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Full Length Research Paper

Determination of the deflection of vertical components via GPS and leveling measurement: A case study of a GPS test network in Konya, Turkey

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Deflection of the vertical is used in reducing geodetic measurements related to geoid networks (vertical and horizontal directions observations and length measurement, etc.) to ellipsoid plane and in geoid modeling processes. Generally, it is obtained by Astro-Geodetic and gravimetric techniques. These techniques are very complex and time-consuming. Ellipsoidal coordinates of points are easily obtained thanks to the widespread use of Satellite Positioning Techniques (GNNS) such as GPS in geodesy. When the orthometric heights of points are determined via geometric leveling, geoid height differences and deflection of the vertical components can be measured faster and much more easily by using GPS and leveling measurements than the other techniques. This study discusses the calculation of the deflection of vertical components via GPS and leveling measurements. The deflection of the vertical components obtained from GPS and leveling measurements were compared with global (EGM96 and CG03C) and local (TG03) geoid models. Deflection of the vertical components ξ (north-south) and η (east-west) were calculated as ξ = -4.15" ± 0.61", η = 8.75" ± 0.69" via a GPS and leveling model; as ξ = -5.64", η = 1.95" via the EGM96 geoid model; as ξ = -4.85", η = 1.82" via the CG03C geoid model; as ξ = -7.47", η = -0.51" via the TG03 model; and as ξ = -3.9", η = 4.6" via Astro-Geodetic deflection of a vertical map of Turkey produced by Ayan (1976). When the values obtained from GPS and leveling measurements were compared with the values produced by the other techniques, the north-south component was found to be approximately consistent, while east-west component differed to same extent. Since very little data on the terrestrial gravity of Turkey was present in the EGM96 and CG03C global geoid models, it was not anticipated that the results obtained via these models would be comparable with other methods.

Key words: Deflection of the vertical, deflection of the vertical components, GPS/Leveling, EGM96, CG03C.

INTRODUCTION

Since the physical earth has a highly complex surface, measurements typically substitute simpler surfaces, in order to facilitate the evaluation of measurements and making of calculations. These surfaces are ellipsoid surfaces, which are defined in geometric terms and geoid surfaces, which are defined in physical terms and constitute one of the equapotential leveling surfaces. A geoid is a sea-level surface with homogenous gravity potential and is always vertical to the direction of the plumb line. It is a part of the leveling surface, which passes partially through the solid earth surface. The curvature of this surface exhibits discontinuity in the places where density changes suddenly. Therefore, this is not a simple analytical surface which can be easily defined in mathematical terms. In country measurements, the geodetic coordinates of points are calculated on an ellipsoid converging to the shape and size of the measurement area. On the other hand, measurements made of the physical earth surface by using measurement tools are related to the actual leveling surface passing though the point and the plumb line direction. An ellipsoid is a simple surface, defined in geometric terms. Although an ellipsoid is designed as a surface converging to a geoid, these two surfaces do not overlap. The difference between the two surfaces is called geoid height, symbolized by the letter N. As well as mapping purposes, practical problems (for instance, the direction of water flow) require geoid knowledge. The position of the geoid in relation to the reference surface can be determined via not only geoid height but also “deflection of the vertical” that can be converted in both directions. Geoid height and deflection of the vertical are two components of disturbed gravity fields.

Deflection of the vertical is defined as the angle between the geodetic zenith direction of a point and the local astronomical zenith direction (Gürkan, 1979). Deflection of the vertical is an important parameter of the local gravity field; it is therefore used in various fields, including the following:

