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Full Length Research Paper

Influence of bond parameters on deformation behaviour of reinforced concrete ties

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In this study, influence of the concrete-reinforcement bond on behaviour of the reinforced concrete tie is considered. The qualitative analysis of elongation and cracking of the tie is given. An original algorithm is suggested for modelling of the elongation of the tie under pure tension. The local behaviour of the concrete-reinforcement bond is approximated by a linear model whose parameters were estimated by minimizing relative error between experimental and calculated results. The global behaviour of the concrete and reinforcement for the analysis is assumed to be linear elastic. Performed assessment of the linear approximation showed that the accuracy of the suggested theoretical model depends on the geometry of the tie and length of the interval of the elongation in which the analysis is performed.

Key words: Concrete-reinforcement bond, cracking, stiffening, pure tension.

INTRODUCTION

It is well known that cracking and deformability of reinforced concrete (RC) depends not only on properties of reinforcement and concrete, but also on concrete-reinforcement bond. Thus, the bond between the concrete and reinforcement is a very important factor contributing to the behaviour of RC structures. The contribution may be dominant in evaluation of serviceability of RC members, especially in bending and tension.

Various working hypothesis are adopted to take into account the influence of the bond in analysis of the RC structures. These hypotheses should correspond to the actual behaviour of the bond undoubtedly. Therefore, understanding of the influence of the bond on behaviour of RC members is an important issue in investigation of RC structures.

The influence of the bond on the behaviour of RC elements can be examined in various ways. Popular numerical methods, analytical approaches and mixed semi-analytical methods are extensively applied. Numerical methods involve finite element (FEM) (Khalfallah and Hamimed, 2005; Khalfallah and Ouchenane, 2007; Lettow et al., 2004; Yu and Ruiz, 2006; Xiao et al., 2009; Zhao, 2011; Youai, 2000) and discrete element methods (Hentz et al., 2004; Wittel et al., 2006; Kim and Yun Mook, 2011). Large amount of data and time consuming post-processing are main disadvantages from the view point of design. Analytical methods involve ordinary differential equations (ODEs) and its exact solutions in explicit forms (Holmyanskii, 1981, 1997; Creazza and Russo, 1999, 2001; Russo and Pauletta, 2006; CEB Task Group 2.5, 2000).

Non-linear ODEs describing the bond may not have exact explicit solution. In this case, the non-linear ODEs and its numerical solutions belong to semi-analytical methods (Holmyanskii 1981, 1997; Russo and Pauletta, 2006). Kaklauskas et al. 2012, and Gribniak et al. 2010 and 2012 have proposed several analytical algorithms for evaluating tension stiffening of the flexural members. Their approaches are based on the experimental results and take into account concrete creep and shrinkage.

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There are several theoretical models, represented by ODEs, for RC elements, which take into account concrete-reinforcement interaction. As a rule, these ODEs are second order. ODEs of the one type are derived with respect to the slip of the reinforcement in concrete (displacement driven approach) (Holmyanskii, 1981, 1997; Creazza and Russo, 1999, 2001; Russo and Pauletta, 2006; CEB Task Group 2.5, 2000). The solution of this ODE is displacement of the reinforcement with respect to the concrete.

ODEs of another type, which can be used to investigate the influence of the concrete-reinforcement interaction, are derived with respect to the shear forces acting in the interface of different layers of a structure (Rzhanicyn, 1986). This method can be called force driven approach. Actually, these ODEs were derived for layered structures - build-up bars; however, they can be applied to investigate the influence of the concrete-reinforcement interaction as well. It should be noted that displacement driven approach has advantage over force driven approach. The former approach allows us to make very general non-linear ODE that involves an arbitrary law of concrete-reinforcement interaction. Such ODE was made by Russo and Pauletta (2006) and Rzhanicyn (1986). It should be emphasised that the exact analytical solution of the non-linear ODE for exponential law of bond is given by Russo and Pauletta (2006).

The present study is devoted to investigate the possibility of applying the assumptions of the linear elastic behaviour of concrete, reinforcement and bond on behaviour of the elongation of a centrically reinforced concrete tie, which is under pure tension. Also, the influence of the bond on deformation behaviour of the tie as well as cracking of the tie was considered. The analysis was conducted analytically. A second order linear ODE with constant coefficient is used as the basis of the analytical methodology (Rzhanicyn, 1986).

The present article is organised as follows: An analytical formulation and theoretical consideration of the problemis first presented, followed by a description of the RC tie under investigation. Thereafter results of analytical study and discussion on these results are presented, and finally, conclusions are drawn.

ANALYTICAL FORMULATION OF THE PROBLEM

Assumption of the model

Cracking and behaviour of the elongation of the tie subjected to pure static tension is investigated. A geometry and loading of the RC tie under investigation is schematically shown in Figure 1a. The RC tie under consideration is subjected by centrally applied external tensile force N_s^0. Contribution of shrinkage strains ε_{sh} of the concrete will be also taken into account. Geometry of the RC tie is characterised by the initial length l_0 and the square-shaped cross-section. The cross-section of the

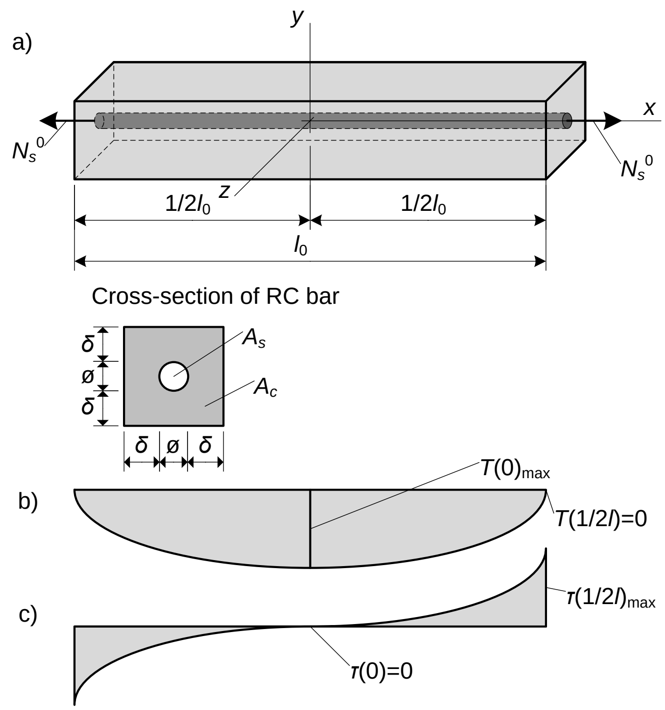

Figure 1. (a) Schematic view of RC tie and its cross-section, (b) distribution of total shear force $T(x)$, (c) distribution of shear force per unit length $\tau(x)$.

RC tie presents composition of the centrally located reinforcing bar with diameter ø and the concrete cover with thickness δ. Thus, the dimension of the cross-section is 2δ+ ø.

In the present article, the model of the build-up bars according to Rzhanicyn (1986) is adopted to investigate influence of the concrete-reinforcement interaction. This model is valid when following assumptions hold:

1. Concrete and reinforcement are isotropic and homogeneous materials in all their volume;
2. Concrete and reinforcement obey Hooke's law;
3. Relationship between stress and slip of the bond is linear;
4. Bond between reinforcement and concrete is distributed uniformly along the surface of the reinforcement;
5. Plane section hypothesis is valid separately for concrete and reinforcement cross-sections, respectively.

