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Full Length Research Paper

Modified damage location indices in beam-like structure: Analytical study

Moatasem M. Fayyadh* and Hashim Abdul Razak

Department of Civil Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia.

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Proposed modifications to two existing algorithms, based on mode shape, for locating of damage in a beam like structure model are presented in this paper. The Curvature Damage Factor (CDF) developed by Wahab and Roeck (1999) computes the change in mode shape curvature between two sets of mode vectors, i.e. undamaged and damaged conditions. The local stiffness indicator (LSI) was developed by Ismail and Abdul Razak (2006) and based on the fourth derivative of the mode shape. The proposed modifications to the mathematical form of the original algorithms aim to improve its sensitivity and overcome its drawback. In order to verify the suitability and necessity for implementing the modification to the existing algorithms, eigenvalue analyses on a finite element model of a beam-like structure model were carried out and the eigenvectors for different cases were obtained. The proposed modified forms of the algorithms exhibited better sensitivity for detecting damage location in addition to the anomalies at the supports being eliminated. The modified algorithms are able to detect the damage wherever its location, applying even to cases of multi damage locations. It is also concluded that the modified algorithms have the sensitivity to detect the damage regardless of its severity. Moreover, procedures for elimination of anomalies have been proposed. The first procedure is based on eliminating the algorithms values at the degrees of freedom which match the nodes along the beam length and it has been applied to all the data presented in this paper. The second procedure is based on eliminating the damage algorithm's values at the supports and this has been applied to the original form of the LSI. Finally, statistical anomalies elimination (SAE) procedure has been proposed and applied to the cases of anomalies along the entire beam length. The SAE elimination procedure has helped to improve the sensitivity by suppressing the anomalies along the beam length.

Key words: Curvature damage factor, modified curvature damage factor, local stiffness indicator, modified local stiffness indicator, statistical anomalies elimination.

INTRODUCTION

In recent years, development of damage detection techniques based on modal parameters has attracted significant attention with regards to civil engineering applications. Damage inspection of structures is important in order to come up with a planned strategy for repair and maintenance works. Numerous research works have been published in the field of damage detection and a variety of methods have been developed and proposed. These methods are mainly based on the relationship between the dynamic characteristics and the damage parameters like crack depth and its location. Cracks on main structural elements can be a major cause of concern since it can lead to structural failure. Thus early crack detection is crucial in order to avoid sudden failure especially when there is the likelihood of overloading on the structure. In general, cracks will cause a reduction in stiffness and correspondingly cause a change in the dynamic parameters like mode shape and its derivatives. Therefore, it is possible to detect the damage location by measuring the change in the mode shape derivatives. Mode shape curvatures are more sensitive to damage and the concept of curvature mode shape was introduced by Pandey et al. (1991). Using a finite element model of a simply supported beam with a

*Corresponding author. E-mail: moatasem.m.f@gmail.com.

reduction in E of 50% at one third of the span; it was demonstrated that the modal curvature was a much more sensitive damage indicator than the modal assurance criteria (MAC) or co-ordinate modal assurance criteria (COMAC) values. This approach was extended by Ratcliffe (1997) using both analytical and experimental results of the curvature of a damaged beam without need of a priori knowledge of the undamaged state. The proposed method applies the Laplace operator on the discrete mode shape and the presence of severe damage was detectable in the form of a jump in the Laplace.

Stubbs et al. (1995) developed a damage index method to locate the damage which utilizes the characteristics of the mode shape curvature for a beam as the main variable in the derived damage localization algorithm based on the relative differences in modal strain energy before and after damage. Wahab and De Roeck (1999) utilized the change in curvature of mode shape to detect the damage location which further reinforces that the change in curvature is more sensitive compared to the mode shape itself. Johan (2003) conducted a study to detect, quantify and locate the damage in a reinforced concrete structure through vibration monitoring. The approach adopted was to evaluate the direct bending and torsional stiffness along the structure from experimental natural frequencies, mode shapes and its derivatives. Beams were gradually damaged and the change of dynamic parameters monitored from the initial to the failure state. The direct stiffness calculated using the modal parameters turned out to be good indicators. Dutta and Talukdar (2004) carried out Eigen value analysis using Lanczos algorithm in an adaptive h-version finite element environment in order to control the discretization error for accurate evaluation of the modal parameters. It was found that there was better localization of damage by considering curvature of mode shapes. Law and Lu (2005) proposed a time domain method in which the parameters of a crack in a structural member were identified from strain or displacement measurement. The dynamic response was calculated based on modal superposition. In the inverse analysis, optimization technique coupled with regularization on the solution was used to identify the cracks. The formulation for identification was further extended to the case of multiple cracks. Computation simulations with sinusoidal and impulsive excitations on a beam with single crack or multiple cracks showed that the method was effective for identifying the parameters of the cracks with a certain degree of accuracy. Ismail and Abdul (2006) used mode shape derivatives to detect the location of damage due to a single crack as well as honeycombs in RC beams known as local stiffness indicator (LSI). LSI was proposed as a damage location indicator. The LSI was obtained by rearranging the equation of free vibration for uniform beams, and applying the fourth order centered finite divided difference formula to regressed mode shape data.

According to LSI damage index, the exact location is around the center of the detected region. Curve fitting with Chebyshev series rationales onto the mode shape also highlighted the points' residuals around the damaged region. Choi et al. (2008) developed two existing algorithms for global non-destructive evaluation and studied localized damage in timber beams using a finite element model. These damage localization algorithms were found to be ineffective in locating multiple damage scenarios and were unable to estimate the severity of damage. The modifications to the damage index algorithms, as well as the development of a hybrid algorithm were proposed by Choi et al. (2008) to overcome the problems. Experimental modal analysis data were used to extract mode shapes for calculating the damage index in the proposed method which utilizes change in modal strain energy between the undamaged and damaged timber beam models. The modified damage index normalizes the mode shape curvature, and the hybrid algorithm combines the modified damage index and change in flexibility algorithm which reflect the changes of natural frequency and mode shape. Detection of damage by using limited number of natural frequencies and/or mode shape was done by Perera et al. (2008) who proposed a new damage detection method called local modal stiffness which can be determined from the frequency response function and depends on both frequency and mode shape. The method was examined by using experimental progress on RC beams with cracking. Kim et al. (2010) offered a new proposal for a hybrid health monitoring system using sequential vibration impedance approaches to detect damage in pre-stress concrete bridge girders. Kopsaftopoulos and Fassois (2010) applied several vibration-based statistical time series as structural health monitoring methods on lightweight aluminum truss structures. Yan et al. (2010) developed a wavelet-based method which not only localized multiple damage sites but also provided information on when the damage occurred. From previous studies, it is apparent that some of the algorithms used to detect and locate damage on steel and reinforced concrete structural elements were less accurate and thus inadequate.

The objective of this study is to propose modifications to two indices for detecting of damage location; one based on the change in curvature of mode shape and the other based on the fourth derivative of mode shape in order to enhance their sensitivity to detect damage location. In addition, this study aims to propose procedures for eliminating the anomalies concomitance the mathematical calculation of the detection of damage location Indices.

Lower modes were found to be more sensitive to the change in the support conditions (Fayyadh and Abdul, 2010; Fayyadh et al., 2011a). A new damage detection index based on the combination between the mode

Table 1. Damage cases adopted in present study.

