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An IMM algorithm based on augmented kalman filter for maneuvering target tracking

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In this paper, in order to increase the accuracy of interacting multiple model (IMM) algorithm in presence of low and medium maneuvers, a new IMM algorithm based on Augmented Kalman Filter (AUKF) has been proposed. The accuracy of the IMM algorithm depends upon having a set of filters with motion models which are similar at all times to the real target situations. One way to increase the accuracy of this estimator is to substitute more accurate filters instead of the Standard Kalman Filter (SKF) in it. In order to improve the performance of IMM algorithm, this paper proposes to replace each SKF in the IMM algorithm with the AUKF. Due to the better accuracy of the AUKF than SKF in presence of low and medium maneuvers, this substitution will improve the performance of IMM algorithm in these maneuvering levels. The Monte-Carlo simulation results show the accuracy of the proposed method.

Key words: Maneuvering target tracking, interacting multiple model (IMM), standard kalman filter (SKF), augmented kalman filter (AUKF).

INTRODUCTION

Standard kalman filter (SKF) is very popular for state estimation problems and provides good tracking accuracy in non-maneuvering mode (Lee et al., 2004, 2005; Bahari et al., 2011). However, it does not have desirable accuracy in presence of maneuver. There are two main approaches to reduce this problem (Lee et al., 2004): the Input Estimation (IE) approaches and the Multiple Model (MM) approaches.

THE IE APPROACH

The IE approach was developed, in order to aid the SKF to cope with the unknown target maneuvers. This approach was introduced by Chan et al. (1979) and includes two main steps. In the first step, the acceleration is estimated by applying the least-square estimation on the measurement residuals, and finally the estimated acceleration in conjunction with the SKF are used to produce new estimation. This approach was based on the constant input assumption and various literatures have developed it on the basis of constant velocity or acceleration consideration (Blair, 1993; Wang and Varshney, 1993). However, this method has not been successful because the real targets move with non-constant velocity or acceleration. To aid the IE approaches to cope with this trouble, Khaloozadeh and Karsaz (2009) have recently proposed a new SKF-based target tracker with IE approach. This method is an Augmented Kalman Filter (AUKF), and it has obtained lots of attention (Bahari and Pariz, 2009; Bahari et al., 2009, 2011; Beheshtipour and Khaloozadeh, 2009; Yang and Ji, 2010) due to the elimination of the constant input assumption in the previous IE approaches. The AUKF has a special form in the state space equation and estimates target acceleration along with the other states, simultaneously. This estimator has satisfactory tracking performance in low and medium maneuver levels.

THE MM APPROACH

The second approach to overcome the shortcoming of
SKF in presence of maneuver includes MM approaches. The IM algorithm is developed due to the behavior of a target cannot be specified by a single model precisely. Among different MM techniques (Krishnamurthy and Evans, 1998; Li and Bar-Shalom, 1996; Bar-Shalom et al., 1989) the Interacting Multiple Model (IM) algorithm is an interesting one to the excellent trade-off between complexity and performance (Mazor et al., 2000; Kim and Kim, 2007). This estimator includes the important areas of application such as: traffic forecasts (Zhang and Liu, 2009), Underwater Target Tracking (Xu et al., 2008), ground target tracking (Guo et al., 2008) and tactical ballistic missile tracking (Cooperman, 2002). However, the major requirement of this estimator is that it should have a set of filters with motion models similar to the real target situations at all times. The main weakness of this algorithm occurs when target undergoes an unknown acceleration levels and none of the existing elemental filters fit well. Therefore, in order to achieve good performance, a large number of models may be required to cover all system behavior patterns. This requirement is not reasonable in real-time applications, due to the lack of computational resources (Li and Bar-Shalom, 1996). Due to the modularity properties of the IM algorithm (Mazor et al., 2000), one way to cope with this trouble is to implement better filter into the IM algorithm instead of SKF (Li and He, 1999). Such substitutions can be found in different literatures (Wu and Cheng, 1994; Boers and Driessen, 2003; Jwo and Tseng, 2009). For example, in the work of Wu and Cheng (1994), a nonlinear IM algorithm has been proposed that are obtained by the replacement of the SKF in the IM algorithm with the Masreliez filter. This nonlinear IM algorithm can effectively handle maneuvering target tracking in presence of non-Gaussian observation noise.