It is well known that, in general, concrete deforms nonlinearly. However, stress-strains relationship of the tensile concrete is closer to the linear elastic law than stress-strains relationship of the compressive concrete. Therefore, the 2nd assumption of linear elastic behaviour of the tensile concrete may serves as an initial approach to investigate the behaviour of the deformation of the RC tie under tensile forces. Mild steel reinforcement can be considered as linear elastic material up to yield point; therefore, 2nd assumption is valid for reinforcement if its stresses do not exceed yield stresses.

The third assumption also is not exact, since stress-slip relationship of the bond is apparently non-linear. However, up to the certain level of the stresses of the bond the stress-slip relationship may be assumed as linear elastic. In addition, the assumptions of the linearity of the stress-strain for the concrete and reinforcement, and linearity of the stress-slip for the bond allow us obtain an exact and explicit solutions of the stress-strain state of both cross-section and bond of the RC tie under consideration. The linear elastic solutions may be used as a first approach to investigate the influence of the interaction of the bond on behaviour of the RC tie under tension. Even though the assumptions about linearity of the bond are not exact, however this assumption is suitable for qualitative analysis of the RC tie.

The fifth assumption of uniform distributed bond between reinforcement and concrete is acceptable if reinforcing bar is long enough.

Governing equations

In the present article, the build-up bars model according to Rzhanicyn (1986) is adopted to investigate an influence of the concrete-reinforcement interaction. According to this model, the differential equation of the total shear force $T(x)$ acting in the bond is as follows:

$$\frac{1}{\xi}\frac{d^2T(x)}{dx^2} = \gamma T(x) + \Delta \tag{1}$$

where ξ is the stiffness or slip modulus of the bond between concrete and reinforcement. It should be noted that the stiffness ξ, in Equation 1, is tangent of an angle between x-axis and the line of the linear force-slip dependence of the bond. In addition, the stiffness ξ, in Equation 1, is related to the total perimeter of the cross-section of the reinforcing bars. Difference of the deformation Δ in Equation 1, is given in Equation 3; while compliance γ, comprising properties of the concrete and the reinforcement, is as follows:

$$\gamma = \frac{1}{E_sA_s} + \frac{1}{E_cA_c} \tag{2}$$

where E_s, E_c, A_s, and A_c are elastic moduli and areas of the cross sections of the reinforcing bars and concrete, respectively.

In Equation 1, the conditional difference of the deformation Δ is as follows:

$$\Delta = -\frac{N_s^0}{E_sA_s} + \frac{N_c^0}{E_cA_c} - \varepsilon_{sh} \tag{3}$$

where N_s^0 and N_c^0 are external axial forces, ε_{sh} is shrinkage strain of the concrete. In Equation 3, the external tensile forces N_s^0 and N_c^0 are assumed to be positive and the compressive forces N_s^0 and N_c^0 are assumed to be negative, the shrinkage strain ε_{sh} of the concrete is assumed to be positive. In addition, it is assumed, that forces N_s^0 and N_c^0 are applied to the centroids of the cross sections of the reinforcement and concrete, respectively. As can be seen from Equation 3, the term Δ takes into account shrinkage strain of the concrete.

The general solution of Equation 1 given by Polyanin and Zaitsev (2002) can be represented as follows:

$$T(x) = C_1\sinh(\lambda x) + C_2\cosh(\lambda x) - \Delta/\gamma \tag{4}$$

where $\lambda = \sqrt{\xi\gamma}$, C_1 and C_2 are integration constants depending on boundary conditions.

Assuming that the origin of the coordinate system $XOYZ$ is in the middle of the block of the RC tie (Figure 1a), then the boundary conditions are following:

(a) the total shear force $T(x)$ equals zero at the ends of the RC tie block, that is, at points $-1/2\cdot l$ and $1/2\cdot l$ (Figure 1b):

$$T(-1/2l) = T(1/2l) = 0 \tag{5}$$

(b) owing to symmetry of the RC tie, the shear force per unit length $\tau(x)$ equals zero at the middle of the block:

$$\tau(x)\big|_{x=0} = \left.\frac{\partial T(x)}{\partial x}\right|_{x=0} = 0 \tag{6}$$

Regarding Equations 5 and 6, we get integration constants C_1 and C_2:

$$C_1 = 0,\ C_2 = \frac{\Delta}{\gamma}\eta \tag{7}$$

where

$$\eta = \frac{1}{\cosh\left(\frac{1}{2}\lambda l\right)} \tag{8}$$

Finally, the total shear force $T(x)$ is as follows:

$$T(x) = \frac{\Delta}{\gamma}\left(\eta\cosh(\lambda x) - 1\right) \tag{9}$$

The normal stresses in the reinforcement bar σ_s and in the concrete σ_c are as follows:

$$\sigma_s(x) = \left(N_s^0 - T(x)\right)\Big/A_s \tag{10}$$

$$\sigma_c(x) = \left(N_c^0 + T(x)\right) / A_c \tag{11}$$

Negative sign of stresses corresponds to the compression while positive sign corresponds to the tension.

Theoretical analysis of the cracking of a reinforced tie

Let us consider cracking of a RC tie under a tensile force N_s^0 and shrinkage strains ε_{sh} of the concrete. We assume that $N_c^0 = 0$. At the beginning of the loading when $N_s^0 = 0$, the RC tie is deformed only due to shrinkage of the concrete. If the rigidity of the reinforcement is not too big with respect to the stiffness of the concrete cross-section, then no cracks appear in the RC tie due to shrinkage of the concrete. Equations 9 and 11 show that if $N_c^0 = 0$, then the stresses of the concrete σ_c are maximum in the middle of the uncracked block of the tie. Since the plane section hypothesis is valid, the maximum normal stresses σ_c appear in the whole cross-section of the concrete located in the middle of the uncracked block of the tie.

Increase of external force N_s^0 yields increase of the stresses of the concrete σ_c. By assuming simple tensile stress fracture criterion, initiation of the cracking is defined upon condition:

$$\sigma_c = f_{ctm} \tag{12}$$

where f_{ctm} is tensile strength of the concrete.

It is obvious that condition (Equation 12) corresponds to the crack opening through the entire cross-section of the RC tie. One crack appearing in the middle of the RC tie divides it into two new smaller blocks. These two new blocks are equal in length since maximum stresses σ_c appear in the middle of the uncracked block. If the tensile force N_s^0 increases further, it causes maximal stresses σ_c of the concrete in the middle of two new blocks. After cracking of two blocks, we have four blocks and three cracks: one old and two new. Further increasing of the tensile force N_s^0 splits all four blocks into 8 blocks and forms new four cracks. Now we have 7 cracks in total. The further increase of N_s^0 causes new cracks. The sequential cracking procedure could be continued until the yield limit of the reinforcement is reached. For clarity, the sequential cracking of the RC tie and corresponding stages of the cracking are shown in Figure 2.