Stiffness damage cases	Stiffness reduction ratio (SRR) (%)	Damage location
Control C	0	N/A
C1	1	Mid-Span EL7
C2	1	Quarter-Span EL3
C3	1	Quarter-Span + Mid-Span EL3 & EL7
C4	1	EL2 + EL5 + EL8

shape vectors and their curvature was developed and verified to have higher sensitivity than existing algorithms (Fayyadh and Abdul, 2011). A new damage severity algorithm was proposed by Fayyadh et al. (2011b) which based on the combination of both natural frequencies and mode shape and it was proven to be better sensitive than exist damage severity algorithms.

FINITE ELEMENT MODELLING

In order to demonstrate the significance and capability of the modified algorithms, one finite-element beamlike structure model was built to represent control and damaged cases. The span length of the beam was 3250 mm with a cross-sectional area of 150 by 250 mm. The sensitivity level of the new algorithm attempted to detect the smallest damage severity and different damage locations. It also attempted to detect damage location in case of multi damage cases. Since the dynamic parameters are related to the stiffness of the structural element (Equation 1), the damage is presented by reducing the modules of elasticity E values:

$$f \propto \sqrt{EI} \qquad (1)$$

Where f is the natural frequency, EI is the flexural rigidity, E is the modulus of elasticity and I is the second moment of inertia.

The stiffness reduction ratio (SRR) which was adopted as the notation for the damage severity can be calculated as:

$$SRR = (1 - E_d/E_c).\ 100\% \qquad (2)$$

Where E_d is the modulus of elasticity for the damage cases and E_c is the modulus of elasticity for the control case.

Two levels of stiffness were adopted as the control where E is 200 KN/mm^2, the SRR was 0% and the damage case where SRR was 1% and E is 198000 KN/mm^2. The damage location algorithms were examined at the same damage level and two different single damage locations; the first single damage was located at the mid-span and the second single damage was located at the quarter-span. In addition, the damage location algorithms were examined when there are multi damage locations. The first multi damage location case is when there is damage located at the quarter span and at the mid span, the second when there is damage located near the support, at the quarter-span and near the mid-span. Total of five damage cases with different single and multi damage locations where adopted with SRR of 1%. Table 1 shows the damage cases adopted in the present study. Figure 1 shows the beamlike structure model for the control beam and different damage cases. Utilizing a general-purpose finite-element package called DIANA TNO that is based on the displacement method; one two-dimensional finite-element model was constructed to represent the beamlike structure model. The beam model was built by using a 4-node plane stress element. Figure 2 shows a typical model for the beam constructed using DIANA TNO software. The physical and material properties of the beam model were Poisson's ratio of 0.2, mass density of 7850 kg/m^3 and Young's modulus of 200,000 MPa for the control case and 198,000 Mpa for the damage case. The self-weight was computed by taking gravitational acceleration as 9.81 m/s^2 in the $-y$ direction (gravity direction). Initially, Eigen analyses were performed so that modal parameters for the control beam model could be approximated. Next, the damage was created on the beam model by changing the value of the modules of elasticity first at the mid-span element (Figure 1b), second at the quarter-span element (Figure 1c), then at quarter-span and mid-span elements (Figure 1d), finally near the support, quarter-span and near the mid-span elements (Figure 1e). At each damaged case, Eigen value analysis was again performed to obtain the modal parameters relevant to the damage case induced. The modal parameters were carried out with the natural frequency as the global characteristic and the mode shape as the local characteristic. The beam was divided into 27 degrees of freedom (DOF); starting from the left support where the first DOF is at 0 distance from the support and the second DOF is at 125 mm from the support.

The next degree of freedom is located at 125 mm from the previous one up to the 27th degree of freedom which is located at the right support (Figure 3). The change in the natural frequency was used to indicate the damage severity based on frequency at each damage case

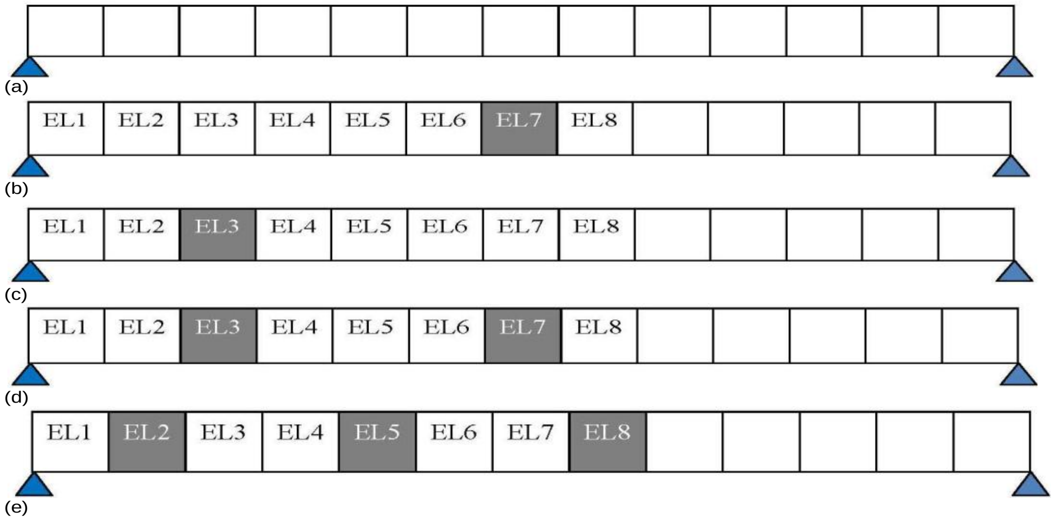


Figure 1. Beam like structure model: (a) Control beam, (b) mid-span damage, (c) quarter-span damage, (d) multi damage EL 3 and 7 and (e) multi damage model EL 2, 5 and 8.

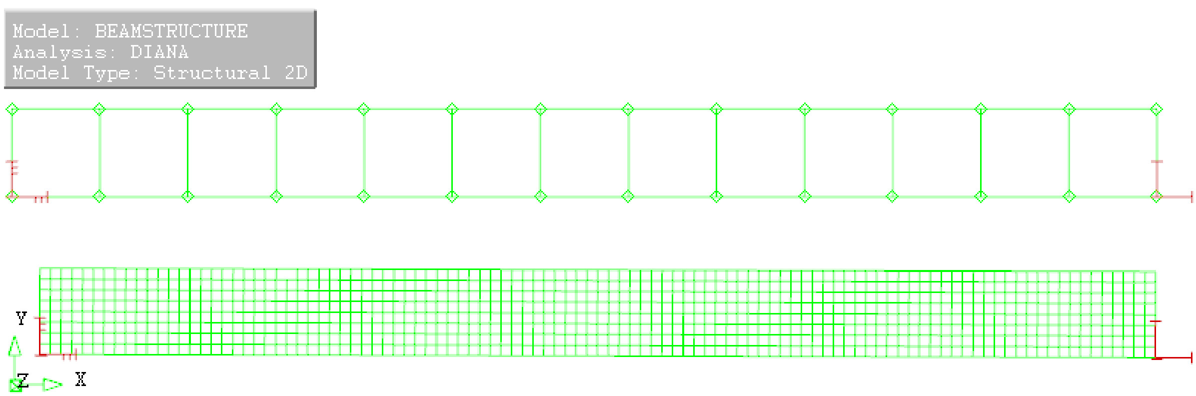


Figure 2. A typical model for the beam like structure model.