By using the above facts, this paper proposes a new maneuvering tracking algorithm which combines the benefits of both AUKF and IM algorithms, simultaneously. In this method, the SKF in the IM algorithm are substituted with the AUKFs. By doing this substitution, the proposed AUKF-based IM (IMMAUKF) algorithm has the complexity comparable with that of the AUKF and the tracking accuracy with that of the IM algorithm. Due to the better accuracy of AUKF than SKF in presence of low and medium maneuvers, the proposed IMMAUKF algorithm yields better accuracy than conventional IM algorithm in these maneuvering levels.

THE PROBLEM STATEMENT

The state and measurement equations of a maneuvering target in a two-dimensional plane are described as follows:

\[ X(n+1) = FX(n) + Cu(n) + Gw(n) \]  
\[ z(n) = HX(n) + v(n) \]  

Where, \( X(n) \) is the state vector, \( z(n) \) is the observation vector, \( u(n) \) is the unknown acceleration vector, \( w(n) \) is the white system driving uncertainty, \( v(n) \) is the white observation uncertainty, \( Q(n) \) is the process covariance matrix, \( R(n) \) is the measurement covariance matrix.

The state and acceleration vectors are defined as:

\[ X(n) = [x(n) \ v_x(n) \ y(n) \ v_y(n)]^T \]

\[ u(n) = [a_x(n) \ a_y(n)]^T \]

In the aforementioned equations, \( F \), \( G \), \( C \) and \( H \) are:

\[
F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}, \quad C = \begin{bmatrix} 1 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \end{bmatrix}
\]

where, \( T \) is the time interval between two consecutive measurements. Due to the uncertainty in the acceleration vector, tracking a maneuvering target is very difficult. One successful method to cope with this trouble is the Augmented Kalman Filter (AUKF) which is described subsequently in ‘The AUKF algorithm’

THE AUKF ALGORITHM

Augmented kalman filter (AUKF) has been developed based on the combination of Fisher and Bayesian uncertainties (Khallowozadeh and Karsaz, 2009). This method considers the acceleration as an additive state term in the state space equation and estimates the acceleration and the original states simultaneously. This method is described as follows:

\[
\begin{bmatrix} X(n+1) \\ u(n+1) \end{bmatrix} = \begin{bmatrix} F & C \\ 0 & I \end{bmatrix} \begin{bmatrix} X(n) \\ u(n) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} w(n)
\]

\[ E\{v(n_i)v^T(n_j)\} = \begin{cases} R(n_i) \quad n_i = n_j \\ 0 \quad n_i \neq n_j \end{cases} \]

\[ E\{w(n_i)w^T(n_j)\} = \begin{cases} Q(n_i) \quad n_i = n_j \\ 0 \quad n_i \neq n_j \end{cases} \]

\[ E\{X(0)X^T(0)\} = \psi, E\{X(0)\} = 0 \]

\[ E\{v(0)\} = 0, E\{v(0)\} = 0 \]

\[ E\{v(0)w(0)^T\} = 0 \]
\[ z(n+1) = [HF \ HC] \begin{bmatrix} X(n) \\ u(n) \end{bmatrix} + HGw(n) + v(n+1) \] (4)

According to the equations (3) and (4), the new augmented state space and measurement equations can be derived as:

\[ X_{\text{Aug}}(n+1) = F_{\text{Aug}} X_{\text{Aug}}(n) + G_{\text{Aug}} W_{\text{Aug}}(n) \] (5)

\[ Z_{\text{Aug}}(n) = z(n+1) = H_{\text{Aug}} X_{\text{Aug}}(n) + V_{\text{Aug}}(n). \] (6)

where,

\[ X_{\text{Aug}}(n) = \begin{bmatrix} X(n) \\ u(n) \end{bmatrix}, F_{\text{Aug}} = \begin{bmatrix} F & C \\ 0 & I \end{bmatrix}, \quad G_{\text{Aug}} = \begin{bmatrix} G \\ 0 \end{bmatrix} \]

\[ W_{\text{Aug}} = w, H_{\text{Aug}} = [HF \ HC] \text{ and } V_{\text{Aug}}(n) = HGw(n) + v(n+1) \]