Now, let us turn our attention to the cracking process of the RC tie or the blocks. Let an event when the new cracks appear in the RC tie be called the cracking stage and denoted by k. Introduced index k is used further for description of the sequential cracking. When RC tie is uncracked, then $k = 0$. The virgin stage is characterised by $k = 0$. When the new crack appears first time, $k = 1$, when the new cracks appear second time, $k = 2$. Opening of the further cracks increases k by 1.

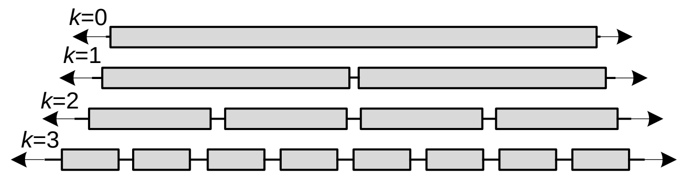

Figure 2. Schematic view of cracking of the RC tie at different stages, k.

If only one new crack appears in the each block of the RC tie, then the relationships between the cracking stage k and number of cracks $n_{crc,k}$ as well as number of blocks $n_{blc,k}$ are as follows:

$$n_{crc,k} = 2^k - 1 \tag{13}$$

$$n_{blc,k} = 2^k \tag{14}$$

A number of new cracks $n_{crc,new\ k}$ at k^{th} cracking stage is as follows:

$$n_{crc,new,k} = 2^{k-1} \tag{15}$$

As can be seen from Equations 13 to 15, the numbers of cracks, new cracks and blocks are power-law functions.

Let us consider ratio between number of the new cracks and total number of the cracks, that is $n_{crc,new,k} / n_{crc,k}$ at k cracking stage:

$$R(k) = \frac{n_{crc,new,k}}{n_{crc,k}} = \frac{2^{k-1}}{2^k - 1} = 0.5 + \frac{1}{2\left(2^k - 1\right)} \tag{16}$$

It is evident that the last member in Equation 16 tends to zero as k tends to infinity. Therefore, $R(k)$ in Equation 16 tends to 0.5 as k tends to infinity. This tendency is clearly illustrated by the graph depicted in Figure 3.

As can be seen from Figure 3, the ratio $R(k)$ tends to limit 0.5 very quickly. At fourth stage $R(k) = 0.53(3)$, at seventh stage $R(k) = 0.504$. Consequently, the number of new cracks, at big enough number k, can be calculated as follows:

$$n_{crc,new,k} \approx 0.5\, n_{crc,k} \tag{17}$$

Calculation of the cracking load and elongation of the reinforcement

Let us calculate the value of the external tensile forces $N^0_{s,crc}$ that causes opening of one crack through the whole cross-section of the concrete in the middle of the certain block of the RC tie. As already mentioned, it is assumed that $N_c^0 = 0$ and the crack opens though the

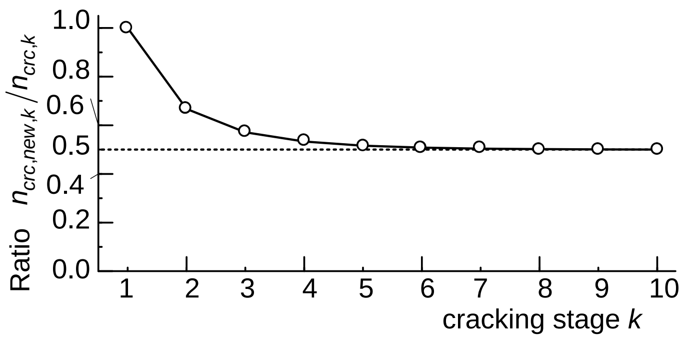

Figure 3. The dependence of the ratio ($n_{crc,new,k} / n_{crc,k}$) on the cracking stage, k.

whole cross-section of the concrete when $\sigma_c = f_{ctm}$. The stresses of the concrete σ_c are maximal when $x = 0$ in *XOYZ* frame of reference, that is, in the middle of the block. On the basis of the made assumptions, Equation 12 is as follows:

$$A_c f_{ctm} = T(x)\big|_{x=0} \tag{18}$$

By putting Equation 3 into Equation 9 and taking into account Equation 18, we get the cracking tensile force $N^0_{s,crc}$:

$$N^0_{s,crc} = E_s A_s \left(\frac{f_{ctm} A_c \gamma}{1-\eta} - \varepsilon_{sh} \right) \tag{19}$$

Having stresses of the reinforcement, Equation 10, the elongation of the reinforcement of the one (i^{th}) uncracked block under the load $N^0_{s,crc}$ can be calculated on the basis of the following relationship:

$$\Delta l_{s,i} = \int_{-1/2l}^{1/2l} \varepsilon_s(x) dx = \frac{1}{E_s} \int_{-1/2l}^{1/2l} \sigma_s(x) dx \tag{20}$$

After integration of Equation 20, we get the elongation $\Delta l_{s,i}$

$$\Delta l_{s,i} = \frac{1}{E_s A_s} \left(N^0_s l + \omega \right) \tag{21}$$

where $\omega = \frac{\Delta}{\gamma} \left(l - 2\frac{\eta}{\lambda} \sinh\left(\tfrac{1}{2}\lambda l\right) \right)$.

The total elongation of the cracked RC tie equal to the sum of all $\Delta l_{s,i}$ ($\Sigma \Delta l_{s,i}$). However, the elongation $\Delta l_{s,i}$ by Equation 21 takes into account shrinkage of the concrete. Therefore, at the initial of the loading, when applied force is small enough, $\Delta l_{s,i} < 0$ if $\varepsilon_{sh} > 0$, that is, if the concrete shrinks. Consequently $\Sigma \Delta l_{s,i} < 0$ as well. It does not meet experimental data that equal zero at the beginning of the loading. The calculated elongation $\Delta l_{s,i}$ will seem more natural if it also equals zero at the beginning of the loading. For this sake, the initial elongation $\Delta l_{s,0}$ of the reinforcement caused only by shrinkage of the concrete should be subtracted from the total elongation $\Sigma \Delta l_{s,i}$ of the RC tie.

The initial elongation $\Delta l_{s,0}$ can be calculated according to Equation 21 by taking conditions $N_s^0 = 0$ and $l = l_0$, where l_0 is an initial length of the uncracked RC tie. Taking into account these conditions and Equations 3 and 8, the elongation $\Delta l_{s,0}$ is as follows:

$$\Delta l_{s,0} = \frac{1}{E_s A_s} \frac{-\varepsilon_{sh}}{\gamma} \left(l_0 - \frac{2}{\lambda} \tanh\left(\tfrac{1}{2}\lambda l_0\right) \right) \tag{22}$$

If the RC tie is divided into $n_{blc,k}$ blocks, the total elongation of the reinforcement of the whole RC tie is as follows:

$$\Delta l_s = \left(\sum_{i=1}^{n_{blc,k}} \Delta l_{s,i} \right) - \Delta l_{s,0} \tag{23}$$

For clarity, the algorithm for calculation of the elongation of the reinforcement of the RC tie is given as follows:

I. $k := 0$; $n_{blc,k} := 1$; $l := l_0$, $N_{s,old}^0 := 0$.

II. **wile** $\sigma_s \leq f_y$:

1. Evaluation of $N^0_{s,crc}$ by (23).
2. **For** $p := 1$ to n_p **do**:
a. $N_s^0 := N_{s,old}^0 + p / n_p \, (N^0_{s,crc} - N_{s,old}^0)$.
b. $n_{blc,k} := 2^{(k-1)}$.
c. evaluation of Δl_s by (23).
3. $k := k + 1$
4. $l := l_0 / 2^k$.
5. $n_{blc,k} := 2^k$.
6. Evaluation of Δl_s by (23).
7. $N_{s,old}^0 := N_s^0$.

