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Full Length Research Paper

ARL performance of residual control charts for trend AR(1) process: A case study on peroxide values of stored vegetable oil

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For the purpose of process control, quality assurance engineers in a vegetable oil factory wonder the performance of the Shewhart, CUSUM, and EWMA residual control charts for peroxide values that show both serial autocorrelation between adjacent observations (autocorrelation) and upward linear trend. To deal with autocorrelated process data, a primary method is to apply these charts to the uncorrelated residuals of an appropriate time series model fitted to the data. In the relevant literature, although performances of the residual charts have been widely studied for autocorrelated processes, there exists no study that shows how these charts' performances change by the addition of a particular type of trend in the autocorrelated data. In the present paper, average run length performances of these charts are computed for peroxide data from two batches, for which trend stationary first order autoregressive (trend AR(1) for short) model is a representative model.

Key words: Statistical process control, autocorrelation, peroxide value, vegetable oil, trend AR(1) model.

INTRODUCTION

The standard assumptions that are usually cited in justifying the use of control charts are that the data generated by the in-control process are normally and independently distributed by mean of μ and standard deviation of σ. When there is significant autocorrelation in a process, traditional control charts with *iid* (independent and identically distributed) assumption will be ineffective. In addition to various control charts developed for monitoring autocorrelated processes, three general approaches are recommended; (i) fit ARIMA model to data then apply traditional control charts such as Shewhart, CUSUM, and EWMA to process residuals (that is, forecast errors), (ii) monitor the autocorrelated observations by modifying the standard control limits to account for the autocorrelation, (iii) eliminate the autocorrelation by using an engineering controller (Montgomery, 1997). While applying traditional charts to process residuals, the residuals are assumed to be statistically uncorrelated. To construct a residual chart, an appropriate time series model is fitted to the autocorrelated data and the residuals are plotted in a control chart. For this reason all of the well-known control schemes can be transformed to the residual control schemes. The main advantage of a residual chart is that it can be applied to any autocorrelated data whether the process is stationary or not.

In addition to autocorrelation, some types of industrial processes such as chemical processes also exhibit a particular kind of trend behavior. In chemical processes linear trend often occurs because of settling or separation of the components of a mixture. Such process data is usually modeled by a trend AR(1) model. Although many different residual monitoring approaches have been proposed in the relevant literature, their performances have not been yet declared for a trend AR(1) process.

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However, quality engineers of an vegetable oil company wonder how results will these charts give for trend AR(1) process and therefore they want to test the performances of Shewhart, CUSUM, and EWMA residual charts for peroxide values of a particular type of vegetable oil to select a specific statistical monitoring approach in the given situation. Because of being good performers for autocorrelated processes, these three residual charts are used in this study for autocorrelated and trending peroxide data. Average run length (ARL) which is defined as the number of observations that must be plotted before a point indicates an out-of-control condition is used as performance criterion. For a desired chart when the process has no mean shift the ARL should be large, and when a mean shift occurs the ARL should be small to indicate the occurrence of the mean shift quickly (Zhang, 2000).

MATERIALS AND METHODS

Control charts for residuals

The shewhart chart

The center line (CL), upper control limit (UCL), and lower control limit (LCL) of the Shewhart $\bar{x}$ and R chart for the 3 standard deviations from the center-line are given below:

$$UCL = \bar{\bar{x}} + 3\frac{\sigma}{\sqrt{n}} \qquad (1)$$

$$CL = \bar{\bar{x}} \qquad (2)$$

$$LCL = \bar{\bar{x}} - 3\frac{\sigma}{\sqrt{n}} \qquad (3)$$

Where x_i are sample of size *n*, $\bar{x} = (x_1 + x_2 + ... + x_n)/n$, $\bar{\bar{x}} = (\bar{x}_1 + \bar{x}_2 + ... + \bar{x}_m)/m$, m is the number of subgroups, $R = x_{\max} - x_{\min}$, $\bar{R} = (R_1 + R_2 + ... + R_m)/m$, and $\hat{\sigma} = \bar{R}/d_2$, where d_2 is the mean of the distribution of the relative range that can be refered from statistical tables. If the production rate is too slow to allow sample sizes greater than one, individual measurements are used. For the control chart for individual measurements, the following parameters are required:

$$UCL = \bar{x} + 3\frac{\overline{MR}}{d_2} \qquad (4)$$

$$CL = \bar{x} \qquad (5)$$

$$LCL = \bar{x} - 3\frac{\overline{MR}}{d_2} \qquad (6)$$

Where $\overline{MR}$ is the average moving range and *MR* is the range between consecutive observations (Montgomery, 1997; Oakland, 2003). If the observations are autocorrelated, the formulations given above are modified in order to construct a Shewhart residual chart. The residual e_t from a time series model of $\{x_t\}$ is defined as:

$$e_i = x_t - \hat{x}_t \qquad (7)$$

Where $\hat{x}_t$ is the prediction of x_t from the time series model at time *t*. In a similar manner, the CUSUM residual and EWMA residual charts are constructed by applying traditional CUSUM and EWMA charts respectively to $\{e_t\}$ (Montgomery, 1997; Zhang, 2000; Montgomery and Runger, 1999; Montgomery and Johnson, 1976). The ARL of the Shewhart charts can be found from:

$$ARL = (1/p) \qquad (8)$$

Where *p* is the probability of exceeding the control limits by any sample point. Thus, if the process is in-control ARL_0= (1/ α) where α is the probability of type I error, but if it is out-of-control, ARL_1 = (1/(1- β)). The probability of not detecting this shift on the first subsequent sample or the β risk which is the type II error is

$$\beta = \Phi\left[\frac{UCL - (\mu_0 + k\sigma)}{\sigma/\sqrt{n}}\right] - \Phi\left[\frac{LCL - (\mu_0 + k\sigma)}{\sigma/\sqrt{n}}\right] \qquad (9)$$

Where Φ denotes the standard normal cumulative distribution and *k* is the multiple of the standard deviation of the statistics plotted on the chart.