(1) Transformation of astronomical coordinates into geodetic coordinates.
(2) Transformation of astronomical azimuth into geodetic azimuth.
(3) Reduction of horizontal and vertical angles to a spheroid (ellipsoid surface).
(4) Net calculations of geodetic networks: positioning of geodetic theodolites and leveling instruments according to the real vertical.
(5) Geoid detection: Current, high-level global geo-potential models have reached a level where they can be used in local applications. Geoid heights and deflection of the vertical components (ξ, η) can be calculated at a specific level of accuracy, depending on the local capacities of the model (whether or not it includes gravimetric data).
(6) Transformation from ellipsoidal heights to local (orthometric, normal) height systems: Satellite measurements are based on a geometric reference ellipsoid (WGS-84). However, continental heights are related to geoid. Since heights from the sea-level are preferred in practice, ellipsoidal heights have to be converted into geoid-based height systems.
(7) Geophysics studies: Deflection of the vertical and geoid heights are directly affected by the mass (density) distribution of the earth. Geodesists try to model the earth surface, while applied geophysicists use such data in exploration for crude oil, natural gas and mineral ores (Acar, 1999; Turgut and Acar, 2005).
(8) Deflection of the vertical (generally defined in terms of two components: north-south and east-west direction) is obtained via astro-geodetic and gravimetric techniques. Astro-geodetic technique adopts astronomical coordinates (Φ, Λ) and geodetic coordinates (φ, λ) while gravimetric technique is based on Stokes formula, using abnormalities of the earth’s gravitational field as the input data (Ayan, 1978; Arslan and Yılmaz, 2005).

In addition to the two techniques mentioned above, global geo-potential and local gravimetric models, as well as combined techniques (GPS-Leveling, GPS-Gravimetric etc.), can be used to obtain such figures. Global geoid models, such as CG03C and EGM96, are developed by using the gravitational information of the whole world. Geoid heights can be calculated by using the potential harmonic coefficients of the global geoid models. On the other hand, local geoid models vary, depending on the geodetic data resources of the country they are used in.

For regions with no gravity information, the orthometric heights obtained via geometric leveling and the ellipsoidal heights obtained via GPS can be used in combination. In previous studies carried out by Soler et al. (1989); Vandenberg (1999); Magilevsky and Melzer (1994); Acar and Turgut (2005); Tse and Iz (2006) and Akkul (2007), it was shown that traditional measurement techniques and new measurement techniques can be used in combination to calculate the deflection of the vertical components. The present study aimed to determine whether or not deflection of the vertical components of an existing Leveling Network, in the Konya Province of Turkey, could be calculated via GPS measurements and geometric leveling measurements. Konya province is a rich application area in terms of the large amount of leveling and GPS heights data produced by many previous studies and the potential to therefore calculate deflection of the vertical components using these data.

This study also aimed to introduce the techniques used in the calculation of the deflection of vertical components and to compare the deflection of the vertical components obtained via GPS/leveling in a selected test network with the values obtained via global geoid models (EGM96 and CG03C) and a local geoid (TG03) model.

Deflection of the vertical

Deflection of the vertical is the angular difference between the real plumb line direction and the normal to a surface (that is, “mathematical plumb line direction”) of a reference surface (Figure 1). Deflection of the vertical has two dimensions: north-south (ξ) and east-west (η) (Figure 1).

Calculation of deflection of the vertical components via GPS and geometric leveling

The relationship between geoid height and deflection of the vertical is presented in Figure 2.

The differential relationship between geoid height and deflection of the vertical is defined through the following formulae (Heiskanen and Moritz, 1984):

$$dN = -\varepsilon . ds \quad (1)$$

Or

$$\varepsilon = -\frac{dN}{ds} \quad (2)$$

Deflection of the vertical on any geodetic azimuth (α) direction can be calculated as follows, by using north-south and east-west components:

$$\varepsilon = \xi . \cos\alpha + \eta . \sin\alpha \quad (3)$$

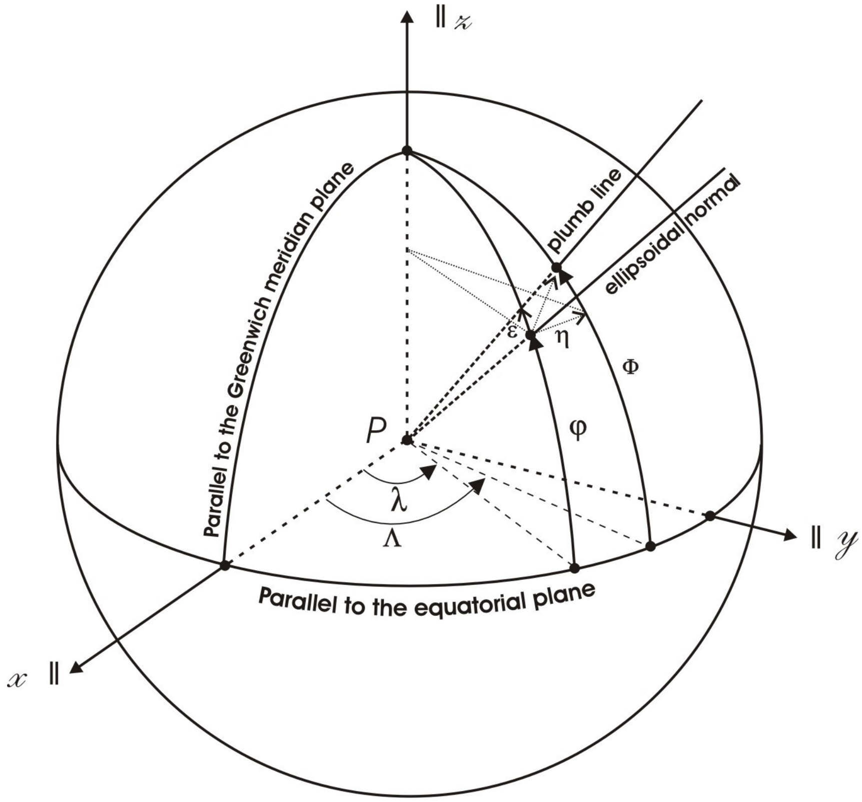

Figure 1. Deflection of the vertical and its components (Üstün, 2006).

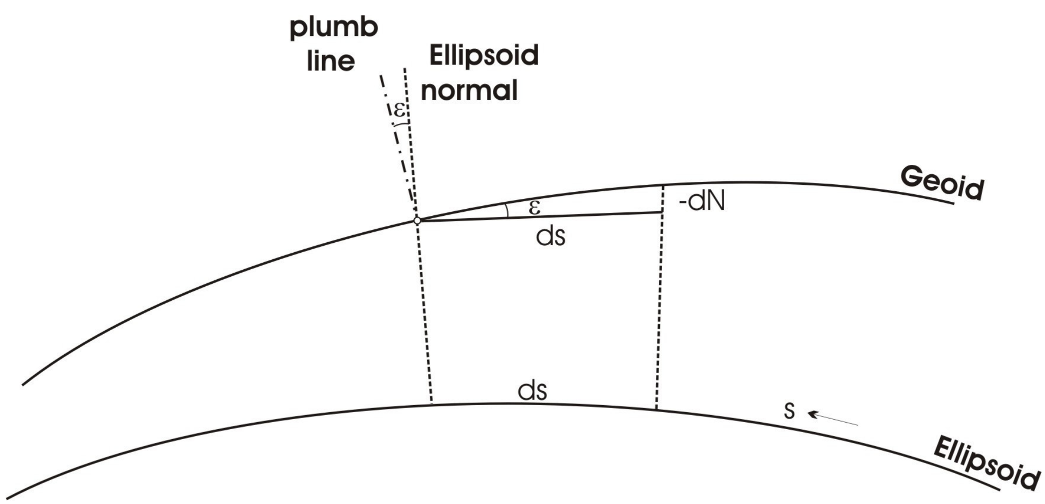

Figure 2. Relationship between geoid height and deflection of the vertical (Heiskanen and Moritz, 1984).

When formulae (2) and (3) are combined, following result is obtained;

$$-\frac{dN}{ds} = \xi.\cos\alpha + \eta.\sin\alpha \tag{4}$$

When the differential elements in formula (4) are replaced by the difference values obtained in geodetic measurements, the result will be as follows;

$$-\frac{\Delta N}{\Delta s} \approx \xi.\cos\alpha + \eta.\sin\alpha \tag{5}$$

Table 1. Standard deviation values.

σ_ε	$\sigma_{\Delta H} = \pm 1mm$ $\sigma_{\Delta h} = \pm 1mm$	$\sigma_{\Delta H} = \pm 5mm$ $\sigma_{\Delta h} = \pm 1mm$	$\sigma_{\Delta H} = \pm 5mm$ $\sigma_{\Delta h} = \pm 5mm$	$\sigma_{\Delta H} = \pm 10mm$ $\sigma_{\Delta h} = \pm 5mm$	$\sigma_{\Delta H} = \pm 10mm$ $\sigma_{\Delta h} = \pm 10mm$
S = 1 km	±0.29”	±1.05”	±1.46”	±2.31”	±2.92”
S = 2 km	±0,15”	±0,53”	±0,73”	±1,15”	±1.46”
S = 5 km	±0,06”	±0.21”	±0,29”	±0.46”	±0.58”
S = 10 km	±0,03”	±0,11”	±0,15”	±0.23”	±0.29”
S = 20 km	±0,01”	±0,05”	±0,07”	±0.12”	±0.15”