At the beginning (item I), it is assumed that the RC tie is uncracked. As can be seen from item II, the elongation Δl_s is calculated while stresses of the reinforcement do not exceed yield stresses f_y. In while loop (item II), the elongation of the reinforcement is calculated ($n_p + 1$) times: before appearing of the new cracks n_p times at different N_s^0 (item 2.) and one time after appearing of the new cracks (item 6). It should be noted, that the elongations Δl_s is calculated at the same cracking force $N^0_{s,crc}$ (item 1a) before new cracking (item 1c) and after it (item 6).

DESCRIPTION OF THE TESTING SAMPLE

The properties of the concrete and reinforcement as well as geometrical parameters of the RC tie under

Table 1. Main material parameters for modelling.

Parameter	Parameter symbol	Numeric value
Concrete		
Modulus of elasticity	E_c	28 GPa
Mean compressive strength	f_{cm}	35 MPa
Mean tensile strength	f_{ctm}	3.0 MPa
Reinforcing steel		
Modulus of elasticity	E_s	210 GPa
Yield stresses	f_Y	630 MPa
Diameter of reinforcement bar (mm)	$\varnothing$	16

Table 2. Parameters of the RC ties under investigation.

Type of the RC tie	Initial length l_0 (m) of the RC tie	Cover's thickness, δ (mm)	Multiplier β	Stiffness of bond ξ (GN/m)	Shrinkage strains of the concrete, ε_{sh} (10^{-3})
(1)	0.96	48, (3$\varnothing$)	Varies: $\beta \in [1, 10^3]$	Varies: $\xi \in [0.99, 990]$	0
(2)	0.96	Varies: $\delta \in \{16, 32, 48\}$ or $\delta \in \{1\varnothing, 2\varnothing, 3\varnothing\}$	Varies: $\beta \in [1, 10^3]$	Varies: $\xi \in [0.99, 990]$	0
(3)	0.96	48, (3$\varnothing$)	10^3	990	0
(4)	Varies: $l_0 \in \{0.96, 1.8, 2.4\}$	48, (3$\varnothing$)	10^3	990	0
(5)	Varies $l_0 \in [0.96, 2.5]$	48, (3$\varnothing$)	10^3	990	0
(6)	0.96	48, (3$\varnothing$)	Varies: $\beta \in \{10, 10^2, 10^3\}$	Varies: $\xi \in [9.9, 99.0, 990]$	0
(7)	0.96	48, (3$\varnothing$)	10^3	990	Varies: $_{sh} \in \{0, 0.1, 0.2, 0.3\}$

investigation were chosen in such a way that the results of analytical analysis could be compared with experimental results published in Elfgren and Noghabai (2001a, b).

To investigate an influence of the bond on the behaviour of the deformation of the RC tie under tensile force the following parameters were varied: thickness of concrete cover δ, stiffness of the bond ξ, initial length of the RC tie l_0, and shrinkage strain of the concrete ε_{sh}. The properties of used materials are given in Table 1. The types of the RC ties under examination and its parameters are given in Table 2.

Let us consider stiffness ξ of the bond in more detail. There are proposed a lot of stress-slip relationships for the bond. In the present article, the bond properties were adopted on the basis of the stress-slip relationship of the bond according to Model Code (MC, 1990) as follows:

$$\tau_{bnd}(s) = \tau_{bnd,\max}(s/s_1)^{\alpha} \text{ when } 0 \le s \le s_1 \qquad (24)$$

where s and s_1 are slip and slip corresponding to

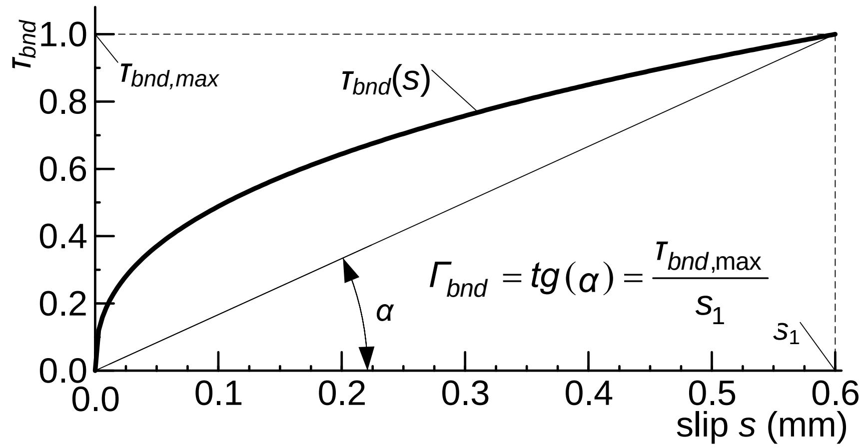

Figure 4. Stress-slip relationship for concrete-reinforcement bond.

the maximum stresses. The units of s and s_1 are mm. In Equation 24, $\tau_{bnd,max}$ is the maximal bond stresses. In the present article, the values of α, s_1, and $\tau_{bnd,max}$, in Equation 24, are adopted according to requirements of MC (1990) for unconfined concrete and good bond conditions: $\alpha = 0.4$, $s_1 = 0.6$, and $\tau_{bnd,max} = 2\sqrt{f_{ck}}$, where f_{ck} is characteristic strength of the concrete; $f_{ck} = f_{cm}$ - 8 MPa, where f_{cm} is mean strength of concrete. It should be emphasized that characteristic values of the concrete compressive strength f_{ck} points out to characteristic value of the $\tau_{bnd,max}$. However in MC (1990), this issue is not considered in detail.

It is assumed that the stress-slip relation of the bond is linear. Then stress of the bond τ_{bnd} can be expressed as follows: $\tau_{bnd}(s) = \Gamma_{bnd} \cdot s$, where Γ_{bnd} can be treated as slip modulus of the concrete-reinforcement bond per unit length. The unit of Γ_{bnd} is [Pa / m]. According to the linear law, the shear force per unit length $\tau(s) = \xi \cdot s$. The relationship between the bond stresses τ_{bnd} and the shear force per unit length τ is as follows: $\tau = P \cdot \tau_{bnd}$, where P is a perimeter of the bond zone. Then, taking into account relationships $\tau_{bnd}(s) = \Gamma_{bnd} \cdot s$, $\tau(s) = \xi \cdot s$, and $\tau = P \cdot \tau_{bnd}$, we get $\xi \cdot s = P \cdot \Gamma_{bnd} \cdot s$, which yield $\xi = P \cdot \Gamma_{bnd}$. Term Γ_{bnd} can be also considered as an initial tangent or secant modulus derived from a stress-slip relationship of the bond. Slip derivative of $\tau_{bnd}(s)$, Equation 24, equals infinity at point $s = 0$. Secant modulus of the stress-slip relationship, Equation 24, at s_1 is $\tau_{bnd,max}/s_1$. For more clarity, the stress-slip relationship for concrete-reinforcement bond, by Equation 24, and secant modulus are shown in Figure 4.