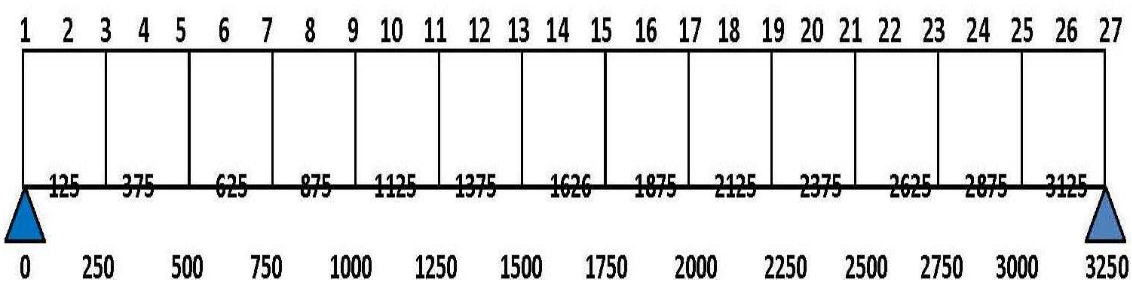


Figure 3. Distribution of 27 DOF along the beam length, top is the degree of freedoms numbering and bottom is the distance in millimetre from the left support.

adopted in the present study as in the following:

$$\text{Change in frequency} = \left(1 - \frac{f_{i,d}}{f_{i,c}}\right) \cdot 100\% \quad (3)$$

Where $f_{i,c}$ and $f_{i,d}$ are the natural frequency at *i*th mode for control and damaged beam, respectively.

The change in frequency values for different damage cases are illustrated in Table 2. The results prove that SRR of 1% corresponds to a very small damage level

Table 2. Change in frequency values at different damage cases.

Damage case	Change in frequency (%)			
	Mode 1	Mode 2	Mode 3	Mode 4
Control C	0.00	0.00	0.00	0.00
C1	0.08	0.01	0.07	0.02
C2	0.04	0.06	0.06	0.03
C3	0.08	0.06	0.13	0.05
C4	0.12	0.10	0.10	0.13

Table 3. MAC values at different damage cases.

Damage case	MAC value			
	Mode 1 (%)	Mode 2 (%)	Mode 3 (%)	Mode 4 (%)
Control C	0	0	0	0
C1	99.999960	99.999978	99.999923	99.999943
C2	99.999984	99.999935	99.999775	99.999869
C3	99.999962	99.999934	99.999839	99.999830
C4	99.999984	99.999926	99.999877	99.998952

whereas at worse case when there is multi-damage at different elements, the maximum FRI value was 0.13% which is too small. Modal assurance criterion (MAC) which is a correlation between experimental mode shapes and curve-fitted mode shapes are used in this study to monitor the change in the mode shape for different damage cases. The correlation for the ith element is given by the following formula:

$$MAC^* = \frac{\left|\sum_{i=1}^{n} \varphi_{i,c} \cdot \varphi_{i,d}\right|^2}{\left(\sum_{i=1}^{n} \varphi_{i,c} \cdot \varphi_{i,c}\right)\left(\sum_{i=1}^{n} \varphi_{i,d} \cdot \varphi_{i,d}\right)} \quad (4)$$

Where φi,c and φi,d are the mode shapes at ith mode for control and damaged cases, respectively.

The MAC values for different damage cases are illustrated in Table 3. The results show an excessively small change in the MAC value which can be neglected in some cases. The maximum change in the MAC value for the fourth mode when there is multi-damage located at different elements was 0.999983, the difference being located at the fifth digit after the decimal point. This proves the very small damage level for SRR of 1% and it also indicates the very small change in the mode shapes which will be used in the form of its derivatives to localize the damage. Any damage location index which can detect the location of such a damage level can be considered a good sensitive index. This allows the conclusion that the SRR of 1% is a quite good enough damage severity to examine the existing algorithms as well as the modified algorithms.

Existing damage algorithms

The results obtained from the finite element analysis were subsequently utilized to verify and compare the sensitivity and accuracy to detect and locate the damage positions, respectively, in this study. The eigenvectors were substituted into the equations for the damage algorithms, namely the curvature damage factor (CDF) and local stiffness indicator (LSI) as well as into the corresponding proposed modified algorithms.

Curvature damage factor (CDF)

The algorthim was proposed by Wahab and De Roeck (1999) where the mode shape curvature at each point is computed from central difference approximation using mode displacement. The change in curvature between two sets of mode vectors that is the control and damaged cases is shown in Equation 5:

$$\text{CDF} = \frac{1}{N}\sum_{j=1}^{N}\left|C_{ci} - C_{di}\right| \quad (5)$$

Where CDF is the curvature damage factor, N is the total number of modes, ‘c’ indicates control case when no load was applied; and ‘d’ indicates damage case when damage load was applied and released and C is the curvature at ith node.

Figures 4 and 5 show the results of CDF according to finite element modelling data for damage located at mid-span and quarter-span respectively. The results were summated for the first four bending modes. The results

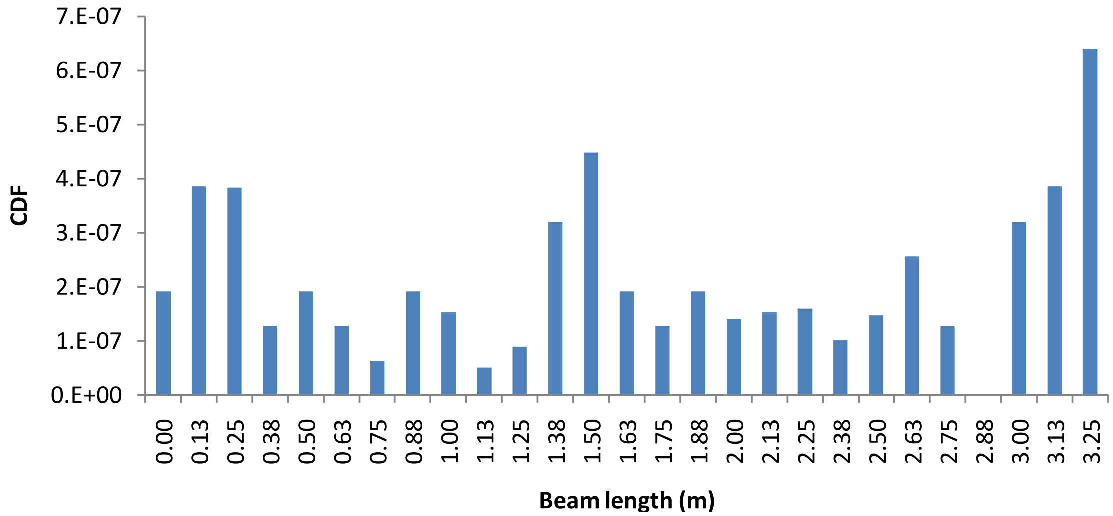


Figure 4. CDF values for damages located at the mid-span.

