Scientific Research and Essays Vol. 6(19), pp. 4125-4132, 8 September, 2011
Available online at http://www.academicjournals.org/SRE
DOI: 10.5897/SRE11.849
ISSN 1992-2248 ©2011 Academic Journals

Full Length Research Paper

An extended TOPSIS method for the multiple attribute group decision making problems based on intuitionistic linguistic numbers

Yan Du[1]* and Jia Zuo[2]

[1]Business Administration School, Shandong Economic University, Jinan Shandong 250014, China.
[2]Information Management School, Shandong Economic University, Jinan Shandong 250014, China.

Accepted 8 August, 2011

With respect to the multiple attribute group decision making problems in which the attribute weights are unknown and the attribute values take the form of the intuitionistic linguistic numbers, an expanded technique for order preference by similarity to ideal solution (TOPSIS) method is proposed. Firstly, the definition of intuitionistic linguistic number and the operational laws are given and distance between intuitionistic linguistic numbers is defined. Then, the attribute weights are determined based on the 'maximizing deviation method' and an extended TOPSIS method is developed to rank the alternatives. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Key words: The intuitionistic linguistic number; multiple attribute group decision making, maximizing deviation method, technique for order preference by similarity to ideal solution.

INTRODUCTION

There are many multi-criteria decision-making (MCDM) problems in real-life. Since the ambiguity of people's thinking and the complexity of objective things, most of these MCDM problems are called fuzzy multiple criteria decision-making (FMCDM) problems in which the ratings and the weights of these criteria are usually expressed by linguistic terms, fuzzy numbers or intuition fuzzy numbers (Atanassov, 1986, 1989; Ding and Chou, 2011; Liu, 2011; Liu and Su, 2010; Liu and Wang, 2011; Liu and Zhang, 2010, 2011a, 2011b; Liu et al., 2011; Paksoy et al., 2010; Wei, 2010; Xu, 2007). The intuitionistic fuzzy set (IFS) can deal with fuzzy information considering both the membership and non-membership of information. So, after its definition was given by Atanassov (1986, 1989), the intuitionistic fuzzy set theory has been developed rapidly, its theory and method have been widely considered from researchers. Later, Atanassov and Gargov (1989) and Atanassov (1994) further introduced the interval-valued intuitionistic fuzzy set (IVIFS) which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than crisp numbers. Xu (2007) investigated the interval-valued intuitionistic fuzzy MADM with the information about attribute weights are incompletely known or completely unknown, a method based on the ideal solution was proposed. Wang (2006) investigated the interval-valued intuitionistic fuzzy MADM with incompletely known weight information. A nonlinear programming model is developed, and ranking is performed through the comparison of the distances between the alternatives and idea/anti-idea alternative.

Shu et al. (2006) gave the definition and operational laws of intuitionistic triangular fuzzy number and proposed an algorithm of the intuitionistic fuzzy fault-tree analysis. Zhang and Liu (2010) used the triangular fuzzy number to denote the membership degree and the non-membership degree and proposed the triangular intuitionistic fuzzy number, then the weighted arithmetic averaging operator and the weighted arithmetic average operator was defined and an approach to multiple attribute group decision making (MAGDM) with triangular

*Corresponding author. E-mail:yandu18@gmail.com.

intuitionistic fuzzy information was developed. Wang (2008) gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic trapezoidal fuzzy number. Wang and Zhang (2008) gave the definition of expected values of intuitionistic trapezoidal fuzzy number and proposed the programming method of multi-criteria decision-making based on intuitionistic trapezoidal fuzzy number with incomplete certain information. Wang and Zhang (2009a) developed the Hamming distance of intuitionistic trapezoidal fuzzy numbers and intuitionistic trapezoidal fuzzy weighted arithmetic averaging (ITFWAA) operator, then proposed multi-criteria decision-making method with incomplete certain information based on intuitionistic trapezoidal fuzzy number. Wang and Zhang (2009b) proposed some aggregation operators, including intuitionistic trapezoidal fuzzy weighted arithmetic averaging operator and weighted geometric averaging operator and defined expected values of intuitionistic trapezoidal fuzzy number. Based on expected values, score function and accuracy function of intuitonitsic trapezoidal fuzzy numbers were defined and a ranking of the whole alternative set can be attained. Wan and Dong (2010) proposed the expectation and expectant score by the coordinates of gravity center of intuitionistic trapezoidal fuzzy number, and defined ordered weighted aggregation operator and hybrid aggregation operator for intuitionistic trapezoidal fuzzy numbers.