The optimal target maneuver estimator for the abovementioned augmented system is:

\[ \hat{X}_{\text{Aug}}(n+1 | n+1) = F_{\text{Aug}} \hat{X}_{\text{Aug}}(n | n) + K_{\text{Aug}}(n+1) \]

\[ [Z_{\text{Aug}}(n+1) - H_{\text{Aug}} F_{\text{Aug}} \hat{X}_{\text{Aug}}(n | n)] \] (7)

Where, the kalman gain is computed as:

\[ K_{\text{Aug}}(n+1) = [P_{\text{Aug}}(n+1 | n) H_{\text{Aug}}^T + G_{\text{Aug}} T_{\text{Aug}}(n)] R_{\text{Aug}}^{-1}(n+1) \] (8)

\[ P_{\text{Aug}}(n+1 | n+1) = P_{\text{Aug}}(n | n) - P_{\text{Aug}}(n+1 | n) H_{\text{Aug}} \times \]

\[ H_{\text{Aug}}^T H_{\text{Aug}}^{-1} H_{\text{Aug}} P_{\text{Aug}}(n+1 | n) \] (9)

\[ P_{\text{Aug}}(n+1 | n) = F_{\text{Aug}} P_{\text{Aug}}(n | n) F_{\text{Aug}}^T + \]

\[ G_{\text{Aug}} Q_{\text{Aug}}(n) G_{\text{Aug}}^T \] (10)

The augmented process noise covariance and the measurement noise covariance and the cross-covariance between them are calculated according to the following equations:

\[ Q_{\text{Aug}}(n) = E[W_{\text{Aug}}(n)W_{\text{Aug}}^T(n)]= \]

\[ E[w(n)w^T(n)] = Q \] (11)

\[ R_{\text{Aug}}(n) = E[V_{\text{Aug}}(n)V_{\text{Aug}}^T(n)]= \]

\[ HGQG^T H^T + R(n) \] (12)

\[ T_{\text{Aug}}(n) = E[W_{\text{Aug}}(n)V_{\text{Aug}}^T(n)]= QG^T H^T \] (13)

More details about AUKF can be found in (Khaloozadeh and Karsaz, 2009, Bahari et al., 2009a,b; Beheshtipour and Khaloozadeh, 2009; Yang and Ji, 2010).

**The IMMAUKF algorithm**

In many target tracking problems, it is difficult to describe the behavior of the target by using a single model completely. In order to cope with this trouble, Multiple Model (MM) approaches have been proposed. The MM approaches use a bank of filters, where each filter is matched to a specific target motion mode and the overall estimation is computed by the weighted sum of the estimates from each filter (Punithakumar et al., 2008). Among different MM approaches the Interacting Multiple Model (IMM) algorithm is an interesting one and it has applied to many important target tracking problems. Due to the modularity properties of the IMM algorithm, one major direction to improve the performance of this algorithm is to implement more accurate set of filters instead of the SKF in it (Li and He, 1999). By use of this fact, this paper proposes to substitute each SKF in the IMM algorithm with the AUKF. Due to the better accuracy of AUKF than SKF in presence of low and medium maneuvers, this substitution will improve the performance of IMM algorithm in these maneuvering situations. The proposed AUKF-based IMM (IMMAUKF) algorithm has the complexity comparable with that of the AUKF and the performance with that of the IMM algorithm. Figure 1 shows the structure of the proposed method.

