

Full Length Research Paper

The photoelectric effect on graphene

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We derive the differential probability of the photoelectric effect realized at the very low temperature of double graphene in the strong magnetic field (The magnetic photoelectric effect). The relation of this effect to the elementary particle physics, nuclear physics and Einstein gravity is mentioned. Our approach is the nanalogue of the Landau theory of the diamagnetism. Understanding of the photoelectric effect on graphene enables the understanding of the photosynthesis in the chlorophyll in magnetic field.

Key words: Mono-layer graphite, Schrödinger equation, photons, photoeffect.

INTRODUCTION

The photoelectric effect is a quantum electronic phenomenon in which electrons are emitted from matter after the absorption of energy from electromagnetic radiation. Frequency of radiation must be above a threshold frequency, which is specific to the type of surface and material. No electrons are emitted for radiation with a frequency below that of the threshold. These emitted electrons are also known as photoelectrons in this context. The photoelectric effect was theoretically explained by Einstein who introduced the light quanta. Einstein writes (Einstein, 1905) in accordance with the assumption to be considered here, the energy of light ray spreading out from point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing and which can only be produced and absorbed as complete units. The linear dependence on the frequency was experimentally determined in 1915 when Robert Andrews Millikan showed that Einstein formula

$$\hbar\omega = \frac{mv^2}{2} + A \quad (1)$$

was correct. Here $\hbar\omega$ is the energy of the impinging photon and A is work function of concrete material. The work function for Aluminium is 4.3 eV, for Beryllium 5.0 eV, for Lead 4.3 eV, for Iron 4.5 eV and so on (Rohlf, 1994).

The work function concerns the surface photoelectric

effect where the photon is absorbed by an electron in a band. The theoretical determination of the work function is the problem of the solid state physics. On the other hand, there is the so called atomic photoeffect (Amusia, 1987), where the ionization energy plays the role of the work function. The system of the ionization energies is involved in the tables of the solid state physics.

The idea of the existence of the Compton effect is also involved in the Einstein article. He writes (Einstein, 1905) the possibility should not be excluded, however, that electrons might receive their energy only in part from the light quantum. However, Einstein was not sure, that his idea of such process is realistic. Only Compton proved the reality of the Einstein statement.

Equation 1 represents the so called one-photon photoelectric effect, which is valid for very weak electromagnetic waves. At present time of the laser physics, where the strong electromagnetic intensity is possible, we know that the so called multiphoton photoelectric effect is possible. Then, instead of equation (1) we can write

$$\hbar\omega_1 + \hbar\omega_2 + \dots + \hbar\omega_n = \frac{mv^2}{2} + A \quad (2)$$

The time lag between the incidence of radiation and the emission of a photoelectron is very small, less than 10^{-9} seconds. As an analogue of the equation (2), the multiphoton Compton effect is also possible.

$$\gamma_1 + \gamma_2 + \dots \gamma_3 + e \rightarrow \gamma + e \tag{3}$$

Einstein in his paper introduced the term "energy quanta" termed "photons" by chemist G. N. Lewis, in 1926. Later Compton, in his famous experiment proved that light quanta have particle properties, or photons elementary particles. At present time, the attention of physicists is concentrated to the planar physics at zero temperature and in the strong magnetic field. Namely, in the graphene physics which is probably new revolution in this century physics with assumption that graphene is the silicon of this century.

In 2004, Andre Geim, Kostia Novoselov (2004, 2005), (Kane, 2005) and co-workers at the University of Manchester in the UK delicately cleaving a sample of graphite with sticky tape produced something that was long considered impossible: a sheet of crystalline carbon just one atom thick, known as graphene.

More than 70 years ago, Landau (1937) and Peierls (1934, 1935) performed a proof that the 2-dimensional crystal is not thermodynamically stable and cannot exist. They argued that the thermodynamical fluctuations of such crystal leads to such displacements of atoms that these displacements are of the same size as the interatomic distances at the any finite temperature. The argument was later extended by Mermin (1968) and it seemed that many experimental observations supported the Landau-Peierls-Mermin theory. So, the "nonpossibility" of the existence of graphene was established.

On the other hand, Geim's group was able to isolate graphene and was able to visualize the new crystal using a simple optical microscope. At present time, there are novel methods how to create graphene sheet. For instance, Dato et al. (2008) used the plasma reactor, where the graphene sheets were synthesized by passing liquid ethanol droplets into an argon plasma. However, Landau-Peierls-Mermin proof is of the permanent heuristical and historical meaning.

Graphene is the benzene ring (C_6H_6) stripped out from their H-atoms. It is allotrope of carbon because carbon can be in the crystalline form of graphite, diamond, fullerene (C_{60}) and carbon nanotube.