The CUSUM chart

The basic purpose of a CUSUM chart is to track the distance between the actual data point and the grand mean. Then, by keeping a cumulative sum of these distances, a change in the process mean can be determined, as this sum will continue getting larger or smaller. These cumulative sum statistics are called the upper cumulative sum (C_t^+) and the lower cumulative sum (C_t^-):

$$C_t^+ = \max[0, x_t - (\mu_0 + K) + C_{t-1}^+] \qquad (10)$$

$$C_t^- = \max[0, (\mu_0 - K) - x_t + C_{t-1}^-] \qquad (11)$$

Where μ_0 is the grand mean and *K* is the slack value (reference value) which is often chosen about halfway between the target μ_0 and the out-of-control value of the mean μ_1 that we are interested in detecting quickly (Montgomery, 1997; Oakland, 2003; Wetherill and Brown, 1991). So, if the shift is expressed in standard deviation units as $\mu_1 = \mu_0 + \delta\sigma$ (or $\delta = |\mu_1 - \mu_0|/\sigma$), then *K* is one-half the magnitude of the shift or $K = (\delta\sigma)/2 = (|\mu_1 - \mu_0|)/2$.

It is important to select the right value for *K*, since a large value of *K* will allow for large shifts in the mean without detection, whereas a small value of *K* will increase the frequency of false alarms. Normally, *K* is selected to be equal to 0.5 σ.

The tabular CUSUM is designed by choosing values for the reference value *K* and the decision interval *H* so that $K = k\sigma$ and $H = h\sigma$. Using *h*=4 or *h*=5 and *k*=1/2 will generally provide a CUSUM that has good ARL properties against a shift about 1 σ in the process mean (Montgomery, 1997).

For CUSUM residual chart, the residuals are calculated by using Equation (7) where e_t shows normal distribution with mean zero and with constant variance. Then, conventional CUSUM control chart can be applied to the residuals using the formulas given in Equation (10) and Equation (11). Several techniques can be used to calculate the ARL of a CUSUM. For a one sided CUSUM (that is, C_t^+ or C_t^-) with parameters *h* and *k,* Siegmund's approximation is

$$ARL = \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2} \tag{12}$$

for $\Delta \neq 0$, where $\Delta = \delta^* - k$ for the upper one-sided CUSUM C_t^+, $\Delta = \delta^* - k$ for the lower one-sided CUSUM C_t^-, $b = h + 1.166$, and $\delta^* = (\mu_1 - \mu_0)/\sigma$. If $\Delta = 0$, one can use $ARL = b^2$. The quantity δ^* represents the shift in the mean, in the units of σ, for which the ARL is to be calculated. Therefore, if $\delta^* = 0$, we would calculate ARL_0 from Equation (12), while if $\delta^* \neq 0$, we would calculate the value of ARL_1 corresponding to a shift of size δ^*. To obtain the ARL of two-sided CUSUM from the ARLs of the two one-sided statistics, say ARL^+ and ARL^-, the following formula given in Equation (13) is used (Montgomery, 1997).

$$\frac{1}{ARL} = \frac{1}{ARL^+} + \frac{1}{ARL^-} \tag{13}$$

The EWMA chart

Like CUSUM chart, EWMA is suitable for detecting small process shifts. EWMA chart uses smoothing constant λ (Shu et al., 2005). The EWMA is a statistic for monitoring the process that averages the data in a way that gives less and less weight to data as they are further removed in time. By the choice of weighting factor λ, the EWMA control procedure can be made sensitive to a small or gradual drift in the process. The formulation of the EWMA statistic is given in Equation (14).

$$z_t = \lambda x_t + (1-\lambda) z_{t-1} \tag{14}$$

Where z_t is the moving average at time *t*. The value of λ can be between zero and one, but it must often chosen between 0.05 and 0.3. The initial value of *z* (that is z_0) is set to the grand mean (μ_0) (Montgomery, 1997; Oakland, 2003; Wetherill and Brown, 1991). If the observations x_t are independent random variables with variance σ^2, then the variance of z_t will be

$$\sigma_{z_t}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda} \right) \left[1 - (1-\lambda)^{2t} \right] \tag{15}$$

The center line and control limits for the EWMA control chart are as follows:

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]} \tag{16}$$

$$CL = \mu_0 \tag{17}$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]} \tag{18}$$

Where *L* is the number of standard deviations from the center-line. There are two main approaches for computing ARL for an EWMA sequence. The first approach is based on the fact that ARL must satisfy the Fredholm integral equation (Crowder, 1987). The second approach is based on the flexible and relatively easy to use Markov chain approach, originally proposed by Brook and Evans (1972) (Pham, 2006). We used second approach to calculate the ARL of EWMA control scheme.