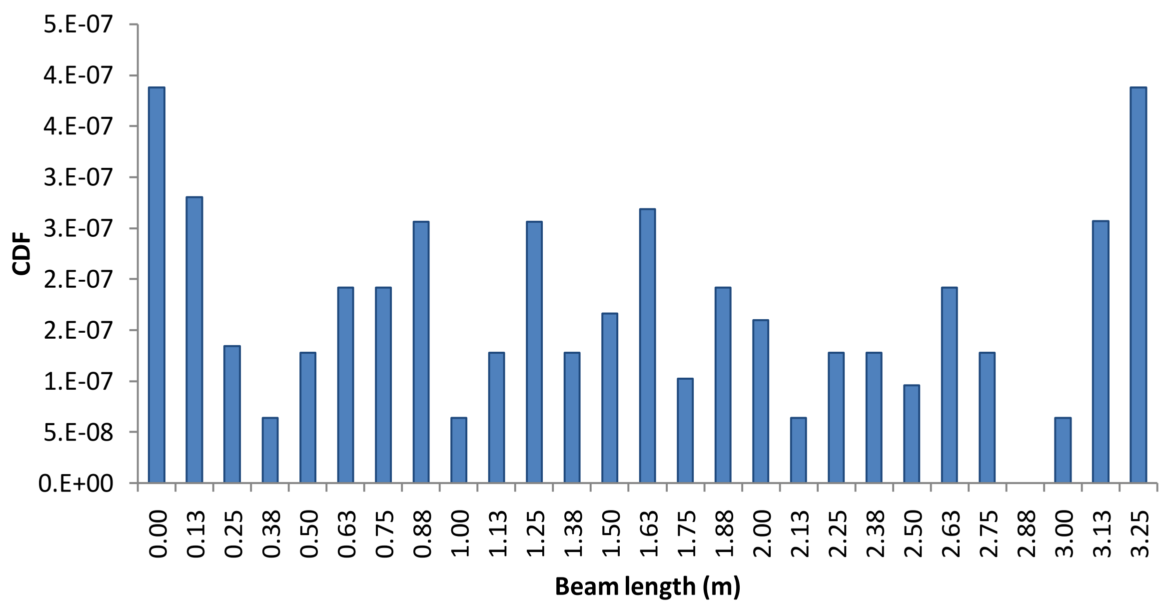


Figure 5. CDF values for damages located at the quarter-span.

showed that the CDF correlated well when the damage location was at the mid-span while it was less sensitive when the damage is located at the quarter-span. Furthermore, values of CDF in all the cases returned high values at the supports which is an anomaly which indicates a flaw in the algorithm. Thus CDF in its original form is rather unreliable when the damage location is near the support. In order to investigate the ability of CDF to detect the damage in multi damage locations, Figures 6 shows the CDF values for multi damage located at quarter-span and mid-span, and Figure 7 shows CDF values for the case of multi damage locations near the left support, quarter-span and near the mid-span. The results showed that CDF had some sensitivity to detect the damage locations in cases of multi damage. However, there still are anomalies in CDF values along the beam length.

Local stiffness indicator (LSI)

The algorithm was proposed by Ismail and Abdul (2006) and based on the equation for free vibration of the Euler beam as follows:

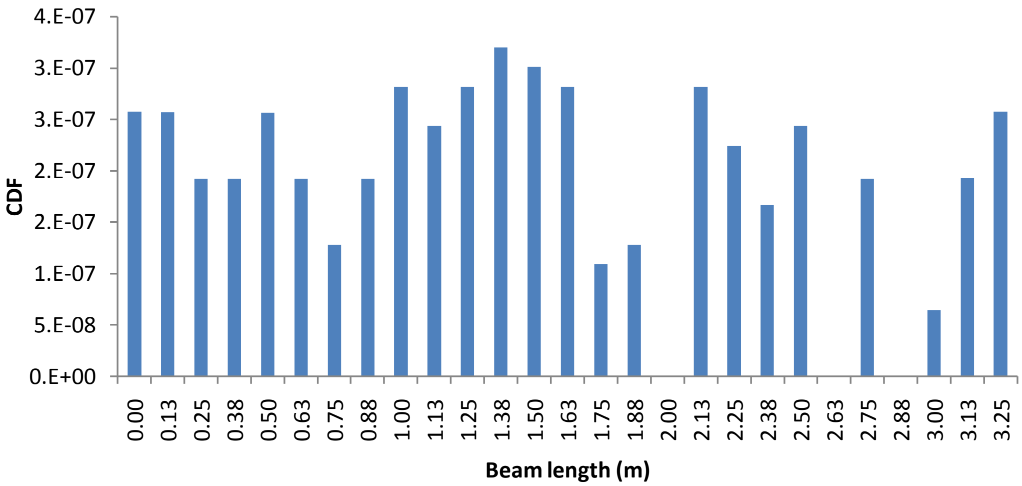


Figure 6. CDF values for multi-damages located at quarter and mid span.

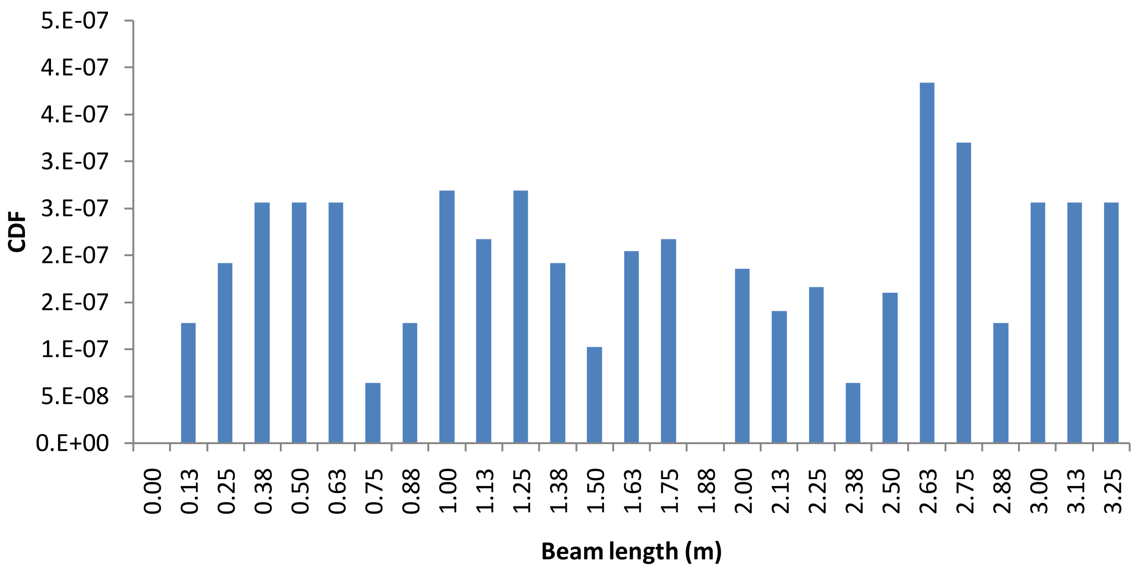


Figure 7. CDF values for damage case c4.