Graphene unique properties arise from the collective behaviour of electrons. The electrons in graphene are governed by the Dirac equation. The Dirac fermions in graphene carry one unit of electric charge and can be manipulated using electromagnetic fields. Strong interactions between the electrons and the honeycomb hexagonal lattice of carbon atoms mean that the dispersion relation is linear and given by $E = vp$, v is called the Fermi-Dirac velocity, p is momentum of a pseudoelectron with pseudospin. The linear dispersion relation follows from the tight-binding Hamiltonian where the Fermi velocity in graphene is only about 300 times less than the speed of light.

The Dirac equation in graphene physics is used for the

so called pseudospin of pseudoelectron formed by the hexagonal lattice. The graphene is composed of the system of hexagonal cells and it geometrically means that graphene is composed from the systems of two equilateral triangles. If the wave function of the first triangle sublattice system is φ_1 and the wave function of the second triangle sublattice system is φ_2 , then the total wave function of the electron moving in the hexagonal system is superposition $\psi = c_1\varphi_1 + c_2\varphi_2$, where c_1 and c_2 are appropriate functions of coordinate x and functions φ_1, φ_2 are functions of wave vector k and coordinate x . This function is evidently the Bloch function, because the potential in graphene is periodic. After insertion of the function $\psi = c_1\varphi_1 + c_2\varphi_2$ into the Schrödinger equation, we get equations for the partial functions φ_1, φ_2 . After the definition of the new spinor function (Lozovik et al., 2008)

$$\chi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \tag{4}$$

It is possible to prove that this spinor function is the solution of the Pauli equation in the nonrelativistic situation and Dirac equation of the generalized case. The corresponding mass of such effective electron is proved to be zero. The pseudoelectron is not an electron of QED and pseudospin is not the spin of QED, because QED is the quantum theory of the interaction of real electrons and photons where the mass of electron is the mass defined by of classical mechanics and not by collective behaviour of hexagonal sheet called graphene. If we accept the formalism of pseudoelectrons, then, the interaction of photon with pseudoelectron is an analogue with the photodesintegration (Levinger, 1960) of deuteron being the bound state of neutron and photon.

$$\gamma + D \rightarrow n + p \tag{5}$$

where the dissociation energy of the process is 2.225 MeV. If we use this analogy in the photoelectric effect in graphene and we must accept the idea that the initial process in the photoelectric effect in graphene is the photodesintegration of pseudoelectron and then the interaction of the photon with the real electron. So, we generalize the Einstein equation to the novel form

$$\hbar\omega = \frac{mv^2}{2} + A_d + A \tag{6}$$

where the desintegration energy necessary for the desintegration of pseudoelectron must be determined

experimentally. The today graphenic theory does not solve this problem and the corresponding experiment was not performed. We suppose that this missing experiment is crucial in the graphene physics. The analogue proces occurs in superconductivity where photon desintegrates the Cooper pair in order to realize emission of an electron from a superconductor. After dissociation of pseudoelectron, we use the Schrödinger equation in order to establish the photoeffect in graphene in the magnetic field. Author (Pardy, 2008) considers in his e-print only the second step. We derive the diferential probability of the photoelectric effect realized at the very low temperature graphene. The relation of this effect to the elementary particle physics of LHC, nuclear physics and Einstein gravity is mentioned.

The quantum theory of the photoelectric effect on graphene in strong magnetic field

The photoelectric effect in graphene is presented here 100 years after well known Einstein article leading to the nobel prize for him. The photoelectric effect in semiconductor by onset of magnetic field is discussed by Kleinert et al. (2008) where the Bloch functions in a magnetic field are considered as the adequate for the solution of the problem. The calculation of photoeffect of graphene including both electron-electron and electron-photon interactions on the same footing is performed for instance by Park et al. (2010). The crucial object is the spectral function with the self-energy which accounts for the many-body interactions going beyond the mean-field picture.

Ratnikov et al. (2008a) define graphene as a two-dimensional zero gap semiconductor with zero conduction and valence band overlap to determine ground state of energy of current carriers. Then they calculate the transition of graphene on a substrate to a semimetallic state (Ratnikov et al. 2008b). The Ratnikov at al. theory can be applied to graphene in magnetic field and bilayer graphene in magnetic field. However, it is not immediately possible to apply the Ratnikov et al. theory to calculate the photoeffect in magnetic field. So, we use the Landau approach applied in Landau diamagnetism.