Theoretically, the stiffness ξ can vary between $P \cdot \tau_{bnd,max}/s_1$ and infinity. To investigate the influence of the stiffness ξ on the elongation of the reinforcement, we assume that ξ may vary within interval $[(P \cdot \tau_{bnd,max}/s_1), \infty]$. For given parameters of the concrete and reinforcement (Table 1) the bond properties are following:

Maximal stresses of the bond $\tau_{bnd,max} = 2\sqrt{f_{cm}} = 2\sqrt{35} =$ 11.83 MPa.

Slip modulus per unit length $\Gamma_{bnd} = \tau_{bnd,max}/s_1 = 11.83 \cdot 10^6 / 0.6 \cdot 10^{-3} = 19.72$ GPa.

Perimeter of the bond zone of reinforcement cross-section $P = 5.027$ cm^2.

Since the stiffness ξ is unknown, it would be more convenient to introduce a multiplier β that shows how many times the stiffness ξ is greater or lover than its basic value $P \cdot \Gamma_{bnd}$. Finally, for adopted cross-section, the stiffness of the bond is as follows: $\xi = P \cdot \beta \cdot \Gamma_{bnd}$, where β is a multiplier. The properties of the bond, the parameters β and ξ, for adopted cross-sections are given in Table 2.

MODELLING RESULTS

Assessment of accuracy of the theoretical modelling

The stiffness ξ of the bond for certain RC prism under pure tension is assessed hereafter on the basis of the experimental results that were presented in Elfgren and Noghabai (2001a, b). Due to non-linear stress-slip relation of the bond, the stiffness ξ is unknown and may vary within interval $[(P \cdot \tau_{bnd,max}/s_1), \infty]$. Let us estimate the stiffness ξ by minimizing the maximal relative difference:

$$d_m = \max\left|\frac{N^0_{s,exp}(\Delta l_s) - N^0_{s,clc}(\Delta l_s)}{N^0_{s,exp}(\Delta l_s)}\right|,\ \Delta l_s \in [0, a] \qquad (25)$$

where $N^0_{s,exp}$ and $N^0_{s,clc}$ are experimental and calculated axial forces of the reinforcement. In other words, we are seeking such a value of the stiffness ξ, with which the relative difference is minimum in certain interval $[0, a]$. The value of the term a may be chosen in accordance with the experimental data. The properties of the materials for RC tie under review are given in Table 1, the RC ties' types are (1) and (2) (Table 2).

As already mentioned, it is convenient to use the multiplier β to characterise how many times the stiffness ξ is greater than basic value $P \cdot \Gamma_{bnd}$. Therefore, the influence

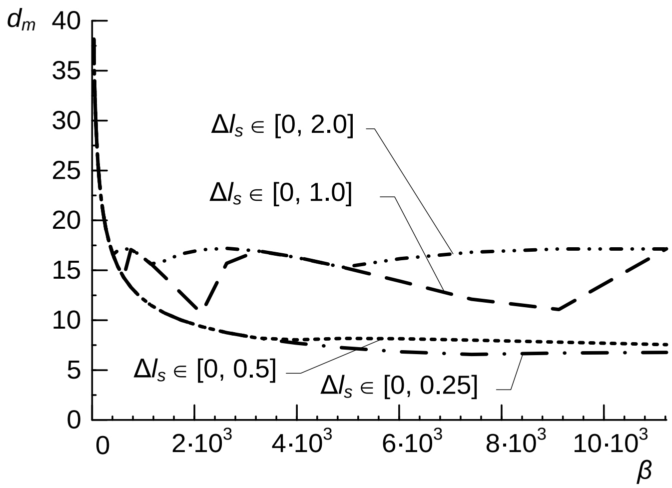

Figure 5. The dependence of the calculated relative difference dm on multiplier β at different intervals [0, a], type of the RC tie is (1).

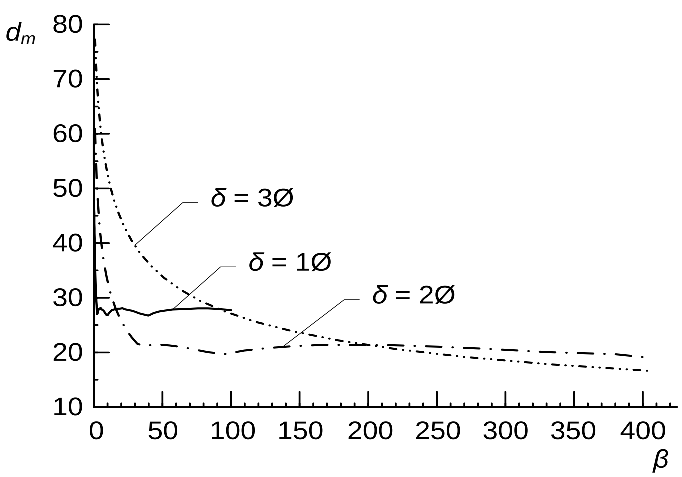

Figure 6. The dependence of the calculated relative difference d_m on multiplier β at different cover thickness δ, when $\Delta l_s \in [0, 2.0]$ mm, type of the tie is (2).

of the stiffness ξ on the relative difference d_m is presented by multiplier β which is a multiplier in the relation $\xi = \beta{\cdot}P{\cdot}\Gamma_{bnd}$. This influence for different intervals [0, a], $a \in \{0.25, 0.5, 1.0, 2.0\}$ mm, as $\delta = 3Ø$ (RC tie's type is (1)), and different cover thicknesses $\delta \in \{1Ø, 2Ø, 3Ø\}$, as $\Delta l_s \in [0, 2.0]$ mm (RC tie's type is (2)) is depicted in Figures 5 and 6. As can be seen from these figures, d_m decreases with increasing β. In general, we cannot claim that limits of d_m exist as β tends to infinity. However, it is possible to claim that variation of d_m is not significant at a certain value of β.

Let β^* be a value at which the variation of d_m is not significant in the interval [β^*, ∞] conditionally. Then, as can be seen from Figure 5, the variation of d_m, in the interval [β^*, ∞], is bigger for narrow interval of Δl_s, $\Delta l_s \in [0, a]$, or in other words, for small values of a (except interval [0, 1.0]). And vice versa, variation of d_m is smaller for bigger value of a. Since the slip modulus $\Gamma_{bnd} \to \infty$ as $s \to 0$, then $\beta^* \to \infty$ as $a \to 0$.

As can be seen from Figure 6, the variation of d_m in certain interval [β^*, ∞], is smaller for thinner covers than for thicker. For instance, from Figure 6, it can be seen that variation of d_m is not significant in interval $\beta \in [\beta^*, \infty]$ when $\beta^* = 20$ for cover 1Ø, and when $\beta^* = 150$ for cover 2Ø. Our analysis showed that variation of d_m is not significant when $\beta^* = 1000$ for cover 3Ø. From Figure 6, it can also be seen that β^* is less for thick covers than for thin. Since $\xi = \beta^*{\cdot}P{\cdot}\Gamma_{bnd}$ then, on the basis of the performed analysis, it is possible to claim that for given RC tie the stiffness ξ decreases with decreasing the thickness of the cover and the width of the interval $\Delta l_s \in [0, a]$.