This procedure involves dividing the interval between LCL and UCL into $p = 2m + 1$ subintervals of width 2δ, where $\delta = (UCL - LCL)/(2p)$. When the number of subintervals *p* is sufficiently large, the finite approach provides an effective method that allows ARL to be effectively evaluated. The EWMA statistic (z_t) is said to be in transient state *j* at time *t* if $H_j - \delta < z_t < H_j + \delta$ for $j = -m, \ldots, -1, 0, +1, \ldots, +m$ where H_j represents the midpoint of the *j*th subinterval. The EWMA statistic is in the absorbing state if $z_t \notin [LCL, UCL]$. An approximation for ARL is given by

$$ARL = d^T Q g \tag{19}$$

Where *d* is the (*p*, 1) initial probability vector, $Q = (I - P)^{-1}$ is the fundamental (*p*, *p*) matrix, *P* is the (*p*, *p*) transition-probabilities matrix and *g=I* is a (*p*, 1) vector of 1s. The initial probability vector *d* contains the probabilities that the statistic z_t starts in a given state. The transition probability matrix *P* contains the one-step transition probabilities. The generic element p_{ij} of *P* represents the probability that the statistic z_t goes from state *i* to state *j* in one step. This probability can be calculated by:

$$p_{ij} = \Phi\left(\frac{H_j + \delta - (1-\lambda)H_i}{\lambda}\right) - \Phi\left(\frac{H_j - \delta - (1-\lambda)H_i}{\lambda}\right) \quad (20)$$

The process model

The first order autoregressive model, AR(1), is a representative model for autocorrelated processes. In these processes, the current observation is correlated with its previous observation. Past studies emphasize the role of AR(1) model in process control (Guh, 2008; Box and Jenkins, 1976). An AR(1) model can be expressed as follows:

$$x_t = \xi + \phi x_{t-1} + \varepsilon_t \quad (21)$$

Where *t* is the time of sampling, x_t is the sample value at time *t*, ξ is the constant, ϕ is the autoregressive coefficient $(-1 < \phi < 1)$, and ε_t is the independent random error term (common cause variation) at time *t* following $N(0, \sigma_\varepsilon^2)$. If an increasing linear trend exists in such a process, its representative model is called the trend AR(1) model (Guh, 2008):

$$X_t = x_t + dt \quad (22)$$

Where *d* is the trend slope in terms of *t*. Let a trend AR(1) process with an upward mean shift be denoted by (Guh, 2008)

$$Z_t = X_t + \delta_\mu \quad (23)$$

Where δ_μ is the magnitude of upward mean shift. In this study, our aim is to test the Shewhart, CUSUM, and EWMA residual charts for a real trend AR(1) process with an upward shift in the mean. To the best of our knowledge, this is the first study that attempts to test both autocorrelated and trended data on the tree well known residual control charts.

Vegetable oils, which are indispensable for human nutrition, are very sensitive to the effects of temperature, sun light, oxygen and some other metal ions. Oil has some chemical transmutations when it is exposed to these effects and this causes deterioration in quality of oil. Fatty acid rich vegetable oils would be uneatable because of the aldehyde, ketone, and fatty acids with small molecules which are ensued after the oxidation reaction that is observed under the inappropriate storage conditions. Because of this reason the stored vegetable oil from same lot may be supplied to the consumers with different qualities. It is important for vegetable oils to provide its quality during the retention period.

Peroxide value (PV) is a measure of total peroxides in olive oil expressed as meq O_2/kg oil and hence it is a major guide of quality (Stepanyan et al., 2005). In other words, peroxide value (PV, meq O_2kg^{-1} oil) is a measure of oxidative rancidity and a guide to olive oil quality (Nouros et al., 1999). The rate of oxidation depends on a number of factors including the availability of oxygen, presence of light and temperature (Pristouri et al., 2010).

RESULTS

Appendixes 1 and 2 depict the peroxide values of two different lots from two different warehouses of the local oil factory. Peroxide value should be monitored continuously to provide maintaining the desired quality of stored vegetable oil during the retention period. By this way the stored vegetable oil is prevented from being supplied to the consumers with different qualities because of the storage conditions of different warehouses. The peroxide values are measured with 6 h period (4 times in a day) in a day during the 125 days (totally 500 observations for each lot). For the vegetable oil peroxide values should be between 0 to 10.0 meq O_2/kg. Regarding the sample and partial autocorrelation of each data we identified trend AR(1) model for the first lot:

$$X_t = 0.9128 X_{t-1} + 0.0854t + e_t \quad (24)$$

Where σ_e =0.02135, the variance of AR(1) process is γ_0 =0.002733, $\phi = 0.9128$, d =0.0854 and for the second lot:

$$X_t = 0.4846 X_{t-1} + 0.0937t + e_t \quad (25)$$

Where σ_e =0.039126, γ_0 = 0.002001, $\phi = 0.4846$, and d =0.0937. There exist strong positive autocorrelation in the first lot (autoregressive coefficient=0.9128) and moderate positive autocorrelation in the second lot (autoregressive coefficient= 0.4846). Degree of autocorrelation varies depending on the storage conditions.

The ARLs of the residual charts - Shewhart individual, Shewhart $\bar{x}$ and *R*, CUSUM, and EWMA - were computed using the formulas given earlier. The peroxide values of two lots are supposed to be in-control. To model assignable causes, a sustained shift of magnitude δ_μ is induced in the mean of peroxide values in Equation (24) and (25) starting at the first observation. Different shift magnitudes δ_μ = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 were considered. For the sake of simplicity, we classified shift magnitudes in three groups as small (δ_μ = 0.5, 1.0), moderate (δ_μ = 1.5, 2.0), and large (δ_μ = 2.5, 3.0). For different magnitudes of the process mean shift, the samples were monitored with these charts.