Similar to the conventional IMM algorithm, the proposed IMMAUKF consists of four major steps: mixing, filtering, calculating of each mode probability, and state fusion (Bar-Shalom et al., 1989). One cycle of this method is summarized as follows:

**Step 1: Mixing of the estimates**

In this step, the previous cycle of estimated states by each filter and its corresponding error covariances are merged together. In the following equations, the \( \hat{X}_{\text{Aug}}^i(n-\|l\|-1) \) and \( \hat{p}_{\text{Aug}}^i(n-\|l\|-1) \) are the estimated state of the \( i \) th model (\( i=1,2,...,N \), \( N \) = number of models) and its corresponding error covariance, respectively.
These variables are obtained by use of the AUKF at time $n - 1$ after the measurement update. One cycle of this step is as follows:

$$
\mu_{(i|j)}(n-1|n-1) = \frac{1}{C_j} P_{(i|j)} \mu_i(n-1)
$$

(14)

$$
C_j = \sum_{i=1}^{N} P_{ij} \mu_i(n-1)
$$

(15)

$$
\hat{X}_{Aug}^j(n-1|n-1) = \sum_{i=1}^{N} \hat{X}_{Aug}^i(n-1|n-1) \mu_{(i|j)}(n-1|n-1)
$$

(16)

$$
\bar{P}_{Aug}(n-1|n-1) = \sum_{i=1}^{N} P_{ij}(n-1|n-1) (\hat{P}_{Aug}^i(n-1|n-1) +
(\hat{X}_{Aug}^j(n-1|n-1) - \hat{X}_{Aug}^i(n-1|n-1)) (\hat{X}_{Aug}^i(n-1|n-1) - \hat{X}_{Aug}^j(n-1|n-1))^T)
$$

(17)

In above equations, $\mu_{(i|j)}(n-1|n-1)$, $P_{ij}$, $C_j$, $\mu_i(n-1)$, $\hat{X}_{Aug}^j(n-1|n-1)$ and $\bar{P}_{Aug}(n-1|n-1)$ are mixing probability, model transition probability for switching from $i$ th model to $j$ th one, normalization constant, model probability of the $i$ th model at time $n-1$, mixed state estimate matched to $j$ th model and its corresponding mixed error covariance at time $n-1$, respectively.

**Step 2: Filtering algorithm**

The update equations for each model are performed according to the AUKF.

**Step 3: Update model probability**

The probability of the mode in effect is found by the likelihood function $\Lambda_j(n)$. This function and the update model probability of the $j$ th model, $\mu_j(n)$, are calculated as:

$$
A^i(n) = \frac{1}{\sqrt{2\pi S_i^{j}(n)}} \exp\left(-0.5(\bar{Z}_{Aug}^i(n))^T (S_i^{j}(n))^{-1} \bar{Z}_{Aug}^i(n)\right)
$$

(18)

$$
\mu_j(n) = \frac{\Lambda_j(n)C_j}{\sum_{j=1}^{N} \Lambda_j(n)C_j}
$$

(19)

Where, $\bar{Z}_{Aug}^i(n)$ and $S_i^{j}(n)$ are the measurement residual and residual covariance of the $j$ th model. These variables are calculated by AUKF in step 2.

**Step 4: Estimate fusion**

Finally, all the state estimates and the error covariance
from the individual filters are merged by the following equations:

\[
\hat{X}(n|n) = \sum_{j=1}^{N} X_{j}^{Aug}(n|n)\mu_{j}(n) \\
\hat{P}(n|n) = \sum_{j=1}^{N} \mu_{j}(n)\left(P_{Aug}(n|n) + [X_{j}^{Aug}(n|n) - \hat{X}(n|n)]^T\right] \\
\]

Where, \(X_{j}^{Aug}(n|n)\), \(P_{Aug}(n|n)\), \(\hat{X}(n|n)\) and \(\hat{P}(n|n)\) are the estimated state and the covariance of the \(j\)th model calculated in filtering algorithm (step 2), the combined state estimate and its corresponding error covariance, respectively.

**SIMULATION RESULTS**

In this section the efficiency of the proposed method is verified by simulation results. Conventional IMM algorithms use two or three models with different structures and noise levels to describe the behavior of the target. For the proposed IMMAUKF algorithm, two models have been considered: One AUKF model with low process noise corresponding to non-maneuvering mode \((M_1)\), and another AUKF model with high process noise corresponding to the maneuvering mode \((M_2)\).

The \((M_1)\) and \((M_2)\) have the following parameters:

**M_1**: The covariance matrices of \((M_1)\) and the elements of augmented state error matrix are \(Q_{M_1} = (0.1)^2 I\) and \(P_{Aug}^{M_1}(0,-1) = 10\). Also, the initial augmented state \(X_{Aug}^{M_1}(0,-1)\) is randomly selected.

