

Short Communication

On a new branch of geometry

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Accepted 28 September, 2009

In this study, the following theorem was proved: There exists a spherical triangle whose interior angle sum is equal to two right angles. MSC: 51 M04, 08C99. PACS: 02.40.Dr, 02.40. Ky.

Key words: Euclid, elements, postulates, non-Euclidean geometries, algebraic applications.

INTRODUCTION

The fifth Euclidean postulate problem in geometry is 2300 years old. The investigations devoted to this problem gave birth to two consistent models of non- Euclidean geometries which are widely applied in quantum mechanics and general theory of relativity. Algebraic application is one of the powerful tools to study the properties of geometry. By assuming the laws of abstract algebra, the famous classical problems in geometry were shown impossible to solve. In this work, by applying classical algebra, the author attempts to establish the above mentioned theorem.

RESULTS

In the spherical construction as shown in Figure 1, small letters denote the sum of the interior angles in triangles and quadrilaterals. Let a, b, c, d, e, e, g, h, i, j, k, l, t, u, v', and w refer, respectively, to the sum of the interior angles in biangle AFED, triangle AES, triangle FOD, triangle AFR, quadrilateral FESO, triangle FRD, biangle ABCD, triangle OBD, triangle ASC, triangle RCD, quadrilateral BCSO, triangle ARC, triangle FAB, triangle FBC, triangle FCE and in biangle EDC. The angles ABC, BCD, CDE, DEF, EFA, FAB, AOB, ARC, ESC, AOR, ORS and RSD are all straight angles and so their measures are all equal to 180 degree.

Let v be the value of this 180 degree (1)

Using (1), $x + y + z + m = 5v + a$ (2)

$x + y + z = 3v + a$ (3)

$y + z + m = 3v + c$ (4)

$$x + y = v + d \quad (5)$$

$$y + z = v + e \quad (6)$$

$$m + z = 2v + f \quad (7)$$

$$5v + g = n + p + q + r \quad (8)$$

$$3v + h = n + p + q \quad (9)$$

$$3v + l = p + q + r \quad (10)$$

$$v + j = n + p \quad (11)$$

$$v + k = p + q \quad (12)$$

$$2v + l = r + q \quad (13)$$

$$x + r = 2v + t \quad (14)$$

$$2v + u = y + q \quad (15)$$

$$z + p = 2v + v' \quad (16)$$

$$2v + w = m + n \quad (17)$$

Adding (2) to (17), $4x + 4y + 6z + 2m + g + h + i + j + k + l + u + w = 4n + 4p + 6q + 2r + a + b + c + d + e + f + t + v'$

Applying (2), (3) in LHS and (8), (9) in RHS, $a + b + i + j + k + l + u + w + 2z = C + d + e + f + t + v' + g + h + 2q$

Assuming (1), $t + u + v' + w = 4v$

$$2v = a + g$$

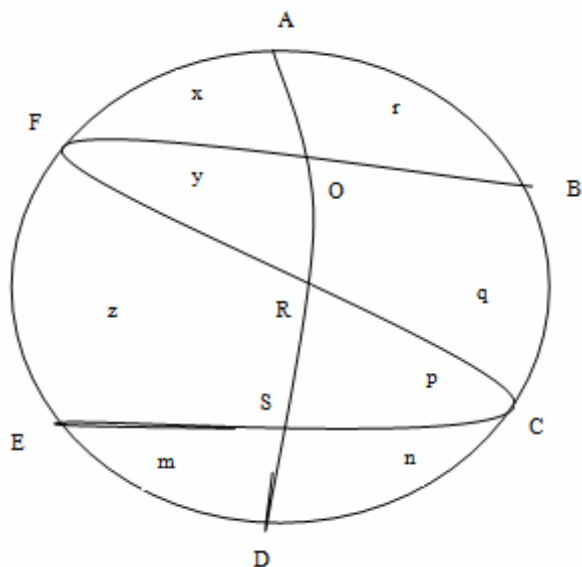


Figure 1. A spherical construction.

Adding the above three equations (eqns.), $2z + 2u + 2w + i + j + k + l = 2v + 2g + 2q + c + d + e + f + h$

$$3v + c = y + z + m \tag{4}$$

$$v + d = x + y \tag{5}$$

$$v + e = y + z \tag{6}$$

$$2v + f = m + z \tag{7}$$

$$3v + h = n + p + q \tag{9}$$

$$p + q + r = 3v + l \tag{10}$$

$$n + p = v = j \tag{11}$$

$$p + q = v + k \tag{12}$$

$$r + q = 2v + l \tag{13}$$

Adding the above ten eqns., $2v + 2u + 2w + 2p + 2r = x + 3y + z + 2m + 2g$

$$x + y + z + m = 5v + a \tag{2}$$

$$2y + 2q = 4v + 2u \text{ from } \tag{15}$$

$$6v + 2i = 2p + 2q + 2r \tag{10}$$

Adding the above four eqns., $2i + 2w = v + m + g$

(10) + (17) gives, $2p + 2q + 2r + 2m + 2n = 10v + 2i + 2w$

From (8) we have, $10v + 2g = 2n + 2p + 2q + 2r$

Adding the above three eqns., $m = v = 180 \text{ degree}$ [from (1)] (18)

That is, the sum of the interior angles of the spherical triangle ESD is equal to two right angles ----- (19) and hence the proof of the theorem.

DISCUSSION

Needless to say, we have derived (19) without assuming Euclid's parallel postulate. This establishes that the fifth Euclidean postulate can be deduced from the first four postulates (Effimov, 1972; Smilga, 1972). But the mere existence of consistent models of non-Euclidean geometries demonstrates that the fifth postulate is a special case. So this is a serious problem. Our application of number theory and the laws of algebra are consistent. Further studies will unlock the hidden treasures of mathematics.

ACKNOWLEDGMENT

The author wishes to thank the late professor of mathematics Palaniappan Kaliappan of Nallamuthu Gounder Mahalingam College, Pollachi, Tamil Nadu 642003, India for his kind encouragement.

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