The residual charts are designed, in such a way, that they have approximately the same ARL for in-control situation, around 370. For the EWMA chart *L*=2.5 is chosen for $\lambda = 0.05$ to have the in-control ARL close to 370. Lucas and Saccucci (1990) give tables that help the user to select *L* and λ. For the Shewhart chart *L*=3 is chosen (Montgomery, 1997) and for the CUSUM chart, *k* and *h* are set to 0.5 and 4.77, respectively, to have the in-control ARL close to 370. Hawkins (1993) gives tables to

Table 1. ARLs for trend AR(1) processes ($\phi = 0.9128$).

ϕ	Mean Shift	EWMA	Shewhart Individual	Shewhart $\bar{x}$ and R	CUSUM
0.9128	0.0	368.6425	371.4650	370.4650	371.2724
	0.5	35.0000	261.9981	370.0000	46.1533
	1.0	34.0526	98.9720	98.9746	30.6169
	1.5	28.8528	98.9312	98.9338	28.8279
	2.0	28.5929	98.8885	98.8908	28.4613
	2.5	28.3328	98.8450	98.8484	28.1275
	3.0	10.8827	1.0423	1.0386	23.5784

Table 2. ARLs for trend AR (1) processes ($\phi = 0.4846$).

ϕ	Mean Shift	EWMA	Shewhart Individual	Shewhart $\bar{x}$ and R	CUSUM
0.4846	0.0	369.1422	371.2050	371.1150	371.4852
	0.5	51.4766	75.9659	75.9893	51.6865
	1.0	16.6363	75.9039	75.9274	17.7445
	1.5	15.7655	15.8736	15.9287	15.7739
	2.0	10.8076	15.7297	15.7847	10.5883
	2.5	8.8577	15.5857	15.6416	8.1054
	3.0	6.4056	1.3707	1.3481	6.4394

select *k* and *h* for Two-Sided Tabular CUSUM chart to have in-control ARL close to 370.

To illustrate how these charts signal, we easily computerized design procedures of the charts with MATLAB 7.4.0, and applied them to the peroxide values of sample vegetable oils stored by the local oil factory. Design of the charts for this sample data was completed in less than 1s of CPU time on a personal computer (AMD turion, 1.79 GHZ, 2.87 GB Ram). To model assignable causes, a sustained shift of magnitude δ_μ is induced in the mean of Z_t in Equation (23) starting at the initial start-up of the system. The ARL values are displayed in Table 1 for the first sample set (from the first lot) and are displayed in Table 2 for the second sample set (from the second lot), respectively.

DISCUSSION

Tables 1 and 2 show that when process exhibit moderate positive autocorrelation ($\phi = 0.4846$), EWMA and CUSUM charts perform almost the same and better than Shewhart type charts for small to moderate mean shifts, which in turn revealed that these two charts are more appropriate in a correlated environment with moderate autocorrelation as they provide a higher probability of coverage than the Shewhart type charts, while Shewhart type charts perform better for large shifts.

The literature concerning the control of small changes in the process average the EWMA and CUSUM control charts are recommended, hence the purpose here is to investigate which of them better detects an out of control signal in some range of standard deviation. For small shifts, EWMA a bit outperforms CUSUM. Similarly, when a strong positive autocorrelation ($\phi = 0.9128$) exist in process data, the EWMA and CUSUM charts performs better than the Shewhart type control charts for small to moderate shifts. Shewhart type charts are not satisfactory in this case, while Shewhart type charts perform better for large shifts. For small and large shifts, EWMA outperforms CUSUM.

Results indicate that the Shewhart type control charts are efficiently complemented by CUSUM and EWMA charts when there is interest in detecting small to moderate changes in peroxide values. It was possible to notice that EWMA control chart was more efficient in all analysis accomplished with the changes in the order of small shifts. It was also observed for this particular case study, the Shewhart type control charts behaving in a similar way; and, therefore, any of these two charts can be used for monitoring the peroxide values of vegetable oil for the given situations. Table 3 summarizes these results.

Table 3. Best performed control charts for trend AR (1) process.

δ \ ϕ	**Strong positive** ($\phi = 0.9128$)	**Moderate positive** ($\phi = 0.4846$)
Small	EWMA	EWMA, CUSUM
Moderate	EWMA, CUSUM	EWMA, CUSUM
Large	Shewhart $\bar{x}$ and *R* Shewhart individual	Shewhart $\bar{x}$ and *R* Shewhart individual

After comparing the ARL performances of given charts each other for the given experimental data, their performances are compared with results that are presented in the recent literature for the analytically calculated ARL results from simulated data. The most notable finding of this case study is that the ARL performance of the residual charts used for trend AR(1) process modeled from experimental data in the current study are distinct from analytical ARL values presented in preliminary studies for AR(1) process (see (Zhang, 2000) for analytically calculated ARL performance of residual charts for autocorrelated process observations). If the ARL results calculated for experimental data which are obtained by measuring the peroxide values of vegetable oil, are compared with the results presented by Zhang (2000) the following results are extracted. For strong positive autocorrelation ($\phi = 0.9128$) the calculated ARL performance of EWMA and CUSUM residual charts for the experimental data obtained from first lot are better then the analytically calculated ARL performances of these charts for all cases. For Shewhart type charts, experimental ARL performance is better than its analytical ARL performance for moderate mean shifts. For moderate positive autocorrelation the results indicate that the experimental ARL performance of EWMA and CUSUM charts are better then their analytically calculated ARL performances for small to moderate shifts. For Shewhart type charts, experimental ARL performance is good for all cases.

