very complex, or some existed methods are not easy to be implemented. We hope the controller will be as simple as possible. To overcome these weaknesses, the design of controllers is based on the tridiagonal matrix stability theory to control chaos in the Lorenz system. The controller is easy to be implemented and this method can avoid large amount of the complex computations.

The organization of the paper is as follows. In the next section, we analyze chaos in the Lorenz system parameters of PMSM. Later on, we introduce basic theory of impulsive differential equation. Also, we made use of theory of impulsive differential equation to devise an effective scheme to control chaos in PMSM. Furthermore, some numerical simulations are done to test

the effectiveness of scheme. Finally, some conclusions are drawn.

CHAOS IN THE LORENZ SYSTEM

The system model of Lorenz system can be described as follows (Lorenz, 1963):

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz \end{cases} \quad (1)$$

where $X = [x, y, z]^T \in R^3$ is variables of system (1), a, b and c are system (1) parameters. Due to

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -122/3 < 0,$$

system (1) have dissipative structure when $a=10$, $b=8/3$, $c=28$.

When the Lyapunov exponents are more than zero when system (1) parameters within limited bound, at the same time, the Lorenz system is in a chaotic state. When $a=10$, $b=8/3$, $c=28$, the system (1) exhibits a chaotic behavior, and the projections of the chaotic attractor are shown in Figure 1 (Wang and Wang, 2008).

The theory of tridiagonal structure matrix stability

Lemma 1 (Liu and Zhang, 2007): If the nonlinear system has the following forms of tridiagonal structure:

$$\dot{x} = \begin{bmatrix} -k_1 & f_1(x) & & & \\ -f_1(x) & -k_2 & & \ddots & \\ & \ddots & \ddots & \ddots & \\ & & & -f_{n-1}(x) & -k_n \end{bmatrix}, \quad (2)$$

where the state variables $x = [x_1, x_2, \dots, x_n]^T \in R^n$, $f_j(x)$ ($1, 2, \dots, n-1$) are function about x , $k_i \in R^+(i=1, 2, \dots, n)$. The state variables x of nonlinear system will be global asymptotic stabilized at origin point.

Proof: A positive definite function as follows,

$$V = \frac{1}{2} x^T x, \quad (3)$$

then the derivative of V is given by

$$\dot{V} = -\sum_{i=1}^n K_i x_i^2 < 0. \quad (4)$$

Therefore V is the Lyapunov function and variables x of nonlinear system will be global asymptotic stabilized at origin point (Liu and Zhang, 2007).

Control chaos in the Lorenz system

The Lorenz system has one of equilibrium point $P_0(0,0,0)$. In this paper, the system (1) is stable to $P_0(0,0,0)$.

Due to variables of system (1) $X = [x, y, z]^T \in R^3$, set $Y = [x', y', z']^T \in R^3$, transforms a position by matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

namely

$$X = AY, \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z' \\ y' \\ x' \end{bmatrix},$$

Substituting these transformed variables into system (1), system (1) can be rewritten as:

$$\begin{cases} \dot{x}' = z'y' - bx' \\ \dot{y}' = cz' - y' - x'z' \\ \dot{z}' = a(y' - z') \end{cases} \quad (5)$$

Consider system (5) exist equilibrium $P(x'_0, y'_0, z'_0)$, so

$$\begin{cases} z'_0 y'_0 - bx'_0 = 0 \\ cz'_0 - y'_0 - x'_0 z'_0 = 0 \\ a(y'_0 - z'_0) = 0 \end{cases} \quad (6)$$

Let $x'' = x' - x'_0$, $y'' = y' - y'_0$, $z'' = z' - z'_0$. System (5) is transformed to

$$\begin{cases} \dot{x}'' = y''z'' + y''z'_0 + z''y'_0 - bx'' + z'_0 y'_0 - x'_0 = -bx'' + y''(z'' + z'_0) + y'_0 z'' + z'_0 y'_0 - x'_0 \\ \dot{y}'' = cz'' + cz'_0 - y'' - y'_0 - x''z'' - x''z'_0 - x'_0 z'' - x'_0 z'_0 = -(z'' + z'_0)x'' - y'' + (c - x'_0)z'' - y_0 - x'_0 z'_0 + cz'_0 \\ \dot{z}'' = ay'' - az'' + ay'_0 - az'_0 \end{cases} \quad (7)$$

The controlled system (5) can be written as

$$\begin{cases} \dot{x}'' = -bx'' + y''(z'' + z'_0) + y'_0 z'' + u_1 \\ \dot{y}'' = -(z'' + z'_0)x'' - y'' + (c - x'_0)z'' + u_2 \\ \dot{z}'' = ay'' - az'' + u_3 \end{cases} \quad (8)$$

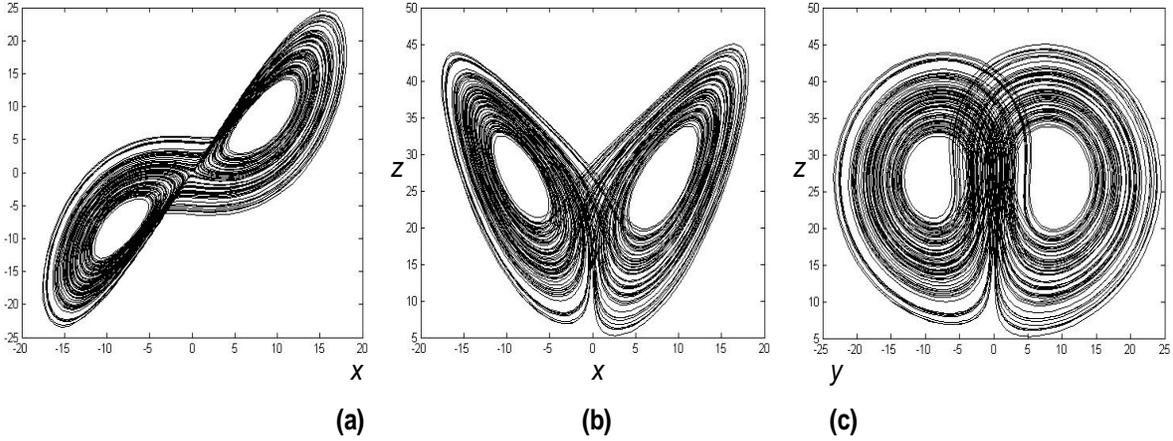


Figure 1. The projections of Lorenz attractor.

where known item of system (7) are added to controller, u_1, u_2, u_3 are controller.