$$\frac{d^4 y}{dx^2} - \lambda^4 y = 0 \quad (6)$$

Re-arranging the Equation 6 in the following form:

$$\lambda^4 = \left| \frac{y^4}{y} \right| \quad (7)$$

In addition, applying the fourth order centred finite difference:

$$y^4 = (y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}) / h^4 \quad (8)$$

Where y^4 is the fourth derivative. Thus, the local stiffness indicator is defined as:

$$LSI = \lambda^4 \quad (9)$$

The eigenvectors from the finite element model were extracted and substituted into the aforementioned equations to determine the LSI at each node. The occurrence of damage in the beam will cause a change in the LSI value at the damage location as compared to the undamaged beam where the values should remain constant throughout its length. Figures 8 and 9 present the values of LSI for the finite element beamlike structure

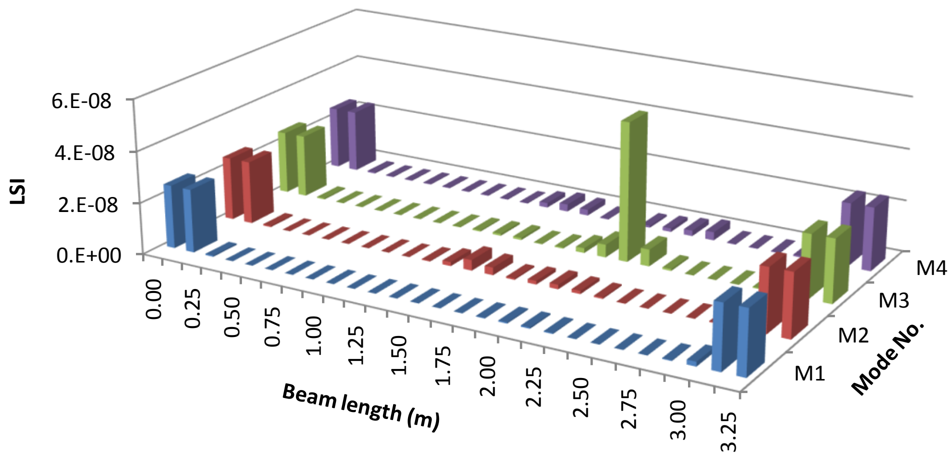
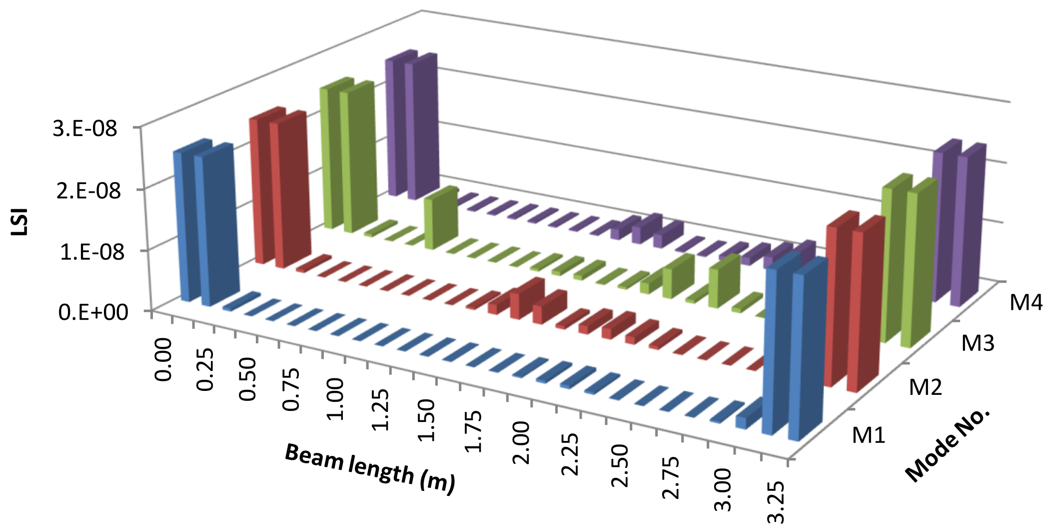
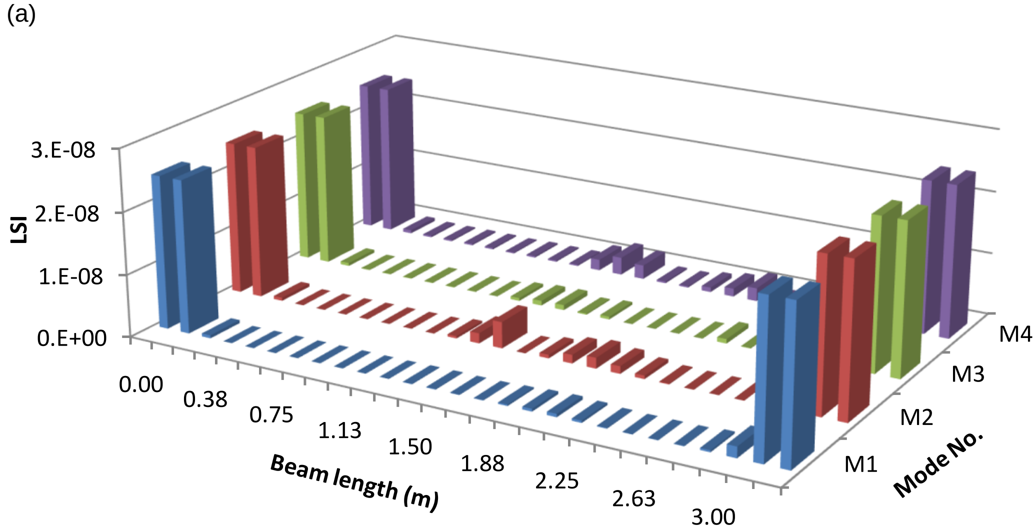


Figure 8. LSI values for damages located at mid-span.



(a)



(b)

Figure 9. LSI values for damages located at quarter-span (a) and LSI index for multi-damages located at quarter and mid.

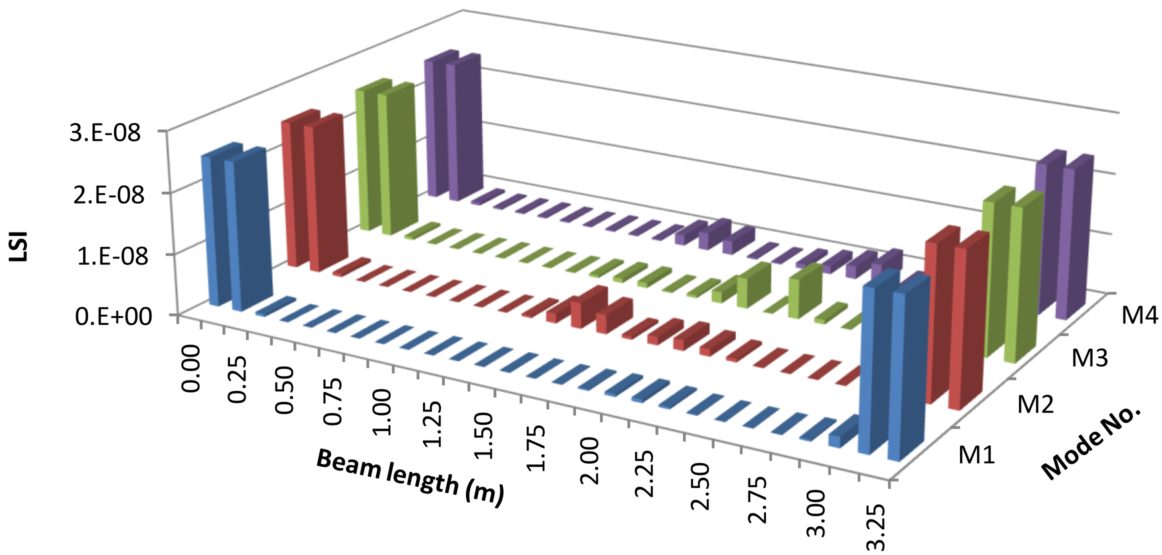


Figure 10. LSI index for damage located at elements EL2, 5 and 8.