On the earth’s surface, for any A and B points close to each other, geoid heights are defined in terms of ellipsoidal (h) and orthometric heights (H), using the following formulae:

$$N_A = h_A - H_A \tag{6}$$

$$N_B = h_B - H_B \tag{7}$$

Subtraction of formula (6) from formula (7) gives the geoid height difference (ΔN_{AB}) between point A and point B, as follows:

$$\Delta N_{AB} = N_A - N_B = \Delta h_{AB} - \Delta H_{AB} \tag{8}$$

Finally, when formula (8) is inserted within formula (5), the following formula is obtained;

$$-\frac{\Delta h_{AB} - \Delta H_{AB}}{\Delta s} \approx \xi . \cos\alpha + \eta . \sin\alpha \tag{9}$$

The symbol ΔH, on the left side of this formula, refers to geometric leveling and the symbol Δh, on the right side of the same, refers to the values obtained from GPS measurements. In this case, formula (9) is a two-variable equation (ξ and η). In this equation, alpha can be calculated through geodetic coordinates measured in points A and B. In order to calculate the deflection of the vertical components for any point A, one requires secondary points, such as B and C. Deflection of the vertical components of point A can be calculated using the ellipsoidal and orthometric heights of point pairs of (A, B) and (A, C). In addition, it is also possible to calculate the deflection of the vertical components by solving the deflection of the vertical components of any point with the help of the values pertaining to three or more points scattered around the calculation point, and by using conditional measurement adjustment (Helmert) (Tse and Iz, 2006).

Ellipsoidal heights and orthometric height differences are assumed to be uncorrelated. When an error distribution rule is applied to formula (9), the second term in formula (10) is a quartic term and can be omitted.

$$\sigma_\varepsilon^2 = \frac{1}{\Delta s^2}\left(\sigma_{\Delta H}^2 + \sigma_{\Delta h}^2\right) + \left(\frac{\Delta h - \Delta H}{\Delta s^2}\right)^2 \sigma_{\Delta s}^2 \tag{10}$$

Therefore, the formula can be written as follows (Tse and Iz, 2006):

$$\sigma_\varepsilon^2 = \frac{1}{\Delta s^2}\left(\sigma_{\Delta H}^2 + \sigma_{\Delta h}^2\right) \tag{11}$$

Assuming that the systematic model errors of GPS and geometric leveling measurements are corrected, the measurement accuracy of both techniques will be relatively high. Theoretical standard deviation values calculated for sΔs = 1 km and different $\sigma_{\Delta H}$ and $\sigma_{\Delta h}$ are listed in Table 1.

As shown in Table 1, the standard deviation used in the calculation of deflection of the vertical was directly proportional to the measurement errors of GPS and geometric leveling measurements, and was inversely proportional to lengths. Moreover, the standard deviation values obtained were relatively low.

APPLICATION

Test network and its characteristics

The system was applied using a 15-point test network developed at the Alaaddin Keykubat Campus of Selçuk University (Konya, Turkey). The height of the network points was approximately 1100 m and the study field was a relatively flat surface (Figure 3).

Field studies and calculations

Height differences between the test network points were calculated via the geometric leveling technique, taking as the basis the leveling reference points of the national leveling network. Assuming that measurements were uncorrelated, orthometric height differences between the points were calculated via adjustment technique according to P = 1/S_{km} weight model. Adjusted height