From Tables 1 and 2, it is clear that when the process is positively and strongly autocorrelated, even the shift is as small as δ_μ = 0.5, the autocorrelation has a big impact on the Shewhart type charts. When δ_μ = 0.5, the ARL for the Shewhart individual and Shewhart $\bar{x}$ and R charts are increased to 261.9981 and 370 from values of 75.9659 and 75.9893. This effect turns in direct contradiction with the large mean shift δ_μ = 3.0. Contrary effects are observed with the change in autocorrelation coefficient for EWMA and CUSUM charts.

In analytical calculations, that are performed by using simulated data, the test data are generated from a known mathematical model and the control charts' performances are tested for these data. But in the present study, control charts' performances are tested for the real data by using a mathematical model that best fits the observed peroxide values. However the fitted model's residuals naturally would be different from the residuals calculated from a model with known parameters and with some assumptions. This causes differences in the calculated ARL performance between real and analytical applications. In other words, the ARL results calculated for experimental data differs from the ARL values that are calculated for simulated data. In analytical studies, the ARL values decreases uniformly from small shifts to the large shifts and the differences are clearly observed. But in real applications during this downturn the differences may not be clear for some cases. For example, in the present study, from Table 1 it is clearly observed that for the strong positive autocorrelation the EWMA charts ARL performance is not distinctive for 1.5σ-2.5σ shifts and etc. Also note that the existence of the trend affects the model fitting and parameter selection for the fitted model.

Conclusion

It is important to know the performances of residual charts for autocorrelated processes in the presence of different patterns. Although the performances of well known residual charts for AR(1) process were studied widely in the early studies, their performances for AR(1) process with upward linear trend remains a mystery to researchers. On peroxide values of a specific vegetable oil produced by a local company, it was observed that EWMA and CUSUM residual charts behaved in a similar way and therefore, any of these two charts can be used for detecting small and moderate shifts in a strong and moderate positive autocorrelated environment, while Shewhart $\bar{x}$ and *R* chart provides a higher probability of coverage for large shifts in the same environment. Company quality engineers can employ these charts for identifying if the storage conditions sustain the oil's conformance during the retention period.

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Appendix 1. Peroxide values of vegetable oil. Peroxide values for the first lot ($\phi = 0.9128$).

Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})
1.00	0.0500	12.75	1.1747	24.50	2.0545	36.25	3.1197	48.00	4.0760	59.75	5.0716	71.50	6.2133	83.25	7.0850	95.00	8.1386	106.75	9.1058
1.25	0.1196	13.00	1.1545	24.75	2.0734	36.50	3.1119	48.25	4.1204	60.00	5.0511	71.75	6.1876	83.50	7.0856	95.25	8.1765	107.00	9.1480
1.50	0.1537	13.25	1.1709	25.00	2.0948	36.75	3.1391	48.50	4.1160	60.25	5.0580	72.00	6.1810	83.75	7.1171	95.50	8.2156	107.25	9.1711
1.75	0.1428	13.50	1.2012	25.25	2.0910	37.00	3.1626	48.75	4.1233	60.50	5.0866	72.25	6.2147	84.00	7.1663	95.75	8.2614	107.50	9.1543
2.00	0.1532	13.75	1.2202	25.50	2.1251	37.25	3.1826	49.00	4.1214	60.75	5.0756	72.50	6.2098	84.25	7.1936	96.00	8.2735	107.75	9.1402
2.25	0.1611	14.00	1.2732	25.75	2.2119	37.50	3.1528	49.25	4.1722	61.00	5.1351	72.75	6.2156	84.50	7.2184	96.25	8.3177	108.00	9.1763
2.50	0.1818	14.25	1.2802	26.00	2.2386	37.75	3.1667	49.50	4.1809	61.25	5.1491	73.00	6.2520	84.75	7.2485	96.50	8.3354	108.25	9.1846
2.75	0.2115	14.50	1.3140	26.25	2.2261	38.00	3.1985	49.75	4.1670	61.50	5.1306	73.25	6.2720	85.00	7.2961	96.75	8.3157	108.50	9.2235
3.00	0.2129	14.75	1.3109	26.50	2.2543	38.25	3.2322	50.00	4.2137	61.75	5.1648	73.50	6.3158	85.25	7.3467	97.00	8.3287	108.75	9.2168
3.25	0.2363	15.00	1.2991	26.75	2.2438	38.50	3.2605	50.25	4.2724	62.00	5.2290	73.75	6.3030	85.50	7.3388	97.25	8.3343	109.00	9.2542
3.50	0.3006	15.25	1.3339	27.00	2.2720	38.75	3.2833	50.50	4.3104	62.25	5.2571	74.00	6.3346	85.75	7.3612	97.50	8.3136	109.25	9.2942
3.75	0.3586	15.50	1.3208	27.25	2.2578	39.00	3.2704	50.75	4.3349	62.50	5.2870	74.25	6.3173	86.00	7.3922	97.75	8.3262	109.50	9.3269
4.00	0.3371	15.75	1.3128	27.50	2.3013	39.25	3.3046	51.00	4.3364	62.75	5.3119	74.50	6.3078	86.25	7.4031	98.00	8.3351	109.75	9.3269
4.25	0.3731	16.00	1.3232	27.75	2.3526	39.50	3.3269	51.25	4.3422	63.00	5.3410	74.75	6.3403	86.50	7.4773	98.25	8.3754	110.00	9.3439
4.50	0.3956	16.25	1.3146	28.00	2.4105	39.75	3.3631	51.50	4.3943	63.25	5.3856	75.00	6.3619	86.75	7.5099	98.50	8.4054	110.25	9.3792
4.75	0.3976	16.50	1.3549	28.25	2.4615	40.00	3.4007	51.75	4.4419	63.50	5.4122	75.25	6.3611	87.00	7.5028	98.75	8.4469	110.50	9.3673
5.00	0.4571	16.75	1.3746	28.50	2.4612	40.25	3.4213	52.00	4.4473	63.75	5.4513	75.50	6.3813	87.25	7.5358	99.00	8.4731	110.75	9.3784
5.25	0.4653	17.00	1.3715	28.75	2.4901	40.50	3.4513	52.25	4.4862	64.00	5.4833	75.75	6.3949	87.50	7.5653	99.25	8.4891	111.00	9.3819
5.50	0.4533	17.25	1.3797	29.00	2.5289	40.75	3.4655	52.50	4.5101	64.25	5.5058	76.00	6.4021	87.75	7.5955	99.50	8.5070	111.25	9.4340
5.75	0.4675	17.50	1.3867	29.25	2.5213	41.00	3.4807	52.75	4.5204	64.50	5.5316	76.25	6.4365	88.00	7.5688	99.75	8.5337	111.50	9.4999
6.00	0.4716	17.75	1.4158	29.50	2.5451	41.25	3.4950	53.00	4.5213	64.75	5.5284	76.50	6.4303	88.25	7.6199	100.00	8.5797	111.75	9.5389
6.25	0.4858	18.00	1.3911	29.75	2.5813	41.50	3.4970	53.25	4.5494	65.00	5.5282	76.75	6.4674	88.50	7.5972	100.25	8.6155	112.00	9.5433
6.50	0.5510	18.25	1.4094	30.00	2.5698	41.75	3.5276	53.50	4.5292	65.25	5.5510	77.00	6.4502	88.75	7.6399	100.50	8.6011	112.25	9.5713
6.75	0.5898	18.50	1.4122	30.25	2.5688	42.00	3.5800	53.75	4.5182	65.50	5.5819	77.25	6.4942	89.00	7.6305	100.75	8.6231	112.50	9.5884
7.00	0.6353	18.75	1.4556	30.50	2.6089	42.25	3.5887	54.00	4.5913	65.75	5.5837	77.50	6.5320	89.25	7.6576	101.00	8.6374	112.75	9.5917
7.25	0.6548	19.00	1.5031	30.75	2.6296	42.50	3.6002	54.25	4.6126	66.00	5.6260	77.75	6.5778	89.50	7.6974	101.25	8.6703	113.00	9.6236
7.50	0.6664	19.25	1.5381	31.00	2.6624	42.75	3.6403	54.50	4.6951	66.25	5.6505	78.00	6.5886	89.75	7.7322	101.50	8.6824	113.25	9.6494
7.75	0.6874	19.50	1.5663	31.25	2.7132	43.00	3.6633	54.75	4.7072	66.50	5.7341	78.25	6.6313	90.00	7.7481	101.75	8.7279	113.50	9.6465
8.00	0.7238	19.75	1.5365	31.50	2.7415	43.25	3.7202	55.00	4.7295	66.75	5.7597	78.50	6.6325	90.25	7.7427	102.00	8.7414	113.75	9.6560
8.25	0.7923	20.00	1.5715	31.75	2.7404	43.50	3.7005	55.25	4.7687	67.00	5.7460	78.75	6.6802	90.50	7.7581	102.25	8.7265	114.00	9.6757
8.50	0.8134	20.25	1.5847	32.00	2.7761	43.75	3.6996	55.50	4.7814	67.25	5.7737	79.00	6.6571	90.75	7.7583	102.50	8.7795	114.25	9.7166
8.75	0.8466	20.50	1.6126	32.25	2.7898	44.00	3.7066	55.75	4.7871	67.50	5.8034	79.25	6.6924	91.00	7.7409	102.75	8.8082	114.50	9.7127
9.00	0.8747	20.75	1.6385	32.50	2.7894	44.25	3.7816	56.00	4.8384	67.75	5.8095	79.50	6.7035	91.25	7.7442	103.00	8.8390	114.75	9.7259
9.25	0.8993	21.00	1.6452	32.75	2.8569	44.50	3.8123	56.25	4.8349	68.00	5.8338	79.75	6.7145	91.50	7.7827	103.25	8.8406	115.00	9.7423

Appendix 1. Contd.