System (8) can be rewritten as,

$$\begin{bmatrix} \dot{x}'' \\ \dot{y}'' \\ \dot{z}'' \end{bmatrix} = \begin{bmatrix} -b & z'' + z'_0 & y'_0 \\ -z'' - z'_0 & -1 & c - x'_0 \\ 0 & a & -a \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad (9)$$

Compare system (9) with system (2), if system (9) equals to

$$\begin{bmatrix} \dot{x}'' \\ \dot{y}'' \\ \dot{z}'' \end{bmatrix} = \begin{bmatrix} -b & z'' + z'_0 & 0 \\ -z'' - z'_0 & -1 & -a \\ 0 & a & -a \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}, \quad (10)$$

system (8) will be asymptotic stabilized at equilibrium point $P(x'_0, y'_0, z'_0)$. The equation can be gotten as follows,

$$\begin{bmatrix} -b & z'' + z'_0 & y'_0 \\ -z'' - z'_0 & -1 & c - x'_0 \\ 0 & a & -a \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -b & z'' + z'_0 & 0 \\ -z'' - z'_0 & -1 & -a \\ 0 & a & -a \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}, \quad (11)$$

Hence obtain controller as follows,

$$\begin{cases} u_1 = -y'_0 z'' \\ u_2 = -(a + c - x'_0) z'', \\ u_3 = 0 \end{cases} \quad (12)$$

$z'' = z' - z'_0$, then the final controller is expressed as

follows,

$$\begin{cases} u_1 = -y'_0(z' - z'_0) \\ u_2 = -(a + c - x'_0)(z' - z'_0) \\ u_3 = 0 \end{cases} \quad (13)$$

NUMERICAL SIMULATION

In this section, we will show the effectiveness of the proposed controller, simulation on controlling chaos in the Lorenz system by the presented controller will be conducted. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the computation problem with time step size $h = 0.001$.

Simulation condition

The initial values of chaos PMSM are chosen as $(x'(0), y'(0), z'(0)) = (5, 8, 6)$, at the same time, the parameters of the Lorenz system is chosen as $a = 10, b = 8/3, c = 28$. The controller is put into effect at 10 s after the chaos begins. The Lorenz system is experiencing chaos oscillation before the controller is carried out, the chaotic behavior is suppressed to equilibrium point $P_0(0, 0, 0)$ in about 3 seconds, the results is shown in Figure 2.

Conclusion

This paper proposed the tridiagonal structure matrix stability theory and the design of controllers based on the tridiagonal structure matrix stability theory. The proposed

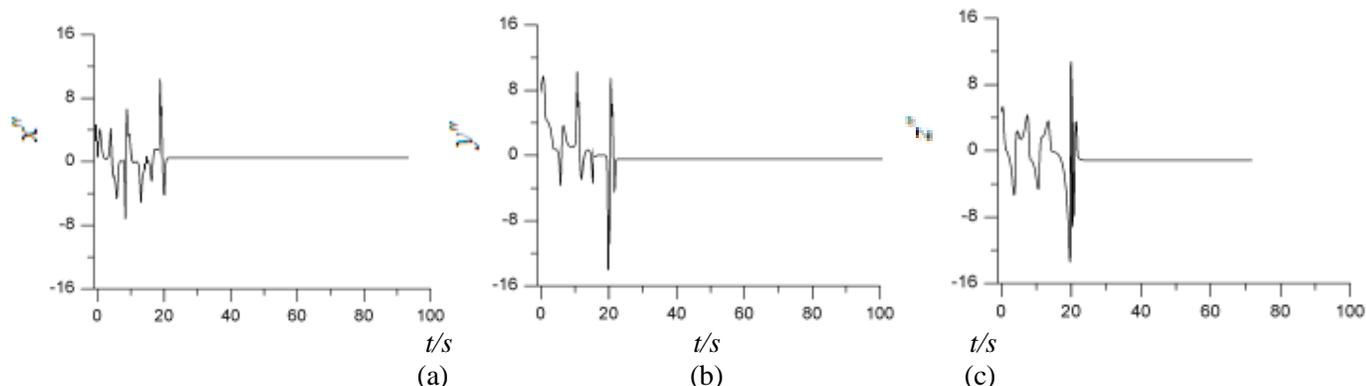


Figure 2. Show system (8) with controlled when $t=20s$.

controller is very simple and the design of controllers needs not a large number of operations. The numerical simulation results demonstrated the effectiveness of the controllers.

ACKNOWLEDGEMENT

This research was supported by the National Natural Science Foundation of China (Nos: 61173183, 60973152, 61272523 and 60573172), the Superior University Doctor Subject Special Scientific Research Foundation of China (No: 20070141014), Program for Liaoning Excellent Talents in University (No: LR2012003), the National Natural Science Foundation of Liaoning province (No: 20082165) and the Fundamental Research Funds for the Central Universities (No: DUT12JB06) and the National Key Project of Science and Technology of China” (No.2011ZX05039-003-4).

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