Finally, the results of group decision making were presented according to the expectation and expectant score. Wei (2010) proposed some aggregation operators including intuitionistic trapezoidal fuzzy ordered weighted averaging (ITFOWA) operator and intuitionistic trapezoidal fuzzy hybrid aggregation (ITFHA) operator and developed an approach to multiple attribute group decision making (MAGDM) with intuitionistic trapezoidal fuzzy information. However, intuitionistic fuzzy set, interval intuitionistic fuzzy set and fuzzy number intuitionistic fuzzy set can only roughly represent criteria's membership and non-membership to a particular concept, such as "good" and "bad", etc, they all have a great limitation (Wang and Li, 2009). At the same time, for people's decision-making process, sometimes it is difficult to give attribute values by quantitative measurement, especially for membership and non-membership degrees of the intuitionistic fuzzy sets and the fuzzy number intuitionistic fuzzy sets to give their quantitative data is difficult while to give linguistic assessment values is easy. However, for a linguistic assessment value, it usually implicit that membership degree is 1, the non-membership degree and hesitation degree of decision makers can not be expressed. On the basis of the intuitionistic fuzzy sets and linguistic assessment sets, Wang and Li (2009) proposed intuitionistic linguistic sets, intuitionistic linguistic numbers, intuitionistic two-semantics and the Hamming distance between two intuitionistic two-semantics and rank the alternatives by calculating the comprehensive membership degree to the ideal solution for each alternative. Intuitionistic linguistic set has a great theoretical significance and value of applications, for being able to represents the membership and non-membership.

As the research on intuitionistic linguistic set and its application to group decision making are relatively little, this paper defined the operational laws of the intuitionistic linguistic numbers, gave the approach for computing the attribute weights based on the 'maximizing deviation method' and an expanded TOPSIS method to rank the alternatives.

PRELIMINARIES

The intuitionistic fuzzy set

Definition 1

Let $X=\{x_1, x_2, \cdots, x_n\}$ be a universe of discourse, then a fuzzy set is:

$$A=\{< x, u_A(x) > x\in X\} \quad (1)$$

Which is characterized by membership function μ $u_A : X \to [0,1]$, where $u_A(x)$ μ indicates the membership degree of the element x to the set A (Zadeh, 1965). Atanassov (1986, 1989) extended the fuzzy set to the IFS and defined it as follows:

Definition 2

An IFS A in X is given by:

$$A=\{< x, u_A(x), v_A(x) > x\in X\} \quad (2)$$

Where $u_A : X \to [0,1]$ and $v_A : X \to [0,1]$, with the condition $0 \le u_A(x) + v_A(x) \le 1, \forall x\in X$. The numbers $u_A(x)$ and $v_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A.

For each IFS A in X, if $\pi(x) = 1 - u_A(x) - v_A(x)$, $\forall x\in X$, then $\pi(x)$ is called the degree of indeterminacy of x to the set A (Atanassov, 1986; 1989). It is obvious that $0 \le \pi(x) \le 1, \forall x\in X$. Let $A=\{< x, u_A(x), v_A(x) > x\in X\}$ and $B=\{< x, u_B(x), v_B(x) > x\in X\}$ be two IFS in the set X and $n \ge 0$, then the operations of IFSs are defined as follows (Atanassov, 1986; Xu, 2007):

$$A+B=\{<x,u_A(x)+u_B(x)-u_A(x)u_B(x),v_A(x)v_B(x)>x\in X\} \quad (3)$$

$$AB=\{<x,u_A(x)u_B(x),v_A(x)+v_B(x)-v_A(x)v_B(x)>x\in X\} \quad (4)$$

$$nA=\{<x,1-(1-u_A(x))^n,(v_A(x))^n>x\in X\} \quad (5)$$

$$A^n=\{<x,(u_A(x))^n,1-(1-v_A(x))^n>x\in X\} \quad (6)$$

The linguistic set and its extension

Suppose that $S=(s_1,s_2,\cdots,s_l)$ is a finite and totally ordered discrete term set, where l is the odd value. In real situation, l is equal to 3, 5, 7, 9 etc. For example, when l = 7, a set S could be given as follows:

$S=(s_1,s_2,s_3,s_4,s_5,s_6,s_7)$ = {very poor, poor, slightly poor, fair, slightly good, good, very good}.