9.50	0.9338	21.25	1.6925	33.00	2.8747	44.75	3.8223	56.50	4.8716	68.25	5.8391	80.00	6.7473	91.75	7.8190	103.50	8.8784	115.25	9.7924
9.75	0.9729	21.50	1.7298	33.25	2.8698	45.00	3.8441	56.75	4.8771	68.50	5.8598	80.25	6.7681	92.00	7.8321	103.75	8.8452	115.50	9.8575
10.00	0.9830	21.75	1.7523	33.50	2.9027	45.25	3.8580	57.00	4.9123	68.75	5.8895	80.50	6.8186	92.25	7.8566	104.00	8.8924	115.75	9.9113
10.25	0.9860	22.00	1.8030	33.75	2.9584	45.50	3.9019	57.25	4.9488	69.00	5.9360	80.75	6.8421	92.50	7.8899	104.25	8.9124	116.00	9.9506
10.50	1.0027	22.25	1.8666	34.00	2.9865	45.75	3.9221	57.50	4.9459	69.25	5.9708	81.00	6.8909	92.75	7.9162	104.50	8.8959	116.25	9.9751
10.75	1.0374	22.50	1.8657	34.25	3.0184	46.00	3.9604	57.75	4.9760	69.50	5.9270	81.25	6.9034	93.00	7.9434	104.75	8.8915	116.50	9.9934
11.00	1.0759	22.75	1.8912	34.50	3.0338	46.25	3.9947	58.00	5.0167	69.75	5.9553	81.50	6.9291	93.25	8.0114	105.00	8.9219	116.75	9.9765
11.25	1.0730	23.00	1.8918	34.75	3.0438	46.50	4.0068	58.25	5.0085	70.00	5.9952	81.75	6.9619	93.50	7.9766	105.25	8.9518	117.00	9.9873
11.50	1.1052	23.25	1.9364	35.00	3.0781	46.75	4.0222	58.50	5.0349	70.25	6.0146	82.00	7.0099	93.75	8.0060	105.50	8.9531	117.25	10.0081
11.75	1.1118	23.50	1.9732	35.25	3.0871	47.00	4.0437	58.75	5.0518	70.50	6.0538	82.25	7.0444	94.00	8.0336	105.75	8.9687	117.50	10.0347
12.00	1.1161	23.75	1.9857	35.50	3.1033	47.25	4.0441	59.00	5.0421	70.75	6.0896	82.50	7.0572	94.25	8.0691	106.00	9.0268	117.75	10.0328
12.25	1.1417	24.00	1.9980	35.75	3.1064	47.50	4.0591	59.25	5.0567	71.00	6.1361	82.75	7.0461	94.50	8.0751	106.25	9.0394	118.00	10.0406
12.50	1.1758	24.25	2.0066	36.00	3.1074	47.75	4.0781	59.50	5.0745	71.25	6.1788	83.00	7.0534	94.75	8.1158	106.50	9.0641	118.25	10.0773

Appendix 2. Peroxide values of vegetable oil. Peroxide values for the second lot ($\phi = 0.4846$).

Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})	Day	Proxide (meq O_2kg^{-1})
1.00	0.100	12.25	1.093	23.50	2.162	34.75	3.262	46.00	4.291	57.25	5.473	68.50	6.343	79.75	7.418	91.00	8.541	102.25	9.612
1.25	0.225	12.50	1.159	23.75	2.140	35.00	3.332	46.25	4.336	57.50	5.466	68.75	6.335	80.00	7.446	91.25	8.565	102.50	9.570
1.50	0.116	12.75	1.222	24.00	2.224	35.25	3.366	46.50	4.397	57.75	5.398	69.00	6.411	80.25	7.501	91.50	8.609	102.75	9.590
1.75	0.124	13.00	1.279	24.25	2.282	35.50	3.365	46.75	4.338	58.00	5.442	69.25	6.487	80.50	7.584	91.75	8.579	103.00	9.661
2.00	0.145	13.25	1.266	24.50	2.332	35.75	3.357	47.00	4.343	58.25	5.440	69.50	6.519	80.75	7.658	92.00	8.634	103.25	9.645
2.25	0.210	13.50	1.300	24.75	2.327	36.00	3.386	47.25	4.391	58.50	5.434	69.75	6.539	81.00	7.662	92.25	8.716	103.50	9.735
2.50	0.213	13.75	1.378	25.00	2.331	36.25	3.396	47.50	4.450	58.75	5.508	70.00	6.561	81.25	7.678	92.50	8.706	103.75	9.748
2.75	0.259	14.00	1.440	25.25	2.353	36.50	3.417	47.75	4.439	59.00	5.548	70.25	6.563	81.50	7.663	92.75	8.806	104.00	9.762
3.00	0.247	14.25	1.383	25.50	2.376	36.75	3.468	48.00	4.457	59.25	5.554	70.50	6.666	81.75	7.669	93.00	8.745	104.25	9.791
3.25	0.281	14.50	1.465	25.75	2.415	37.00	3.449	48.25	4.524	59.50	5.562	70.75	6.650	82.00	7.658	93.25	8.856	104.50	9.800
3.50	0.305	14.75	1.428	26.00	2.500	37.25	3.452	48.50	4.496	59.75	5.559	71.00	6.610	82.25	7.713	93.50	8.818	104.75	9.798
3.75	0.376	15.00	1.401	26.25	2.495	37.50	3.495	48.75	4.584	60.00	5.534	71.25	6.644	82.50	7.779	93.75	8.822	105.00	9.848
4.00	0.345	15.25	1.448	26.50	2.524	37.75	3.501	49.00	4.631	60.25	5.542	71.50	6.663	82.75	7.826	94.00	8.792	105.25	9.915
4.25	0.350	15.50	1.460	26.75	2.604	38.00	3.488	49.25	4.610	60.50	5.575	71.75	6.701	83.00	7.802	94.25	8.751	105.50	9.972
4.50	0.365	15.75	1.523	27.00	2.564	38.25	3.581	49.50	4.660	60.75	5.631	72.00	6.721	83.25	7.821	94.50	8.804	105.75	9.925
4.75	0.428	16.00	1.546	27.25	2.597	38.50	3.596	49.75	4.721	61.00	5.693	72.25	6.726	83.50	7.857	94.75	8.875	106.00	9.970

Appendix 2. Contd.