The quantum mechanical description of the photoeffect is presented in modern monographs as the nonrelativistic one, or relativistic one. We calculate the nonrelativistic process because the wave function of electron is described by the Schrödinger equation of motion. Instead of the Bloch functions in the periodic potential we use the very simple approximation including only the magnetic wave function for electrons moving in the 2-dimensional sheet. This is the main idea of the quantum mechanical description of the photoeffect the description by the appropriate S-matrix element involving the interaction of atom with the impinging photon with the simultaneous generation of the electron, which can be described

approximately by the plane wave

$$\psi_q = \frac{1}{\sqrt{V}} e^{i\vec{q}\cdot\vec{x}}, \quad \vec{q} = \frac{\vec{p}}{\hbar} \quad (7)$$

where p is the momentum of the ejected elektron. (We use text in the boldface expression for vectors). We suppose that magnetic field is applied locally to graphene, so in a sufficient distance from graphene the wave function is of the form of the plane wave (7). This situation has an analogue in the classical atomic effect discussed in a monograph of Davydov (1976). In other words, if the photon energy only just exceeds the ionization energy of atom, then we cannot use the plane wave approximation but the wave function of the continuous spectrum. The probability of the emission of electron by the electromagnetic wave is of the well-known form (Davydov, 1976)

$$dP = \frac{e^2 p}{8\pi^2 \epsilon_0 \hbar m \omega} \left| \int e^{i(\vec{k}-\vec{q})\cdot\vec{x}} (\vec{e} \cdot \vec{\nabla}) \psi_0 dx dy dz \right|^2 d\Omega = C |J|^2 d\Omega \quad (8)$$

where the interaction for absorption of the electromagnetic wave is normalized to one photon in the unit volume, e is the polarization of the impinging proton with the wave vector k , ϵ_0 is the dielectric constant of vacuum, ψ_0 is the basic state of an atom. We have denoted the integral in the $||$ by J and the constant before $||$ by C . We also supposed that the energy of impinging photon is sufficiently big that change of vector k during the desintegration of the pseudoelectron is very small. It is well-known that the description conductivity in graphene can be approximated in the tight-binding interaction Hamiltonian.

We consider here the case with electrons in magnetic field as an analog of the Landau diamagnetism. So, we take the basic function for one electron in the lowest Landau level, as

$$\psi_0 = \left(\frac{m\omega_c}{2\pi\hbar} \right)^{1/2} \exp\left(-\frac{m\omega_c}{4\hbar} (x^2 + y^2) \right), \quad \omega_c = \frac{|e|H}{mc} \quad (9)$$

which is the solution of the Schrödinger equation in the magnetic field with potentials $\vec{A} = (-Hy/2, -Hx/2, 0)$, $A_0 = 0$ (Drukarev, 1988)

$$\left[\frac{p_x^2}{2m} + \frac{p_y^2}{2m} - \frac{m}{2} \left(\frac{\omega_c}{2} \right)^2 (x^2 + y^2) \right] \psi = E \psi \quad (10)$$

We have supposed that the motion in the z-direction is zero and it means that the wave function does not depend on z. So, the main problem is to calculate the integral

$$J = \int e^{i\vec{k}\cdot\vec{x}} (\vec{e} \cdot \vec{\nabla}) \psi_0 dx dy dz \quad \vec{K} = \vec{k} - \vec{q} \tag{11}$$

with the basic Landau function being given by the equation (9). Operator $(\hbar/i)\vec{\nabla}$ is the Hermitean one and it means we can rewrite the last integrals as follows

$$J = \frac{i}{\hbar} \vec{e} \cdot \int \left[\left(\frac{\hbar}{i} \vec{\nabla} \right) e^{i\vec{k}\cdot\vec{x}} \right]^* \psi_0 dx dy dz, \tag{12}$$

which gives

$$J = i\vec{e} \cdot \vec{K} \int e^{-i\vec{k}\cdot\vec{x}} \psi_0 dx dy dz \tag{13}$$

The integral in equation (13) can be transformed to the cylindrical coordinates with

$$dx dy dz = \rho d\rho d\phi dz, \quad \rho^2 = x^2 + y^2 \tag{14}$$

which gives for vector K fixed on the axis z with $K_x = K_y = 0$ and with physical condition $\vec{e} \cdot \vec{k} = 0$, expressing the physical situation where polarization is perpendicular to the direction of the wave propagation. So,

$$J = (i)(\vec{e} \cdot \vec{q}) \int_0^\infty \rho d\rho \int_{-\infty}^\infty dz \int_0^{2\pi} d\phi e^{-iKz} \psi_0. \tag{15}$$

Using

$$\psi_0 = A \exp(-B\rho^2); \quad A = \left(\frac{m\omega_c}{2\pi\hbar} \right)^{1/2}; \quad B = \left(\frac{m\omega_c}{4\hbar} \right) \tag{16}$$

the integral (15) is now

$$J = (-\pi) (\vec{e} \cdot \vec{q}) \frac{A}{B} \int_0^\infty dz e^{-iKz} = (-\pi) \frac{A}{B} (\vec{e} \cdot \vec{q}) (2\pi) \delta(K) \tag{17}$$