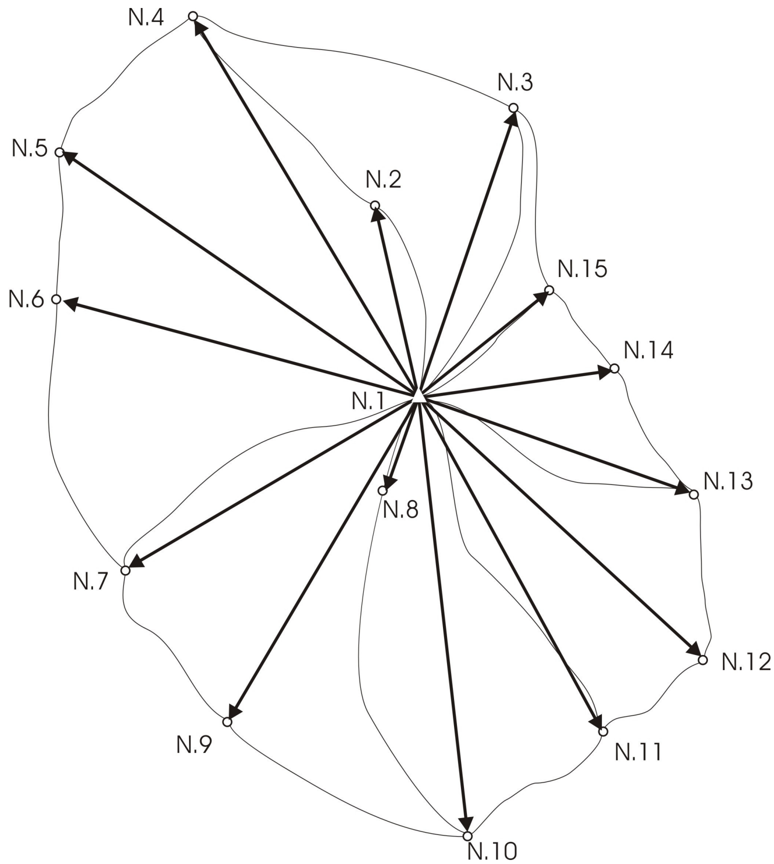

Figure 3. Test network.

Table 2. Orthometric and ellipsoidal height differences.

Length Number	From	To	Jeodezik		Orthometric height differences ΔH (m)	Ellipsoidal height differences Δh (m)
			Azimuth (α) (°)	Distances (s) (m)		
1	N.1	N.2	341,05925	1694,162	-19,907	-19,987
2		N.3	346,58740	2528,995	-44,584	-44,675
3		N.4	339,91027	3247,269	-69,275	-69,362
4		N.5	319,73764	2772,858	-100,514	-100,616
5		N.6	299,42439	1842,437	-81,225	-81,307
6		N.7	259,76721	1183,185	-38,025	-38,066
7		N.8	156,46449	485,699	0,690	0,698
8		N.9	181,83638	1202,129	-1,002	-0,987
9		N.10	151,27492	1970,697	37,714	37,806
10		N.11	127,17294	2210,365	50,332	50,436
11		N.12	107,93301	1910,508	50,077	50,166
12		N.13	91,93777	1928,868	48,002	48,096
13		N.14	75,91260	1590,138	18,778	18,837
14		N.15	59,94529	1257,791	12,701	12,732

differences are given in Table 2.

Two Topcon Pro GPS receivers were used in GPS calculations. One of the receivers was installed on TUTGA (SLCK) point in the campus area, while the other was placed on all points of the network in turn. 1 h sessions were held in static measurement mode.

Table 3. The deflection of vertical components.

Method.	The deflection of vertical components	
	ξ (North-South)	η (East-West)
GPS/Leveling	-4.15” ± 0.61	8.75” ± 0.69
EGM96	-5.64”	1.95”
CG03C	-4.85”	1.82”
TG03	-7.47” ± 1.81”	-0.51” ± 2.06”
Astro-Geodetic	-3.9”	4.6”

Ellipsoidal height differences obtained after evaluation of the measurements by using GPS evaluation software are given in Table 2.

By using the ellipsoidal coordinates obtained via geometric measurements, the length of the geodetic curve between the points and related azimuth values were calculated using the formula developed by Vincenty (1975).

Calculation of deflection of the vertical components

Using the orthometric and ellipsoidal height differences between point A and B, formula (9) was used to calculate the deflection of the vertical components:

$$\frac{1}{\Delta S_{AB}} V_{\Delta h_{AB}} - \frac{1}{\Delta S_{AB}} V_{\Delta H_{AB}} + \cos\alpha_{AB}.\xi + \sin\alpha_{AB}.\eta + \frac{\Delta h_{AB} - \Delta H_{AB}}{\Delta S_{AB}} = 0 \quad (12)$$