Performed assessment showed that stiffness ξ of the bond depends not only on the properties of the reinforcement but also on the thickness of the cover and the width of the interval of the elongation Δl_s in which the elongation of the reinforcement is considered.

Analysis of influence of various factors on elongation of the RC tie

An analysis of influence of various factors on the cracking and elongation of the RC tie is considered using the methodology given earlier. These factors are: the initial length l_0, the shrinkage strains of the concrete, and the stiffness of the bond or value of the multiplier β. Types of the RC ties under investigation are shown in Table 2, materials properties of these ties are given in Table 1.

Let us, at the beginning, consider experimental and calculated N_s^0–($\Delta l_s / l_0$) curves, based on performed analysis of the stiffness ξ (Figure 7). The type of the RC tie is (3) (Table 2). It should be noted that the experimental $N^0_{s,exp}$–($\Delta l_s / l_0$) curve shown in Figure 7 is drawn schematically by smoothing the experimental data given by Elfgren and Noghabai (2001a, b) and the curve shows only characteristics feature of behaviour of the RC tie. In Figure 7, void circular points A, F, G to J, and shaded diamond-like points A^* and B^* denote crack formation phases of the calculated ant experimental N_s^0–($\Delta l_s / l_0$) curves, respectively. Symbol k denotes the cracking stages. It should be noted that in the present article, the term cracking stage denotes an event at which the new cracks form at a load $N^0_{s,crc}$, while the term crack formation phase denotes the process which may involve several cracking stages; which is followed by stable deformation phase. The crack formation phase may consist of one cracking stage as well. For example, in Figure 7, the part of the curve between points A and F is the first crack formation phase involving 5 cracking stage, while the part between points G and H is the second crack formation phase involving the only 6th cracking

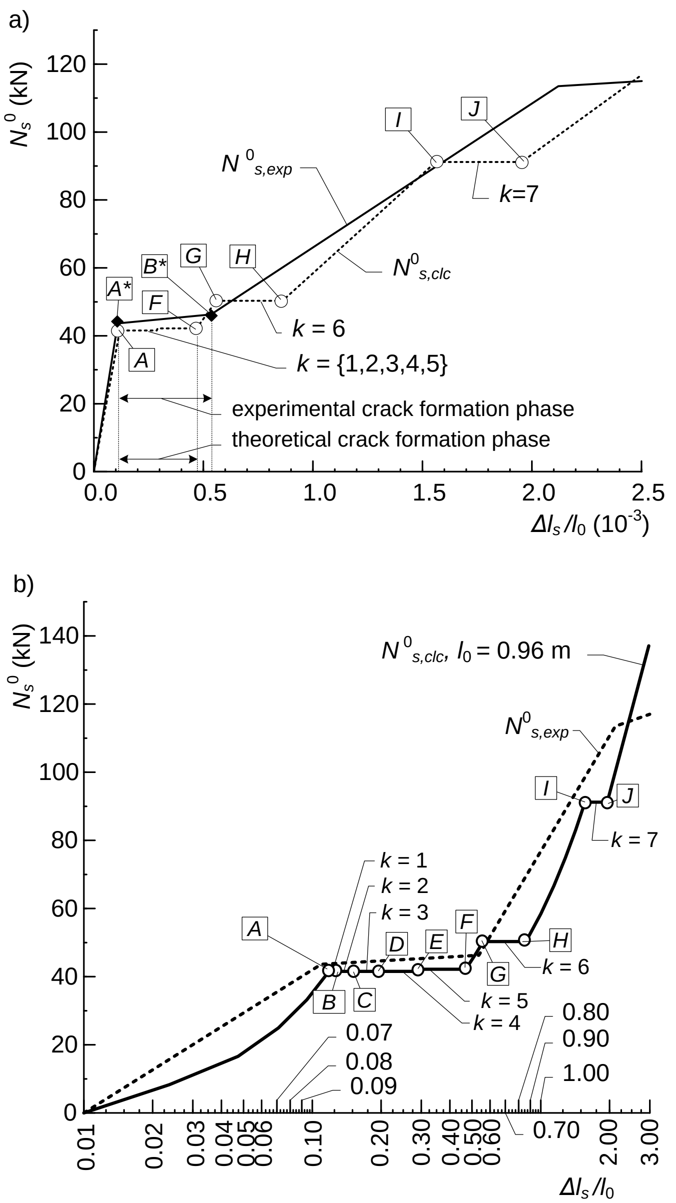

Figure 7. The dependences of the tensile force N_s^0 on relative elongation ($\Delta l_s/l_0$): (a) linear scale, (b) logarithmic scale, type of the RC tie is (3).

stage.

As can be seen from Figure 7, there are several differences between experimental and theoretical N_s^0−($\Delta l_s/l_0$) curves. Firstly, there are three theoretical crack formation phases: the first between points *A* and *F*, the second between *G* and *H*, and the last between *I* and *J*; and only one experimental crack formation phases between points *A** and *B**. Between points *A* and *F* there are five theoretical cracking stages and one theoretical cracking stage in remain crack formation phases. According to Equation 13, 31 cracks formed between points *A* and *F*. Even though there are five cracking stages in the first theoretical crack formation phase and one cracking stage in remain theoretical crack formation phases, 6th and 7th, the length of the each crack formation phase, that is the length of Δl_s at the cracking phases, is similar. It occurs due to lots of new cracks, $n_{crc,new,k}$, formed in 6th and 7th cracking stages, 32 and 64, respectively, according to Equation 15.

It should be noted that the number of cracks $n_{crc,k} = 31$ in the first crack formation phase is unreal comparing to the number of the cracks calculated by the formula $n_{crc,k} = l_0/\mathrm{ceil}(s_{r,max})$, where $s_{r,max}$ is the maximum crack space according to EN 1992 11, here ceil(•) is ceiling functions. According to EN 1992 11, for given 3th type of the tie, $s_{r,max} = 50.277$ cm, then $n_{crc,k} = 1$. Therefore, theoretical number 31 of the cracks is not real in the sense of EN 1992 11 herewith numbers of the new cracks $n_{crc,new,k}$ and the blocks $n_{blc,k}$ are not real in the same sense. Consequently, the cracks and blocks considered earlier are not real; they may be called fictitious cracks and blocks. Corresponding numbers of the cracks and blocks may be called as a total number of the fictitious cracks $n_{crc,k}$, number of the new fictitious cracks $n_{crc,new,k}$ and number of the fictitious blocks $n_{blc,k}$ at k^{th} cracking stage. On the other hand, quite good agreement of the calculated cracking phases with the experimental allows us to claim that the calculated cracking phases can be considered as real. Moreover, the proposed model is suitable to predict a stepped shape of the load-elongation curve of the RC tie. The number of the cracking stages *k*, at the fist cracking phase, reduces significantly with decreasing of the stiffness of the bond ξ or multiplier β. The cracking parameters *k*, $n_{crc,k}$, $n_{blc,k}$ and $s_{r,max}$ for RC tie of type (6) calculated by the proposed method and EN 1992 1 1 at the first cracking phase are given in Table 3.