Usually, for any linguistic set S, it requires that s_i and s_j must satisfy the following additional characteristics (Herrera et al., 1996; Herrera and Herrera-Viedma, 2000):

1) The set is ordered: $s_i \prec s_j$, if and only if $i<j$;
2) There is the negation operator: $neg(s_i)=s_{l-i}$;
3) Maximum operator: $\max(s_i,s_j)=s_i$, if $i\geq j$;
4) Minimum operator: $\min(s_i,s_j)=s_i$, if $i\leq j$;

For any linguistic set $S=(s_1,s_2,\cdots,s_l)$, the relationship between the element s_i and its subscript i is strictly monotone increasing (Herrera et al., 1996; Xu, 2006), so the function can be defined as follows:

$$f:s_i=f(i)$$

Obviously, the function $f(i)$ is the strictly monotone increasing function about subscript i. To preserve all the given information, the discrete linguistic label $S=(s_1,s_2,\cdots,s_l)$ is extended to a continuous linguistic label $\overline{S}=\{s_\alpha \mid \alpha\in R\}$ which satisfied the aforementioned characteristics. The operations are defined as follows (Xu, 2006):

$$\beta s_i=s_{\beta\times i} \quad (7)$$

$$s_i\oplus s_j=s_{i+j} \quad (8)$$

$$s_i/s_j=s_{i/j} \quad (9)$$

$$(s_i)^n=s_{i^n} \quad (10)$$

$$\lambda(s_i\oplus s_j)=\lambda s_i\oplus\lambda s_j \quad (11)$$

$$(\lambda_1+\lambda_2)\tilde{s}_i=\lambda_1 s_i\oplus\lambda_2 s_i \quad (12)$$

The intuitionistic linguistic set (ILS)

Definition 3

An ILS A in X is given by Wang and Li (2009):

$$A=\left\{<x[h_{\theta(x)},(u_A(x),v_A(x))]>| x\in X\right\} \quad (13)$$

Where $h_{\theta(x)}\in\overline{S}$, $u_A:X\to[0,1]$ and $v_A:X\to[0,1]$, with the condition $0\leq u_A(x)+v_A(x)\leq 1,\forall x\in X$. The numbers $u_A(x)$ and $v_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to linguistic index $h_{\theta(x)}$.

For each ILS A in X, if $\pi(x)=1-u_A(x)-v_A(x)$, $\forall x\in X$, then $\pi(x)$ is called the degree of indeterminacy of x to linguistic index $h_{\theta(x)}$. It is obvious that $0\leq\pi(x)\leq 1,\forall x\in X$.

Definition 4

Let $A=\left\{<x[h_{\theta(x)},(u_A(x),v_A(x))]>| x\in X\right\}$ be ILS, the ternary group $<h_{\theta(x)},(u_A(x),v_A(x))>$ is called an intuitionistic linguistic number and A can also be viewed as a collection of the intuitionistic linguistic number. So, it can also be expressed as (Wang and Li, 2009):

$$A=\left\{<h_{\theta(x)},(u_A(x),v_A(x))>| x\in X\right\}$$

In addition, $\pi_A(x)=1-u_A(x)-v_A(x)$ represents the degree of indeterminacy, and it can also be called the intuitionistic linguistic fuzzy degree.

Let $\tilde{a}_1=<s_{\theta(a_1)},(u(a_1),v(a_1))>$ and $\tilde{a}_2=<s_{\theta(a_2)},(u(a_2),v(a_2))>$ be two IL sets and $\lambda\geq 0$, then the operations of ILS are defined as follows (Wang and Li, 2009):

$$\tilde{a}_1 + \tilde{a}_2 =< s_{\theta(a_1)+\theta(a_2)}, (1-(1-u(a_1))(1-u(a_2)), v(a_1)v(a_2)) > \quad (14)$$

$$\tilde{a}_1 \otimes \tilde{a}_2 =< s_{\theta(a_1)\times\theta(a_2)}, (u(a_1)u(a_2), v(a_1)+v(a_2)-v(a_1)v(a_2)) > \quad (15)$$

$$\lambda\tilde{a}_1 =< s_{\lambda\times\theta(a_1)}, (1-(1-u(a_1))^{\lambda}, (v(a_1))^{\lambda}) > \quad (16)$$

$$\tilde{a}_1^{\ \lambda} =< s_{(\theta(a_1))^{\lambda}}, ((u(a_1))^{\lambda}, 1-(1-v(a_1))^{\lambda}) > \quad (17)$$