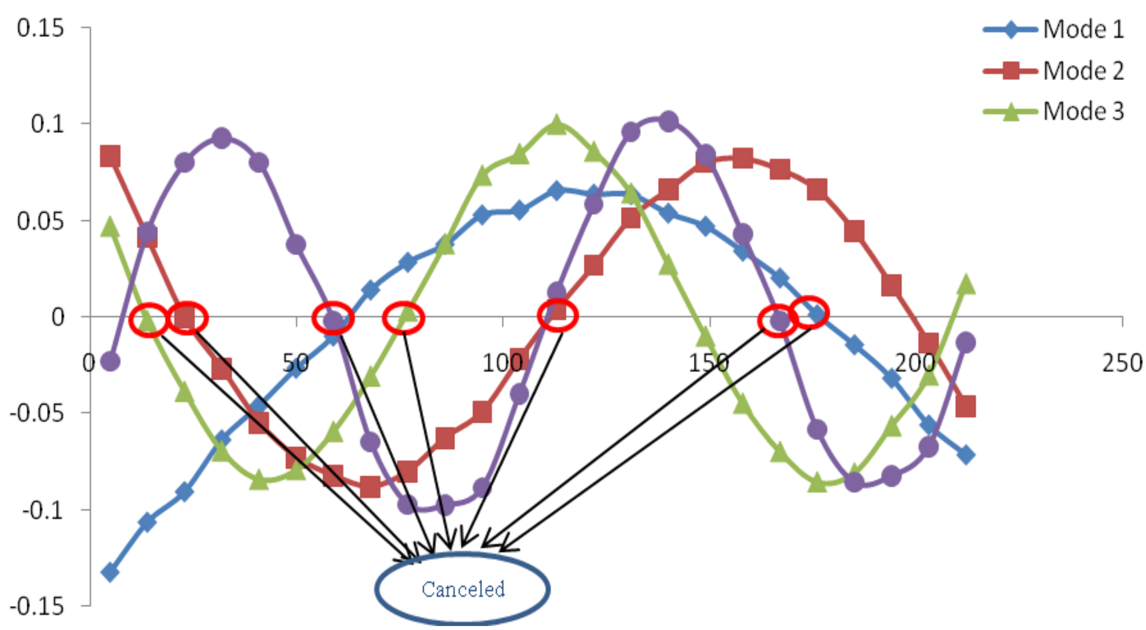


Figure 11. Mode shape patterns for the control case.

model for damage located at mid-span and quarter-span respectively. The results were summated for the first four bending modes. The LSI is a less sensitive damage indicator compared to the CDF for damage location in the regions considered.

For regions at the support, the values appear as anomalies and this is the major drawback of the LSI indicator in its original form. In order to investigate LSI capability to detect multi damage case, Figures 10 and 11 shows the LSI values for the damage cases C3 and C4. The result showed that Modes 2 and 4 detect a damage matching the damage at elements 5 and 8.

PROPOSED ANOMALIES ELIMINATING PROCEDURES

CDF and LSI algorithms results carried out showed some anomalies along the beam length and at the supports. These anomalies can be due to reasons other than the mathematical form of the specific damage location algorithm. In order to understand the reason for such anomalies, the mode shape patterns are highlighted where different modes will have different patterns. Figure 11 presents the mode shape patterns of modes 1 to 4 for a simply supported beam as an example. The mode shape patterns showed there were some degrees of freedoms (DOF) that will match the nodes along the beam length which have mode vector values close to zero. Such thing may cause anomalies at the location of these nodes that is DOF, since multiplying or dividing by values close to zero may cause anomalies. In order to eliminate such anomalies, it is suggested to eliminate the damage detection algorithm value at the DOF that matches the nodes and for each separate mode shape. From Figures 3 to 6, it can be concluded that for mode two, the DOF number 14 at 1625 mm from the left support need to be eliminated; for mode three DOF number 10 and 18 which located at 1125 and 2125 mm, respectively, from the left support have to be eliminated and for the fourth mode DOF number 20 which is located at 2375 mm from the left support also need to be eliminated. Such effects can influence the ability of the damage location algorithm to locate the damage position.

Statistical anomalies elimination (SAE)

Since most of the damage location algorithms are based on the calculations of the second or the fourth derivatives of mode vectors, which will result in very small values that is 10^{-8}, such values during the multiplication or division will manifest as anomalies in the algorithm along beam length. Statistical elimination procedure is suggested in order to overcome this problem. The procedure called the statistical anomalies elimination (SAE) is based on statistical calculations, where the mean and the standard deviation for all algorithms values along the beam length are shown in the equations as follows:

$$\mu = \frac{\sum_{i=1}^{N} \mathrm{MCDF}}{N} \quad (10)$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathrm{MCDF} - \mu)^2} \quad (11)$$

Where μ is the mean, σ is the standard deviation and N is the total degree of freedom along the beam length.

The SAE procedure steps are as follows:

i) Calculate the mean and the standard deviation values for the data set.
ii) Find the upper band limit L.

$$L = \mu + \sigma \quad (12)$$

i) Find the main value σ_n again for the complete algorithm values set, except the values that are more than the upper band limit (L) value.
ii) Subtract the new main value σ_n from all the algorithm values set except the nodes that are higher than the upper band limit (L) value.

$$\mathrm{DLA}_i^f = \mathrm{DLA}_i - \sigma_n \quad (13)$$

Where DLA_i^f is the modified value for damage location algorithm and "i" is the node number. The SAE procedure will be applied to improve the sensitivity in the case of anomalies along the span length.

Modified damage algorithms

In order to overcome the drawbacks of CDF and LSI algorithms, a modification has been performed on the mathematical form of CDF and LSI algorithms. This section will present the modification on each algorithm and will examine modified algorithms at different damage cases. Here, the elimination of the degree of freedoms which are matching the nodes will be considered and included in all the results. The SAE procedure will be considered in some cases when there are anomalies in the algorithm values along the beam length.

Modified curvature damage factor (MCDF)

The curvature damage factor (CDF) accounts for all available mode shapes through the summation of the mode shape curvatures. The values of mode shape curvature are dependent on the shapes of each individual mode.