As can be seen from Table 3, $n_{crc,k}$, according to proposed method, tends to $n_{crc,k}$ according to EN 1992 11. The values of the cracking parameters can be considered as real when $\beta = 10$. It should be noted that *k*, $n_{crc,k}$, $n_{blc,k}$ and $s_{r,max}$ are the same as $5 \le \beta \le 10$. On the other hand, Figure 7 shows that the N_s^0–Δl_s curves, at different stiffness of the bond, are also close to each other and have a typical stepped shape.

The given comparison shows that the presented method is suitable to model not only elongation of the reinforcement but also cracking space and width of the cracks. However, this issue should be examined more carefully by taking into account several parameters, namely elongation of the reinforcement, number of the cracks and its width.

It should be noted that the theoretical dependences of N_s^0 on ($\Delta l_s/l_0$) are piecewise-linear. The points at which the curve N_s^0–($\Delta l_s/l_0$) breaks are located at the beginning and ending of the crack formation phases.

Theoretically, elongation of the RC tie may be described as follows: At the beginning of a loading, the external fore N_s^0, applied to the reinforcement, increases

Table 3. Cracking parameters calculated according to the proposed method and EN 1992 11 at the first cracking phase, type of the RC tie is (6).

Multiplier, β	Number of cracking stage, k	Number of cracks, $n_{crc,k}$	Number of blocks, $n_{blc,k}$	Crack space $s_{r,max}$ (cm)
According to the proposed method				
10	2	3	4	24
100	3	7	8	12
1000	5	31	32	3
According to EN 1992 1 1				
–	–	1	2	50.77

gradually from zero to a load of the first crack $N^0_{s,crc}$. Since plane section hypothesis is valid and cracking condition is $\sigma_c = f_{ctm}$, cracks open through the whole cross-section of the concrete at $N^0_{s,crc}$. The new cracks divide the concrete into several smaller blocks, which are connected with reinforcement only, and reduce the stiffness of the whole RC RC tie. Since the cracks appear suddenly, at $N^0_{s,crc}$, the stiffness decreases suddenly as well. At this stage, the behaviour of the elongation of a RC tie depends on the loading. If displacements are controlled then, the elongation Δl_s remains the same immediately before cracking and at the cracking, and, due to decreasing stiffness of the RC tie, the tensile force N_s^0 decreases as well. In the cracking stage, a jump of the tensile force N_s^0 appears in N_s^0−($\Delta l_s / l_0$) diagram. This case is typical for experimental N_s^0−($\Delta l_s / l_0$) curves when displacement controlling machines are used for the test. Other hypothetical case may occur when force are controlled. In this case, the tensile force N_s^0 remains the same before cracking and after it. We have a jump of the relative elongation ($\Delta l_s / l_0$) of the reinforcement in N_s^0−($\Delta l_s / l_0$) diagram at $N^0_{s,crc}$ due to the suddenly decreased stiffness of the RC tie. The former N_s^0−($\Delta l_s / l_0$) diagram is not as convenient as the last one which is also used in codes (for instance, MC, 1990).

When N_s^0 reach $N^0_{s,crc}$, and first crack occurs, intensive cracking process, so-called the cracking formation phase, begins. If the bond is stiff enough, or a RC tie is long enough, many cracking stages may occur in the first cracking formation phase, in example 5 (Figure 7). However, if the stiffness ξ is small, or a RC tie is short, then only few cracking stages occur even if in the first crack formation phase. In our theoretical curve (Figure 7), the increment of N_s^0 between different cracking stages k, $k \in \{1, 2, 3, 4, 5\}$, is very small. The cracking stages are clearly indicated by logarithmic scale in Figure 7b. Practically, N_s^0 remains unchanged at the cracking stages 1 to 5. Thus, the increment $\Delta N_s^0 \approx 0$, between points A and F.

In general, the relative elongation ($\Delta l_s / l_0$) of the reinforcement depends on initial length l_0. This is illustrated in Figure 9. The cracking stages, denoted by k, are shown in this picture for the beam whose l_0 = 0.96 m as well. At the beginning of the loading, the ratio ($\Delta l_s / l_0$) does not depend on l_0. However, after the first crack formation phase, the ratio ($\Delta l_s / l_0$) is different at different l_0. It should be noted that five cracking stage occur in the First crack formation phase for all three N_s^0−Δl_s curves shown in Figure 9.

Let us consider relative increments of elongations Δl_s of reinforcement at each cracking stage k, $k \in \{1, 2, 3, 4, 5, 6, 7\}$ for the beams whose $l_0 \in \{0.96, 1.8, 2.5\}$. It should be noted that cracking stages under investigation are depicted in Figure 7 for the beam whose l_0 = 0.96 m. Let the elongation of the reinforcement at the beginning and end of the first cracking stage be $\Delta l_s(A)$ and $\Delta l_s(B)$. Then relative increments for the first cracking stage is $\Delta(\Delta l_s(1) / l_0) = (\Delta l_s(B) - \Delta l_s(A)) / l_0$. The relative increments for other cracking stages are defined similarly. These differences $\Delta(\Delta l_s(k) / l_0)$, $k \in \{1, 2, 3, 4, 5, 6, 7\}$, for considered RC tie, $l_0 \in \{0.96, 1.8, 2.5\}$, are shown in Figure 10.

As can be seen from Figure 10, the difference $\Delta(\Delta l_s(k) / l_0)$ increases with increasing cracking stage k, and decreases with increasing the initial length l_0 for considered RC ties The dependence of ($\Delta l_s / l_0$) on initial length l_0 in general is piecewise-linear. This dependence, when the load N_s^0 = 100 kN and type of the bar is (5), is shown in Figure 11. As can be seen from this Figure 11, ($\Delta l_s / l_0$) decreases linearly with increasing l_0. However, at certain value of l_0, the ratio ($\Delta l_s / l_0$) changes steeply. It occurs since addition cracking stage appears at the load N_s^0 = 100 kN. That is, when l_0 < 1.78 m, then k = 7, when l_0 > 1.79 m, then k = 8. Appeared new cracks caused jump of the relative elongation. The last example shows that Δl_s dependence on l_0 may also have discontinuities like the dependence on N_s^0−Δl_s.

The stiffness of a RC tie under tension increases with increasing the stiffness ξ of the bond. In other words, the slope of N_s^0–Δl_s curve increases with increasing ξ. This is illustrated in Figure 8. As can be seen from this picture, $N^0_{s,clc}$ tends to $N^0_{s,exp}$ as ξ, or β, increases. It should be noted that according to Equation 19, the cracking tensile

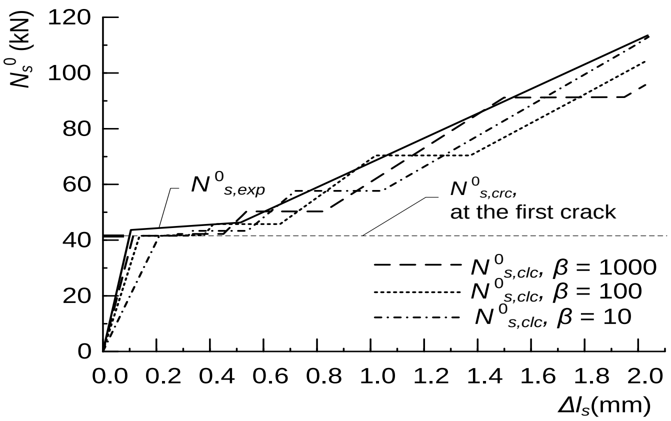

Figure 8. The N_s^0–Δl_s curves at different values of β, type of the RC tie is (6).