Theorem 1

For any two intuitionistic linguistic numbers $\tilde{a}_1 =< s_{\theta(a_1)}, (u(a_1), v(a_1)) >$ and $\tilde{a}_2 =< s_{\theta(a_2)}, (u(a_2), v(a_2)) >$, it can be proved the calculation 'rules' shown as follows:

$$\tilde{a}_1 + \tilde{a}_2 = \tilde{a}_2 + \tilde{a}_1 \quad (18)$$

$$\tilde{a}_1 \otimes \tilde{a}_2 = \tilde{a}_2 \otimes \tilde{a}_1 \quad (19)$$

$$\lambda(\tilde{a}_1 + \tilde{a}_2) = \lambda\tilde{a}_1 + \lambda\tilde{a}_2, \lambda \geq 0 \quad (20)$$

$$\lambda_1\tilde{a}_1 + \lambda_2\tilde{a}_1 = (\lambda_1 + \lambda_2)\tilde{a}_1,\ \lambda_1, \lambda_2\ \geq 0 \quad (21)$$

$$\tilde{a}_1^{\ \lambda_1} \otimes \tilde{a}_1^{\ \lambda_2} = (\tilde{a}_1)^{\lambda_1+\lambda_2},\ \lambda_1, \lambda_2\ \geq 0 \quad (22)$$

$$\tilde{a}_1^{\ \lambda_1} \otimes \tilde{a}_2^{\ \lambda_1} = (\tilde{a}_1 \otimes \tilde{a}_2)^{\lambda_1},\ \lambda_1 \geq 0 \quad (23)$$

The distance between two intuitionistic linguistic numbers

Definition 5

Let $\tilde{a}_1 =< s_{\theta(a_1)}, (u(a_1), v(a_1)) >$ $\tilde{a}_2 =< s_{\theta(a_2)}, (u(a_2), v(a_2)) >$ be intuitionistic linguistic numbers, $d(\tilde{a}_1, \tilde{a}_2)$ is called the distance between $\tilde{a}_1$ and $\tilde{a}_2$:

$$d(\tilde{a}_1,\tilde{a}_2) = |\theta(a_1)u(a_1) - \theta(a_2)u(a_2)| + |\theta(a_1)v(a_1) - \theta(a_2)v(a_2)| + |\theta(a_1)\pi(a_1) - \theta(a_2)\pi(a_2)|$$

Theorem 2

For any intuitionistic linguistic numbers: $\tilde{a}_1$, $\tilde{a}_2$ and $\tilde{a}_3$:

$$d(\tilde{a}_1, \tilde{a}_1) = 0 \quad (24)$$

$$d(\tilde{a}_1, \tilde{a}_2) = d(\tilde{a}_2, \tilde{a}_1) \quad (25)$$

$$d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_2, \tilde{a}_3) \geq d(\tilde{a}_1, \tilde{a}_3) \quad (26)$$

Proof

For Formula 24 and 25, obviously, they are right. For Formula 26:

$$\begin{aligned}
& d(\tilde{a}_1,\tilde{a}_2) + d(\tilde{a}_2,\tilde{a}_3) \\
&= |\theta(a_1)u(a_1) - \theta(a_2)u(a_2)| + |\theta(a_1)v(a_1) - \theta(a_2)v(a_2)| + |\theta(a_1)\pi(a_1) - \theta(a_2)\pi(a_2)| \\
&+ |\theta(a_2)u(a_2) - \theta(a_3)u(a_3)| + |\theta(a_2)v(a_2) - \theta(a_3)v(a_3)| + |\theta(a_2)\pi(a_2) - \theta(a_3)\pi(a_3)| \\
&\geq |\theta(a_1)u(a_1) - \theta(a_3)u(a_3)| + |\theta(a_1)v(a_1) - \theta(a_3)v(a_3)| + |\theta(a_1)\pi(a_1) - \theta(a_3)\pi(a_3)| \\
&= d(\tilde{a}_1,\tilde{a}_3)
\end{aligned}$$

So, $d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_2, \tilde{a}_3) \geq d(\tilde{a}_1, \tilde{a}_3)$.

TOPSIS METHOD FOR GROUP DECISION MAKING BASED ON INTUITIONISTIC LINGUISTIC NUMBERS

In a decision problem, let $E = \{e_1, e_2, \cdots, e_p\}$ be a set of experts, $A = (A_1, A_2, \cdots, A_{\mathrm{m}})$ be a discrete set of alternatives, $C = (C_1, C_2, \cdots, C_{\mathrm{n}})$ be the set of attributes, $W = (w_1, w_2, \cdots, w_n)^T$ be the weighting vector of the attributes, $\sum_{j=1}^{n} w_j = 1, w_j \geq 0$, ω is unknown. $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_p)$ is the weighting vector of the experts, $\sum_{k=1}^{p} \lambda_k = 1$. Suppose that $\tilde{R}^k = \left[\tilde{r}_{ij}^k\right]_{m\times n}$ is the decision matrix, where $\tilde{r}_{ij}^k = \left\langle a_{ij}^k, \left(u_{ijk}, v_{ijk}\right)\right\rangle$ takes the form of the intuitionistic linguistic number, given by the decision maker e_k, for alternative A_i with respect to attribute C_j, and $0 \leq u_{ijk} \leq 1, 0 \leq v_{ijk} \leq 1, u_{ijk} + v_{ijk} \leq 1$, $a_{ij}^k \in S$. The steps of ranking the alternatives based on these conditions are shown as follows:

Step 1: Make integrated matrix

Integrate the matrix $\tilde{R}^k = \left[\tilde{r}_{ij}^k\right]_{m\times n}$ given by decision maker e_k into the integrated matrix $\tilde{R}=\left[\tilde{r}_{ij}\right]_{m\times n}$:

$$\tilde{r}_{ij} = \sum_{k=1}^{p} \lambda_k \tilde{r}_{ij}^{k} \tag{27}$$

Where,

$$\tilde{r}_{ij} = \left\langle a_{ij}, \left(u_{ij}, v_{ij}\right)\right\rangle.$$

Step 2: Calculate attribute weights

The maximizing deviation method is proposed by Wang (1998) to deal with MADM problems with numerical information. For a MADM problem, if the attribute values of each alternative have little differences under an attribute, it shows that such an attribute plays a small important role in the priority procedure. Contrariwise, if some attribute makes the attribute values among all the alternatives have obvious differences, such an attribute plays an important role in choosing the best alternative. So to the view of sorting the alternatives, if one attribute has similar attribute values across alternatives, it should be assigned a small weight; otherwise, the attribute which makes larger deviations should be evaluated a bigger weight, in spite of the degree of its own importance. Especially, if the attribute values of all alternatives are all equal with respect to a given attribute, then such an attribute will be judged unimportant by most decision makers. In other word, such an attribute should be assigned a very small weight. Wang (1998) suggests that zero weight should be assigned to the corresponding attribute. For the attribute C_j, the deviation values of alternative A_i to all the other alternatives can be defined as $D_{ij}(w_j)=\sum_{l=1}^{m} d(\tilde{r}_{ij},\tilde{r}_{lj})w_j$, then $D_j(w_j)=\sum_{i=1}^{m} D_{ij}(w_j)=\sum_{i=1}^{m}\sum_{l=1}^{m} d(\tilde{r}_{ij},\tilde{r}_{lj})w_j$ represents the total deviation values of all alternatives to the other alternatives for the attribute C_j. $D(w_j)=\sum_{j=1}^{n} D_j(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(\tilde{r}_{ij},\tilde{r}_{lj})w_j$ represents the deviation of all attributes to all alternatives. The optimize model is constructed as follows:

$$\begin{cases} \max D(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(\tilde{r}_{ij},\tilde{r}_{lj})w_j \\ s.t \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0, j=1,2\ldots\ldots n \end{cases} \tag{28}$$

We can get:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(\tilde{r}_{ij},\tilde{r}_{lj})}{\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d^2(\tilde{r}_{ij},\tilde{r}_{lj})}}$$

Furthermore, we can get the normalized attribute weight based on this model:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(\tilde{r}_{ij},\tilde{r}_{lj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(\tilde{r}_{ij},\tilde{r}_{lj})} \tag{29}$$

Step 3: Use TOPSIS method to rank the alternatives

TOPSIS (technique for order preference by similarity to ideal solution) is proposed by Hwang and Yoon (1981), and it is a popular approach to MCDM problems. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution.

1) Make the weighted matrix:

$$\tilde{Y}=(\tilde{y}_{ij})_{m\times n}=\begin{bmatrix} w_1\tilde{r}_{11} & w_2\tilde{r}_{12} & \cdots & w_n\tilde{r}_{1n} \\ w_1\tilde{r}_{21} & w_2\tilde{r}_{22} & \ldots & w_n\tilde{r}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_1\tilde{r}_{m1} & w_2\tilde{r}_{m2} & \cdots & w_n\tilde{r}_{mn} \end{bmatrix} \tag{30}$$

Where,

$$\tilde{y}_{ij} = \left\langle b_{ij}, \left(u_{ij}^{'}, v_{ij}^{'}\right)\right\rangle$$

2) Determine the Ideal solution and negative ideal solution:

$$\begin{cases} \tilde{y}_j^{+} =< s_l, (1,0) > \\ \tilde{y}_j^{-} =< s_1, (0,1) > \end{cases} \quad j=1,2,\cdots,n \tag{31}$$

3) Calculate the distance between the alternative A_i and Ideal solution/negative ideal solution.

The distance between the alternative A_i and ideal solution/negative ideal solution is:

$$\begin{cases} d_i^{+} = \sum_{j=1}^{n} d(\tilde{y}_{ij}, \tilde{y}_j^{+}) \\ d_i^{-} = \sum_{j=1}^{n} d(\tilde{y}_{ij}, \tilde{y}_j^{-}) \end{cases} \quad i=1,2,\cdots,m \tag{32}$$

Table 1. Decision Matrix 1.