Instead of reflecting the changes in the curvature due to damage, summation of non-normalized mode shape curvature will distort the damage index in favour of higher modes which results in false damage identifications. To overcome this problem in order to use the index for detecting damage location which is a local phenomenon, it is proposed to calculate the change in curvature at each node for each respective mode considered and compare the values between damage and control cases. According to the proposed change, the equation for the modified curvature damage factor (MCDF) is given as:

$$\mathrm{MCDFij} = \left| \frac{C_{di} - C_{ci}}{C_{ci}} \right| 100\% \quad (14)$$

Where *'i'* is node number and *'j'* is mode shape number. *C* is the curvature at each node *'i'*; for both 'c' the control case where no initial damage load is applied and *'d'* the

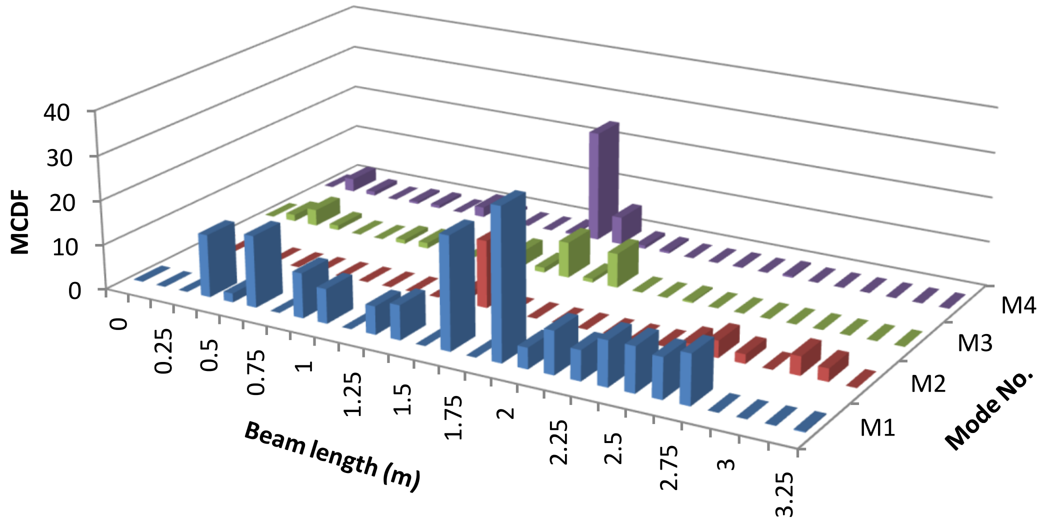


Figure 12. MCDF for damages located at mid-span.

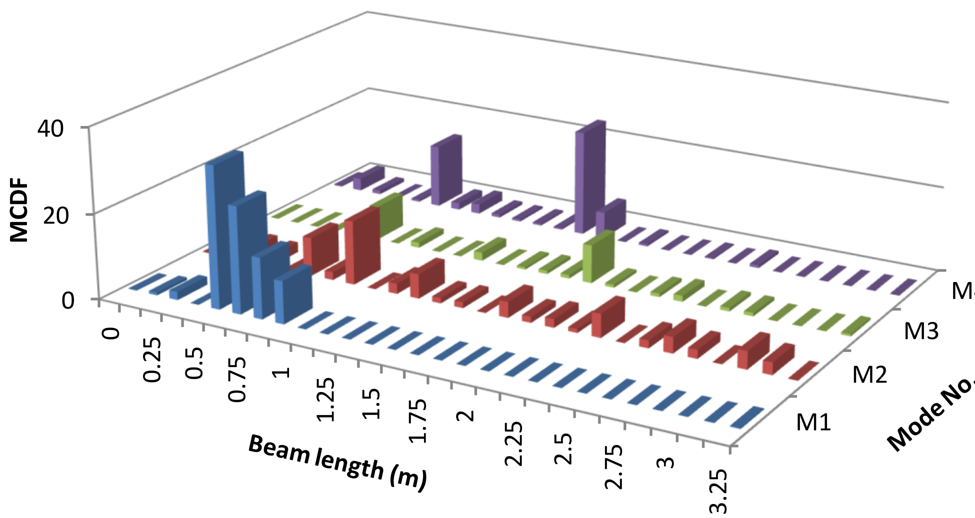


Figure 13. MCDF for damages located at quarter-span.

damage case when initial damage load is applied and released. Figures 12 and 13 shows the MCDF values for damage cases C1 and C2, respectively.

The result showed that the modified CDF was more sensitive for detecting damage location compared to the original CDF. MCDF had the ability to indicate the damage at different locations along the beam length. The results show that all the modes have good sensitivity to detect the damage location in the case of the single damage. Mode one showed some anomalies along the beam length in the damage case C1 and mode two showed some anomalies along the beam length in the damage case C2. The SAE procedure was applied to eliminate the anomalies in MCDF values for Modes 1 and 2 and as shown in Figures 14 and 15 respectively for cases C1 and C2. The results showed that the SAE procedure had increased the sensitivity by eliminating the anomalies along the beam length. The modification on CDF had been examined in order to check its ability and sensitivity to detect multi damage cases and Figures 16 and 17 show the results of MCDF for damage cases C3 and C4 respectively. The results of damage case C3 showed good ability of MCDF to detect multi damage locations, proofs of which are detected damages located at quarter span and mid-span which match the actual damage locations. Modes two to four detect damage located at the zone between the quarter-span and the mid-span which is affected by the actual damage. The

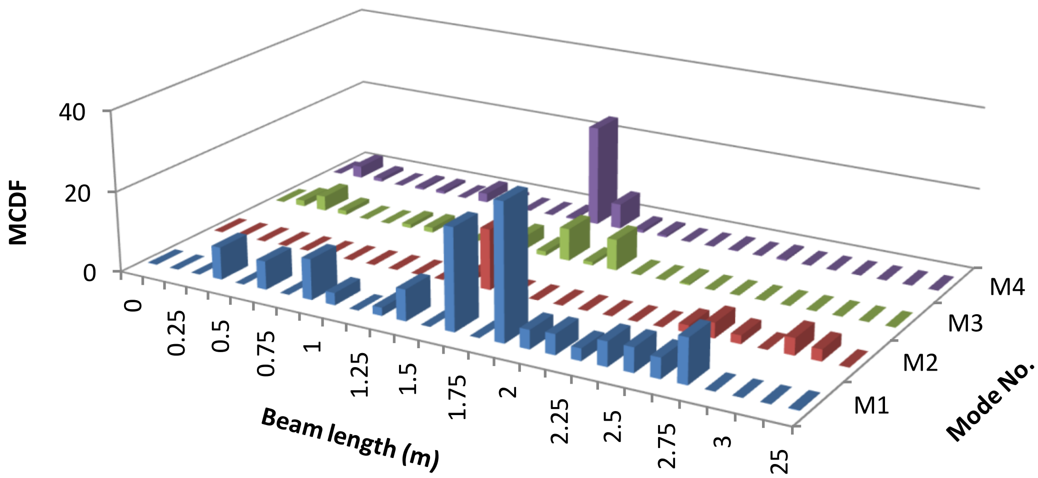


Figure 14. MCDF values for damage case C1 after applying SAE procedure.