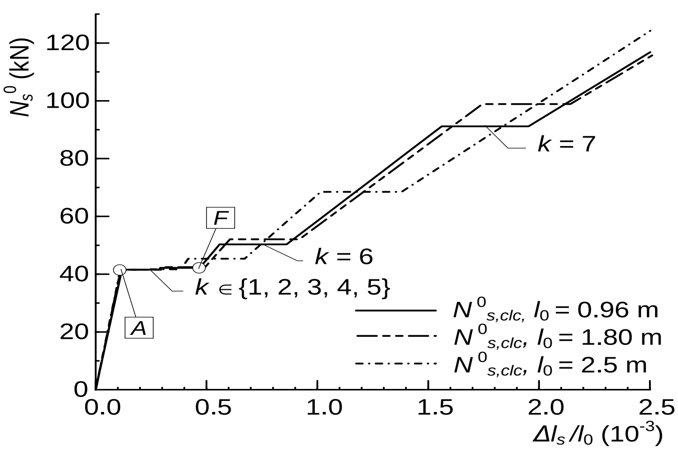

Figure 9. The dependences of the tensile force N_s^0 on relative elongation $\Delta l_s/l_0$ at different initial lengths l_0 of the tie, type of the RC tie is (4).

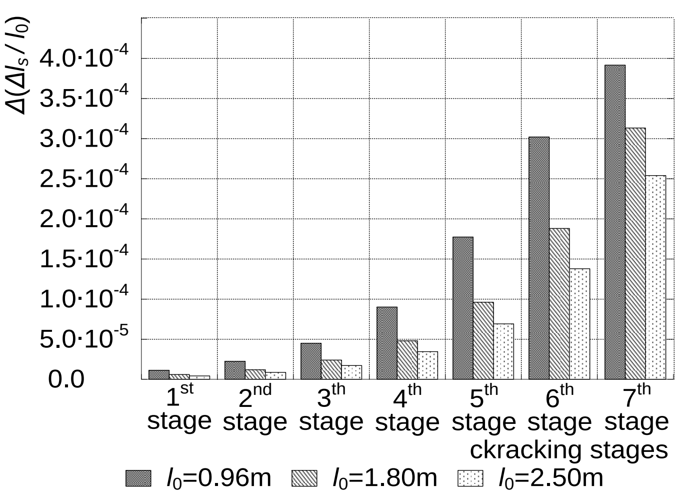

Figure 10. The relative increments $\Delta(\Delta l_s/l_0)$ of cracking stages at different initial length of the RC ties l_0, type of the tie is (4).

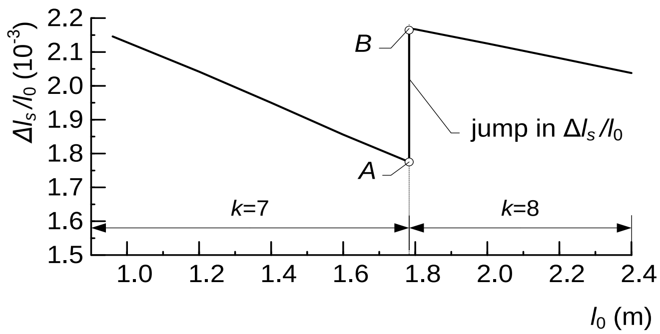

Figure 11. The dependence of the relative elongation $\Delta l_s/l_0$ on the initial length l_0 of the tie at load $N_s^0 = 100$ kN, type of the RC tie is (5).

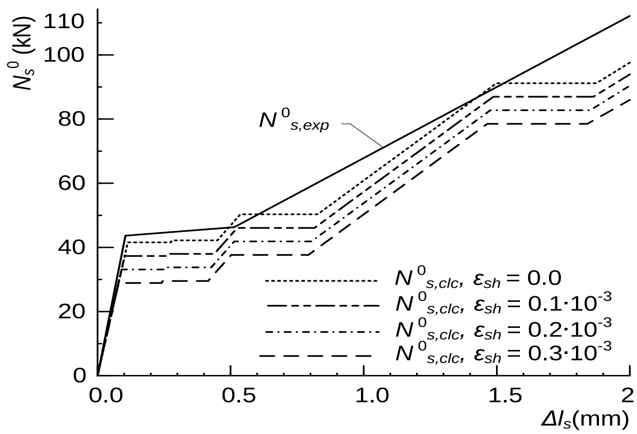

Figure 12. The N_s^0–Δl_s curves at different shrinkage strains ε_{sh} of the concrete, type of the RC tie is (7).

force $N^0_{s,crc}$ depends on the stiffness ξ. However, as can be seen from Figure 8, the load, for considered RC tie, at the first crack does not depend on the stiffness ξ, or β. The shrinkage of the concrete was not taken into account for already investigated RC ties, even though, as is known, ordinary concrete always shrinks. It also affects accuracy of the calculated N_s^0–Δl_s relationship. The dependence of N_s^0 on the elongation Δl_s at different shrinkage strains ε_{sh} of the concrete are shown in Figure 12, type of the RC tie is (7). As can be seen from this picture, $N^0_{s,clc}$ and $N^0_{s,crc}$ decrease with increasing the shrinkage strain ε_{sh}. Also, for considered RC tie, a slope of the N_s^0–Δl_s curves remains the same for different values of the shrinkage strains, while this slope is different at different stiffness of the bond (Figure 8).

Conclusions

A possibility of the analytical modelling of the elongation of the reinforced-concrete tie using linear elastic stress-slip relationship of the bond was investigated analytically by comparing calculated and experimental data of the ties with three different thickness of the cover. It was shown that the linear approximation of the stress−slip relationship of the bond obtained by minimizing the maximal relative difference with respect to experimental results could be applied for description of deformation behaviour of the cracked RC tie in pure tension. However, the number of the cracks is too big in comparison with EN 1992 11. Therefore, the number of the cracks calculated using obtained values of the stiffness by minimizing the relative ratio of the elongation of the tie in a certain interval is not real and could be treated only as fictitious.

Performed analysis showed, however, that the accuracy of the suggested approach depends on the approximation technique of the ascending path of the stress-slip relationship of the bond. Application of the relatively small slip range leads to artificial increase of the bond stiffness, therefore scaling factor of bond stiffness should be taken into account. Moreover this factor depends on the thickness of the cover of the reinforcement.

Cracking tensile force, applied to the reinforcement, decreases with increasing shrinkage strains of the concrete.

Slope of the force-elongation diagram increases with increasing the stiffness of the bond or its slip modulus.

Generally, application of the suggested linear model simplifies the calculation of the elongation of the cracked reinforced concrete tie, however, underestimates values of the longitudinal stiffness.

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