Parameters	(C_1)	(C_2)	(C_3)	(C_4)
A1	$\langle s_5,(0.2,0.7)\rangle$	$\langle s_2,(0.4,0.6)\rangle$]	$\langle s_5,(0.5,0.5)\rangle$	$\langle s_3,(0.2,0.6)\rangle$
A2	$\langle s_4,(0.4,0.6)\rangle$	$\langle s_5,(0.4,0.5)\rangle$	$\langle s_3,(0.1,0.8)\rangle$	$\langle s_4,(0.5,0.5)\rangle$
A3	$\langle s_3,(0.2,0.7)\rangle$	$\langle s_4,(0.2,0.7)\rangle$	$\langle s_4,(0.3,0.7)\rangle$	$\langle s_5,(0.2,0.7)\rangle$
A4	$\langle s_6,(0.5,0.4)\rangle$	$\langle s_2,(0.2,0.8)\rangle$	$\langle s_3,(0.2,0.6)\rangle$	$\langle s_3,(0.3,0.6)\rangle$

Table 2. Decision Matrix 2.

Parameters	(C_1)	(C_2)	(C_3)	(C_4)
A1	$\langle s_4,(0.1,0.7)\rangle$	$\langle s_3,(0.2,0.7)\rangle$	$\langle s_3,(0.2,0.8)\rangle$	$\langle s_6,(0.4,0.5)\rangle$
A2	$\langle s_5,(0.4,0.5)\rangle$	$\langle s_3,(0.3,0.6)\rangle$	$\langle s_4,(0.2,0.6)\rangle$	$\langle s_3,(0.2,0.7)\rangle$
A3	$\langle s_4,(0.2,0.6)\rangle$	$\langle s_4,(0.2,0.7)\rangle$	$\langle s_2,(0.4,0.6)\rangle$	$\langle s_3,(0.3,0.7)\rangle$
A4	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_4,(0.4,0.5)\rangle$	$\langle s_2,(0.3,0.6)\rangle$	$\langle s_4,(0.2,0.6)\rangle$

Table 3. Decision Matrix 3.

Parameters	(C_1)	(C_2)	(C_3)	(C_4)
A1	$\langle s_5,(0.2,0.6)\rangle$	$\langle s_3,(0.3,0.7)\rangle$	$\langle s_4,(0.4,0.5)\rangle$	$\langle s_4,(0.2,0.7)\rangle$
A2	$\langle s_4,(0.3,0.7)\rangle$	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_2,(0.1,0.8)\rangle$	$\langle s_3,(0.4,0.6)\rangle$
A3	$\langle s_4,(0.2,0.7)\rangle$	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_1,(0.1,0.8)\rangle$	$\langle s_4,(0.2,0.7)\rangle$
A4	$\langle s_3,(0.2,0.7)\rangle$	$\langle s_3,(0.1,0.7)\rangle$	$\langle s_4,(0.3,0.6)\rangle$	$\langle s_5,(0.4,0.5)\rangle$

4) Calculate the relative closeness coefficient:

$$C_i = \frac{d_i^+}{d_i^+ + d_i^-}.(i = 1, 2, \cdots, m) \quad (33)$$

5) Rank the alternatives.

Utilize the relative closeness coefficient to rank the alternatives. The smaller C_i is the better alternative is.

EXAMPLE

There are 4 companies (alternatives) $\{A_1, A_2, A_3, A_4\}$. Evaluate the companies from the perspective of technological innovation capability of enterprises, evaluating 'indicators' including: ability of resources into innovation (C_1), innovation management (C_2), innovativeness (C_3) and research and development capabilities (C_4). $\{e_1, e_2, e_3\}$ is the set of experts. $\lambda = (0.4, 0.32, 0.28)$ is the weighting vector of the experts. The evaluations of the 4 companies by the experts are listed in Tables 1 to 3.