**M_2**: The covariance matrices of \((M_2)\) and the elements of augmented state error matrix are \(Q_{M_2} = (15)^2 I\) and \(P_{Aug}^{M_2}(0,-1) = 10\). Also, the initial augmented state \(X_{Aug}^{M_2}(0,-1)\) is randomly selected.

The model transition probability matrix between \(M_1\) and \(M_2\) is considered as follows:

\[
P_{ij} = \begin{cases} 0.95 & \text{if } i = j \\ 0.05 & \text{otherwise} \end{cases}
\]

The accuracy of proposed IMMAUKF algorithm is compared with the conventional IMM algorithm with one CV model and CA model (IMM2) and Wang’s method (Wang and Varshney, 1993). The mathematical description for CV and CA models can be found in different literature (Yu et al., 2009). It should be noted that CV and CA models in the IMM2 algorithm has the same parameters and mode transition probabilities as \(M_1\) and \(M_2\), respectively.

In the following example, the simulation results are given to show the accuracy of the proposed method. This example has been chosen from (Khaloozadeh and Karsaz, 2009).

**EXAMPLE**

In this example, the target initial condition is considered as \(X(0) = \begin{bmatrix} -10 (m) & 20 (ms^{-1}) & 100 (m) & 15 (ms^{-1}) \end{bmatrix}^T\). For time interval \(0 \leq t \leq 100 s\) the target has the acceleration of \(u(t) = [0 g, 0 g]^T\), \(g = 9.8 ms^{-2}\) then it starts to maneuver with \(u(t) = [0.2 g, 0.3 g]^T\) for interval \(100 s < t \leq 300 s\). In this simulation, the sampling time is \(T = 1 s\) and the elements of the measurement noises are selected as \(R_{ij} = (100)^2 m^2\).

Figures 2 to 9 demonstrate the target range estimation and the corresponding errors of each method, target trajectory, errors of target position estimation in X and Y directions, speed trajectory, errors of target speed estimation in X and Y directions, target acceleration estimation in X and Y directions, target course estimation and the corresponding errors, and target azimuth estimation and the corresponding errors, respectively.

Table 1 shows the results of state estimation for the IMMAUKF and IMM2 algorithms, and Wang’s method. In this table, the Root Mean Square Error (RMSE) of different parameters has been computed with Monte-Carlo analysis of 100 runs.

**Conclusion**

In this paper, a new IMM algorithm based on AUKF has been introduced which combines the benefits of both AUKF and IMM algorithm, simultaneously. In this method, the SKFs in the conventional IMM algorithms are substituted with the AUKFs. By doing these substitutions, the proposed IMMAUKF algorithm has the complexity comparable with that of the AUKF and the tracking accuracy with that of the IMM algorithm. Consequently, due to the better accuracy of AUKF than SKF in presence of low and medium maneuvers, the proposed IMMAUKF algorithm yields better accuracy than conventional IMM algorithm in presence of these kinds of maneuvers. The Monte-Carlo simulation results show the effectiveness of the proposed method.
Figure 2. The actual value and estimation of the target range and the corresponding errors.

Figure 3. Target trajectory.
Figure 4. Errors of target position estimation in X and Y directions.

Figure 5. Speed trajectory.
Figure 6. Errors of target speed estimation in X and Y directions.

Figure 7. Errors of target acceleration estimation in X and Y directions.
Figure 8. Mode probability evaluator.

Figure 9. The actual value and estimation of the target Azimuth and the corresponding errors.
Table 1. The RMSE of the estimated parameters.

<table>
<thead>
<tr>
<th>Example</th>
<th>Error</th>
<th>Algorithm</th>
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<tr>
<td></td>
<td></td>
<td>IMMUKF</td>
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<tr>
<td></td>
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<td>Wang</td>
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<td></td>
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<td>IMP1</td>
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<td>Y-position(m)</td>
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<td>Low maneuver</td>
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<tr>
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<td>4.84</td>
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REFERENCES