5.00	0.521	16.25	1.488	27.50	2.609	38.75	3.567	50.00	4.725	61.25	5.736	72.50	6.707	83.75	7.843	95.00	8.931	106.25	9.936
5.25	0.577	16.50	1.416	27.75	2.584	39.00	3.609	50.25	4.760	61.50	5.715	72.75	6.758	84.00	7.877	95.25	8.877	106.50	10.018
5.50	0.578	16.75	1.527	28.00	2.522	39.25	3.650	50.50	4.762	61.75	5.789	73.00	6.754	84.25	7.911	95.50	8.854	106.75	10.022
5.75	0.581	17.00	1.535	28.25	2.509	39.50	3.601	50.75	4.770	62.00	5.827	73.25	6.818	84.50	7.945	95.75	8.910	107.00	10.043
6.00	0.577	17.25	1.622	28.50	2.591	39.75	3.626	51.00	4.783	62.25	5.895	73.50	6.899	84.75	7.931	96.00	8.967	107.25	10.121
6.25	0.605	17.50	1.701	28.75	2.633	40.00	3.633	51.25	4.767	62.50	5.868	73.75	7.004	85.00	8.010	96.25	9.095	107.50	10.030
6.50	0.668	17.75	1.657	29.00	2.672	40.25	3.757	51.50	4.781	62.75	5.885	74.00	6.999	85.25	8.046	96.50	9.067	107.75	10.080
6.75	0.578	18.00	1.721	29.25	2.666	40.50	3.782	51.75	4.826	63.00	5.886	74.25	7.004	85.50	7.951	96.75	9.085	108.00	10.071
7.00	0.678	18.25	1.733	29.50	2.736	40.75	3.854	52.00	4.898	63.25	5.945	74.50	7.042	85.75	7.965	97.00	9.134	108.25	10.062
7.25	0.676	18.50	1.723	29.75	2.782	41.00	3.880	52.25	4.886	63.50	5.999	74.75	7.050	86.00	8.012	97.25	9.114	108.50	10.151
7.50	0.701	18.75	1.810	30.00	2.781	41.25	3.901	52.50	4.969	63.75	6.016	75.00	7.098	86.25	7.984	97.50	9.095	108.75	10.167
7.75	0.758	19.00	1.787	30.25	2.848	41.50	3.881	52.75	4.937	64.00	6.121	75.25	7.078	86.50	8.116	97.75	9.080	109.00	10.187
8.00	0.787	19.25	1.853	30.50	2.803	41.75	3.919	53.00	4.987	64.25	6.074	75.50	7.092	86.75	8.143	98.00	9.188	109.25	10.227
8.25	0.731	19.50	1.910	30.75	2.816	42.00	3.889	53.25	4.951	64.50	6.086	75.75	7.082	87.00	8.183	98.25	9.227	109.50	10.239
8.50	0.720	19.75	1.813	31.00	2.863	42.25	3.909	53.50	4.962	64.75	6.014	76.00	7.085	87.25	8.148	98.50	9.301	109.75	10.253
8.75	0.804	20.00	1.965	31.25	2.860	42.50	3.934	53.75	4.994	65.00	6.045	76.25	7.105	87.50	8.193	98.75	9.270	110.00	10.325
9.00	0.802	20.25	1.992	31.50	2.964	42.75	3.990	54.00	5.048	65.25	6.021	76.50	7.179	87.75	8.282	99.00	9.322	110.25	10.309
9.25	0.786	20.50	2.006	31.75	3.037	43.00	3.954	54.25	5.147	65.50	6.089	76.75	7.199	88.00	8.302	99.25	9.398	110.50	10.334
9.50	0.838	20.75	1.957	32.00	3.037	43.25	3.965	54.50	5.142	65.75	6.088	77.00	7.151	88.25	8.326	99.50	9.393	110.75	10.386
9.75	0.873	21.00	1.964	32.25	2.988	43.50	4.041	54.75	5.132	66.00	6.124	77.25	7.275	88.50	8.342	99.75	9.381	111.00	10.375
10.00	0.874	21.25	1.938	32.50	3.054	43.75	4.137	55.00	5.173	66.25	6.145	77.50	7.304	88.75	8.338	100.00	9.327	111.25	10.411
10.25	0.889	21.50	1.942	32.75	2.978	44.00	4.155	55.25	5.142	66.50	6.195	77.75	7.248	89.00	8.309	100.25	9.419	111.50	10.465
10.50	0.960	21.75	1.969	33.00	3.028	44.25	4.145	55.50	5.180	66.75	6.272	78.00	7.277	89.25	8.428	100.50	9.454	111.75	10.485
10.75	0.925	22.00	2.050	33.25	3.204	44.50	4.167	55.75	5.230	67.00	6.344	78.25	7.324	89.50	8.367	100.75	9.430	112.00	10.597
11.00	1.040	22.25	2.148	33.50	3.198	44.75	4.195	56.00	5.239	67.25	6.349	78.50	7.323	89.75	8.417	101.00	9.478	112.25	10.519
11.25	1.083	22.50	2.189	33.75	3.148	45.00	4.290	56.25	5.261	67.50	6.327	78.75	7.359	90.00	8.451	101.25	9.574	112.50	10.580
11.50	1.081	22.75	2.152	34.00	3.212	45.25	4.273	56.50	5.296	67.75	6.356	79.00	7.405	90.25	8.542	101.50	9.566	112.75	10.612
11.75	1.102	23.00	2.084	34.25	3.211	45.50	4.314	56.75	5.334	68.00	6.347	79.25	7.466	90.50	8.510	101.75	9.569	113.00	10.580
12.00	1.091	23.25	2.153	34.50	3.272	45.75	4.308	57.00	5.411	68.25	6.348	79.50	7.416	90.75	8.572	102.00	9.622	113.25	10.586