The decision steps

1) Utilize the aggregation operator to integrate the matrix $\tilde{R}^k = \left[\tilde{R}_{ij}^k\right]_{m\times n}$, getting the integrated matrix:

$$\tilde{R}=\begin{bmatrix} <s_{4.68},(0.273,0.670)> & <s_{2.6},(0.251,0.658)> & <s_{3.80},(0.204,0.581)> & <s_{4.24},(0.319,0.591)> \\ <s_{4.32},(0.264,0.591)> & <s_{4.36},(0.258,0.558)> & <s_{3.04},(0.360,0.730)> & <s_{3.40},(0.204,0.586)> \\ <s_{3.60},(0.293,0.666)> & <s_{4.28},(0.231,0.670)> & <s_{2.52},(0.429,0.692)> & <s_{4.08},(0.308,0.700)> \\ <s_{4.84},(0.335,0.533)> & <s_{2.92},(0.418,0.663)> & <s_{2.96},(0.243,0.600)> & <s_{3.88},(0.191,0.570)> \end{bmatrix}$$

2) Calculate attribute weights according to Formula 29, we can get:

$$W=(0.191,0.294,0.238,0.277)^T$$

3) Calculate the weighted matrix, we can get:

4) Determine the ideal solution and negative ideal solution according to Formula 31:

$$\tilde{Y}=\begin{bmatrix} <s_{0.895},(0.780,0.926)> & <s_{0.764},(0.666,0.884)> & <s_{0.906},(0.685,0.879)> & <s_{1.491},(0.729,0.865)> \\ <s_{0.826},(0.775,0.904)> & <s_{1.281},(0.672,0.842)> & <s_{0.725},(0.784,0.928)> & <s_{1.403},(0.644,0.863)> \\ <s_{0.689},(0.791,0.925)> & <s_{1.258},(0.650,0.889)> & <s_{0.973},(0.817,0.916)> & <s_{1.475},(0.722,0.906)> \\ <s_{0.926},(0.811,0.887)> & <s_{0.858},(0.774,0.886)> & <s_{0.706},(0.714,0.885)> & <s_{1.455},(0.633,0.856)> \end{bmatrix}$$

$$\begin{cases} \tilde{y}_j^+ =< s_7,(1,0)> \\ \tilde{y}_j^- =< s_1,(0,1)> \end{cases} \quad j=1,2,\cdots,n$$

5) Calculate the distance between the alternative A_i and ideal solution/negative ideal solution according to Formula 32, we can get:

$$d_1^+=35.54504, d_2^+=35.76206, d_3^+=35.8979, d_4^+=35.46698$$

$$d_1^-=6.772948, d_2^-=6.869806, d_3^-=7.28789, d_4^-=6.734347$$

6) Calculate the relative closeness coefficient according to Formula 33, we can get:

$$C_1=0.839951, C_2=0.838858, C_3=0.831243, C_4=0.840423$$

From the aforementioned, we can get $C_4>C_1>C_2>C_3$, so the order of the alternatives is A3 $\succ$ A2 $\succ$ A1 $\succ$ A4.

RESULTS

In this paper, we firstly defined the distance between intuitionistic linguistic numbers, and proved its effectiveness, and then we gave a method for determining the weight based on the maximizing deviation principle and the distance formula which expanded the traditional maximizing deviation method to the intuitionistic linguistic numbers. Then we gave the definition of the ideal solution and negative ideal solution in the intuitionistic linguistic numbers for the multiple attribute decision making problems and then we gave the expanded TOPSIS method to rank the alternatives. Finally, we verified the developed approach by an illustrative example.

DISCUSSION

In order to further illustrate the effectiveness of the method proposed in this paper, we compare with method proposed by Wang and Li (2009) and we get the same order for the alternatives by using attribute weights calculated in this paper , it is A3 $\succ$ A2 $\succ$ A1 $\succ$ A4.The attribute values of the two methods take the form of the intuitionistic linguistic numbers, but the method in this paper can deal with the decision making problems in which attribute weights are unknown and method proposed by Wang and Li (2009) can only solve decision making problems in which attribute weights are real numbers. In addition, the method proposed in this paper is simpler.

Conclusion

An intuitionistic linguistic number is a generalization of intuitionistic fuzzy number and linguistic variables. This paper extends a linguistic variable to an intuitionistic linguistic number by defining the membership and non-membership of a linguistic variable. Then the operational laws of the intuitionistic linguistic numbers are given and the maximizing deviation method is used to calculate the attribute weights and the TOPSIS method is extended to rank the alternatives. Finally, an illustrative example has been given to show the steps of the developed method. It shows that this method is simple and easy to understand and it constantly enriches and develops the theory and method of MAGDM and proposed a new idea for solving the MAGDM problems. In the future, we shall continue working in the extension and application of the method.

ACKNOWLEDGMENTS

This paper is supported by the Humanities and Social Sciences Research Project of Ministry of Education of China (No.10YJA630073), the Natural Science Foundation of Shandong Province (No. ZR2011FM036) and Dr Foundation of Shandong Economic University. The authors also would like to express appreciation to the anonymous reviewers and journal editors for their very helpful comments on improving the paper.