Then,

$$dP = C |J|^2 d\Omega = 4\pi^2 \frac{A^2}{B^2} C (\vec{e} \cdot \vec{q})^2 \delta^2(K) d\Omega \tag{18}$$

Now, let be the angle Θ between direction \mathbf{k} and direction \mathbf{q} and let be the angle Φ between planes $(\mathbf{k}; \mathbf{q})$ and $(\mathbf{e}; \mathbf{k})$. Then,

$$(\vec{e} \cdot \vec{q})^2 = q^2 \sin^2 \Theta \cos^2 \Phi \tag{19}$$

So, the differential probability of the emission of photons from the double graphene in the strong magnetic field is as follows

$$dP = \frac{4e^2 p}{\pi \epsilon_0 m^2 \omega \omega_c} q^2 \sin^2 \Theta \cos^2 \Phi \delta^2(K) d\Omega; \quad \omega_c = \frac{|e| \hbar H}{mc} \tag{20}$$

We can see that our result differs from the result for the original photoelectric effect (Davydov, 1976) which involves still the term

$$\frac{1}{\left(1 - \frac{v}{c} \cos \Theta \right)^4} \tag{21}$$

which means that the most intensity of the graphenic photoeffect is in the direction of the electric vector of the electromagnetic wave ($\Theta = \pi/2, \Phi = 0$). While the nonrelativistic solution of the photoeffect in case of the Coulomb potential was performed by Stobbe (Stobbe, 1930) and the relativistic calculation by Sauter (Sauter, 1931) the graphenic magnetic photoeffect (with electrons moving in the magnetic field and forming magnetic atom) was not still performed in a such simple form. The delta-term represents the conservation law $|\mathbf{k} - \mathbf{q}| = 0$, in our approximation.

DISCUSSION

We have considered the photoeffect in the planar crystal at zero temperature and in the very strong magnetic field. We did not determine the atomic photoeffect on the graphene atoms which needs the knowledge of the wave function of electrons of graphene. We calculated only the process which can be approximated by the Schrödinger equation for electrons orbiting in magnetic field as an analog to the Landau diamagnetic approach.

Ratnikov et al. (2008a) defined graphene as a two-dimensional zero gap semiconductor with zero conduction and valence band overlap to determine ground state of energy of current carriers. Then they calculated the transition of graphene on a substrate to a semi-metallic state (Ratnikov et al., 2008b). The Ratnikov et al. theory can be applied to graphene in magnetic field and bilayer graphene in magnetic field. However, it is not immediately possible to apply the Ratnikov et al. theory to photoeffect in magnetic field with Landau simplicity.

We did not consider the relativistic description in the constant magnetic field. Such description can be realized using the so called Volkov solution of the Dirac equation in the magnetic field instead of the plane wave. The Volkov solution (Volkov, 1935) of the Dirac equation for an electron moving in a field of a plane wave was derived in articles of autor (Pardy, 2003, 2004) and in monograph of Berestetskii et al. (1989). The explicit form of such solution was used by Ritus (Ritus, 1979), Nikishov (1979) and others and by author (Pardy, 2003, 2004, 2007), in the different situation. For instance for the description of the electron in the laser field, synchrotron radiation, or in case of the massive photons leading to the Riccati equation (Pardy, 2004). The new experiments are necessary in order to verify the photoelectric equation in graphene. The photoelectric effect at zero temperature can be realized only by very short laser pulses, because in case of the continual laser irradiation the zero temperature state is not stable. Only very short pulses can conserve the zero temperature of 2D system.

The graphene can be deformed in such a way that the metric of the deformed sheet is the Riemann one (Cortijo et al., 2007). However, the Riemann metric is the metric of general relativity. So, there is the analogy between deformation and Einstein gravity, which is the Sakharov idea. The discussion on this approach was presented also by author (Pardy, 2005). The deformed graphene obviously leads to the modification of the photoeffect in graphene and it can be used as the introduction to the photoelectric effect influenced by the gravitational field. The information on the photoelectric effect in graphene and also the elementary particle interaction with graphene is necessary not only in the solid state physics, but also in the elementary particle physics in the big laboratories where graphene can form the substantial components of the particle detectors. The graphene can be probably used as the appropriate components in the solar elements, the anode and cathode surface in the electron microscope, or as the medium of the memory hard disks in the computers. While the last century economy growth was based on the Edison-Tesla electricity, the economy growth in this century will be obviously based on the graphene physics. We hope that these perspective ideas will be considered at the universities and in the physical laboratories.

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