Formula (12) includes ellipsoidal and orthometric height differences and -as variables- two deflection of the vertical components. Deflection of the vertical components can be calculated by applying a variable-included conditional measurement adjustment (Helmert) approach to error equations (Tse and Iz, 2006):

$$\underline{A}^T \underline{V} + \underline{B}.\hat{\underline{X}} + \underline{W} = 0 \;;\; \underline{N}_1 = \underline{A}^T.\underline{P}^{-1}.\underline{A} \quad (13)$$

Here, normal equations are written as

$$\begin{aligned} \underline{N}_1.\underline{K} + \underline{B}.\underline{X} + \underline{W} &= 0 \\ \underline{B}^T.\underline{K} &= 0 \end{aligned}$$

Or, in matrix form, as

$$\begin{bmatrix} \underline{N}_1 & \underline{B} \\ \underline{B}^T & \underline{0} \end{bmatrix} . \begin{bmatrix} \underline{K} \\ \underline{X} \end{bmatrix} + \begin{bmatrix} \underline{W} \\ \underline{0} \end{bmatrix} = 0 \quad (14)$$

The following calculation steps are taken to solve the equation (Ulusoy, 1990):

Here, $\underline{A}$ refers to a 12 × 24 coefficients matrix; $\underline{V}$ refers to a 24 × 1 correction vector for ellipsoidal and orthometric height differences of the calculation points; $\underline{B}$ refers to a 12 × 2 design matrix; $\underline{X}$ refers to a 2×1 variable vector for deflection of the vertical components of the start point; and $\underline{W}$ refers to measurement a 12 × 1 closure vector. Since there are 12 conditional equations and 2 variable parameters, the solution has 10 degrees of freedom.

Using the method described, the deflections of the vertical components of the test point were found to be ξ = -4.15’’ ± 0.61’’, η = 8.75’’ ± 0.69’’.

Deflections of the vertical components of the test point were also calculated for EGM96 and CGO3C global models and the TG03 local geoid model. The results obtained are shown in Table 3.

As can be seen in Table 3, similar north-south component values were produced by the three techniques listed above. However, these techniques produced (to some extent) different values for the east-west component. Since there was very little data in the EGM96 and CG03C models about the geodetic gravity of Turkey, comparable results were not anticipated (When compared to geoid heights, values for deflection of the vertical were 10 times smaller in numerical terms). However, despite this situation, a significant approximation was recorded in the north-east component and a rough approximation in the other component. On the other hand, incompliance of the results produced in TG03 is related to the small distance between the points. On the other hand, incompliance between the results produced via TG03 is related to the short distance between the network points. It can be concluded, from Table 2, that the error of the geoid height differences should be small and the distance between them should be large if these differences are to be used in the calculation of the deflection of vertical components. In conclusion, the most reliable of these three techniques (geometric, global model and local model) is the GPS/ leveling technique.

Conclusion and Suggestions

This study presented functions for the detection of some physical geodetic figures facilitated by satellite positioning techniques (in line with the technological developments). The results of a case study were presented, to show calculation of the deflection of vertical components using ellipsoidal and orthometric height differences.

To detect the deflection of vertical components via GPS/Leveling data, a 15-point test network was developed at Alaaddin Keykubat Campus of Selçuk University (Konya, Turkey). Ellipsoidal coordinates of the test network were obtained via GPS measurements and the orthometric heights via a geometric leveling techni-que.

Deflection of the vertical components of the test network were calculated using geodetic height differences as the basis, with different heights (ΔN), geodetic azimuth (α) and geodetic curve lengths. Deflection of the vertical components (which were calculated on the basis of the measurements on 14 bases, the length of which varied between 485 m - 3250 km in different directions) were found to be “-4.15” ± 0.61” for the north-south component and “8.75 ± 0.69” for the east-west component. Considering the mean error values, it can be concluded that variable deflection of the vertical components of the point were calculated at a sufficient level of accuracy. In contrast, the accuracy of two other techniques used depended on the amount and accuracy of gravimetric data representing the area. In addition, the short distance between the points and the relatively flat study area makes it more difficult for the models to calculate the deflection of the vertical. The most reliable technique for comparison was the astro-geodetic technique, via which the deflections of vertical components were obtained. The values produced in the present study for the deflecttion of vertical components were found to approximate the corresponding values forecast for Konya in the “Astro-geodetic deflection of the vertical map of Turkey”, produced by Ayan (1976).

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