REFERENCES

Atanassov KT (1986).Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1): 87-96.
Atanassov KT (1989). More on intuitionistic fuzzy sets. Fuzzy Sets and Systems, 33: 37-46.
Atanassov KT (1994). Operators over interval-valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 64:159-174.
Atanassov KT, Gargov G (1989). interval-valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 3: 343-349.
Ding JF, Chou CC (2011). A fuzzy MCDM model of service performance for container ports. Sci. Res. Essays., 6(3): 559-566.
Herrera F, Herrera-Viedma E (2000). Linguistic Decision Analysis: Steps for Solving Decision Problems Under Linguistic Information. Fuzzy Sets and Systems, 115(1): 67-82.
Herrera F, Herrera Viedma E, Verdegay JL (1996). A model of consensus in group decision making under linguistic assessments. Fuzzy Sets and Systems, 79(1): 73-87
Hwang CL, Yoon K (1981). Multiple Attributes Decision Making Methods and Applications, Springer, Berlin Heidelberg, pp. 32-45.
Liu PD (2011). A Weighted Aggregation Operators Multi-attribute Group Decision-making Method based on Interval-valued Trapezoidal Fuzzy Numbers. Expert Syst. Appl., 38 (1): 1053-1060.
Liu PD, Su Y (2010). The multiple attribute decision making method based on the TFLHOWA operator. Comput. Math. Appl., 60(9): 2609-2615
Liu PD, Wang MH (2011). An extended VIKOR method for multiple attribute group decision making based on generalized interval-valued trapezoidal fuzzy numbers. Sci. Re. Essays., 6(4): 760-765
Liu PD, Zhang X (2010). The Study on Multi-Attribute Decision-Making with Risk Based on Linguistic Variable. Int. J. Comput. Intell. Syst., 3(5): 601-609
Liu PD, Zhang X (2011a). Investigation into Evaluation of Agriculture Informatization Level Based on Two-tuple. Technol. Econ. Dev. Econ., 17(1): 74-86
Liu PD, Zhang X (2011b). Research on the Supplier Selection of Supply Chain Based on Entropy Weight and Improved ELECTRE-III Method. International J. Prod. Res., 49(3): 637-646.
Liu PD, Zhang X, Liu WL (2011).A Risk Evaluation Method for the High-Tech Project Investment Based on Uncertain Linguistic Variables. Technol. Forecast. Soc. Change., 78(1): 40-50
Paksoy T, Pehlivan NY, Ozceylan E (2010). Application of Fuzzy mathematical programming approach to the aggregate production/distribution planning in a supply chain network problem. Sci. Res. Essays, 5(22): 3384-3397.
Shu MH, Cheng CH, Chang JR (2006). Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly. Microelectronics Reliability, 46: 2139-2148
Wan SP, Dong JY (2010). Method of intuitionistic trapezoidal fuzzy number for multi-attribute group decision. Control and decision, 25: 773-776.
Wang JQ (2006). Multi-criteria interval intuitionistic fuzzy decision-making approach with incomplete certain information. Control and decision, 21: 1253-1256,1263
Wang JQ (2008). Overview on fuzzy multi-criteria decision-making approach. Control and decision, 23: 601-606.
Wang JQ, Li JJ (2009). The multi-criteria group decision making method based on multi-granularity intuitionistic two semantics. Sci. Technol. Inf., (33): 8-9.
Wang JQ, Zhang Z (2008). Programming method of multi-criteria decision-making based on intuitionistic fuzzy number with incomplete certain information. Control and decision, 23: 1145-1148.
Wang JQ, Zhang Z (2009a). Multi-criteria decision-making method with incomplete certain information based on intuitionistic fuzzy number. Control and decision, 24: 226-230.
Wang JQ, Zhang Z (2009b). Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems. J. Syst. Eng. Elect., 20: 321-326.
Wang YM (1998). Using the method of maximizing deviations to make decision for multi-indices, Syst. Eng. Elect., 7: 24-26, 31.
Wei GW (2010). .Some Arithmetic Aggregation Operators with Intuitionistic Trapezoidal Fuzzy Numbers and Their Application to Group Decision Making. J. Comput., 5: 345-351.
XU ZS (2006). Goal programming models for multiple attribute decision making under linguistic setting. J. Manage. Sci. in China, 9(2):9- 17.
Xu ZS (2007). Models for multiple attribute decision-making with intuitionistic fuzzy information. International Journal of Uncertainty. Fuzziness and Knowledge-Based Systems, 15:285-297.
Zhang X, Liu PD (2010). Method for aggregating triangular intuitionistic fuzzy information and its application to decision making.Technological and economic development of economy, 16: 280-290.
Zadeh L A (1965). Fuzzy sets. Information and Control, 8: 338- 356.