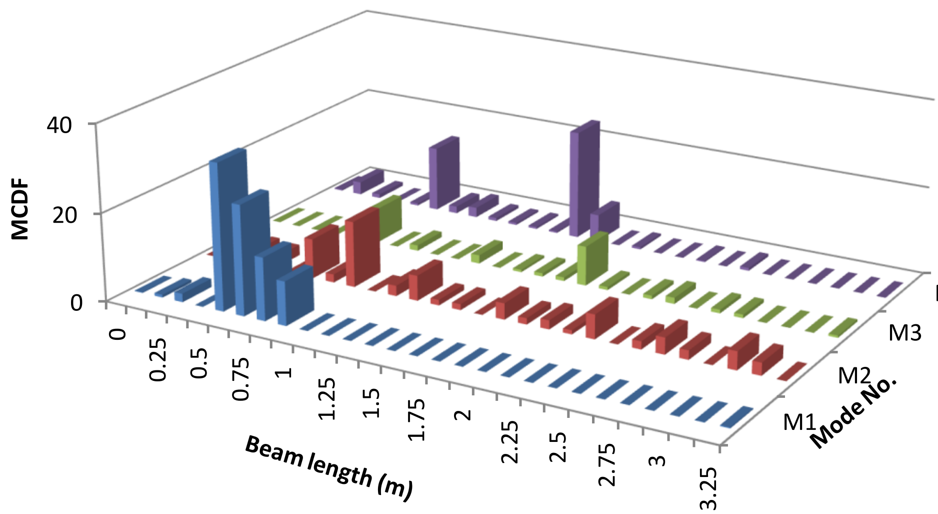


Figure 15. MCDF values for damage case C2 after applying SAE procedure.

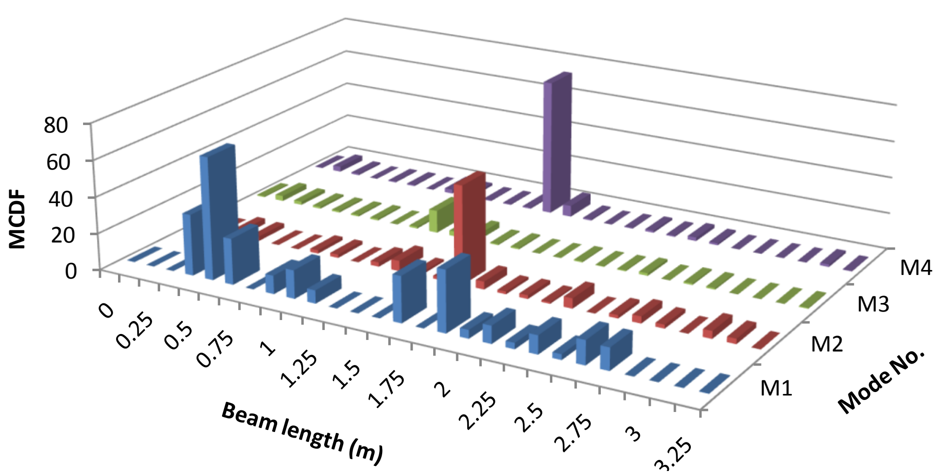


Figure 16. MCDF values for damage case C3.

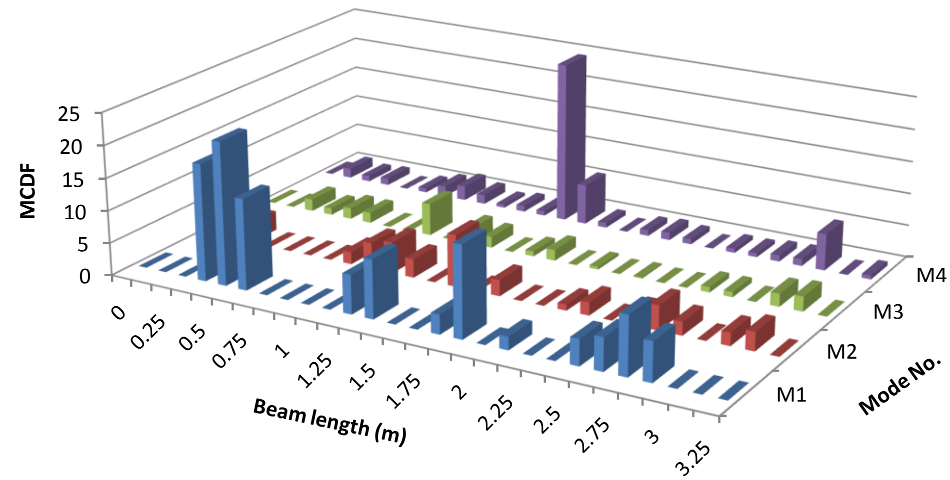


Figure 17. MCDF values for damage case C4.

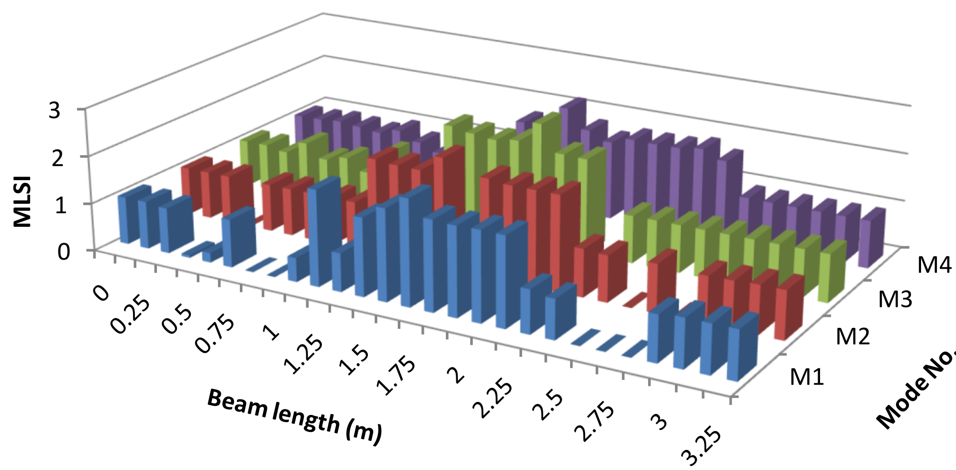


Figure 18. MLSI values for damages located at mid-span.

results of the damage case C4 show that MCDF still has acceptable sensitivity to detect the damage in cases of more than two damage locations.

Modified local stiffness indicator (MLSI)

The local stiffness indicator (LSI) was developed to have indicators for cases when data for the initial state of the structure before damage is unavailable. However it was concluded that there are anomalies due to boundaryconditions presumably due to free vibration equation of Euler beam used for simply supported case.Furthermore, the LSI is based on the fourth derivative of mode shape and any anomaly will be amplified depending on the degree of the derivative, thus making it significant at the support. For cases when the datum data is available and to overcome the anomalies problem, the modified form is given by Equation 8, expressed as a ratio of the LSI for the damage and control cases. In the modified form and if there are anomalies due to boundary conditions at the supports, it will be eliminated by dividing the damage over control:

$$\text{MLSIij} = \frac{LSI_d}{LSI_C} = \left| \frac{y_d^4 * y_c}{y_c^4 * y_d} \right| \qquad (15)$$

Where MLSI is the modified indicator, 'i' is the node number, 'j' is the mode shape number, y^4 is the fourth derivative of mode shape. The subscript'd' is damage case and 'c' is control case. Figures 18 and 19 show the results of MLSI for the finite element results for damage cases C1 and C2.

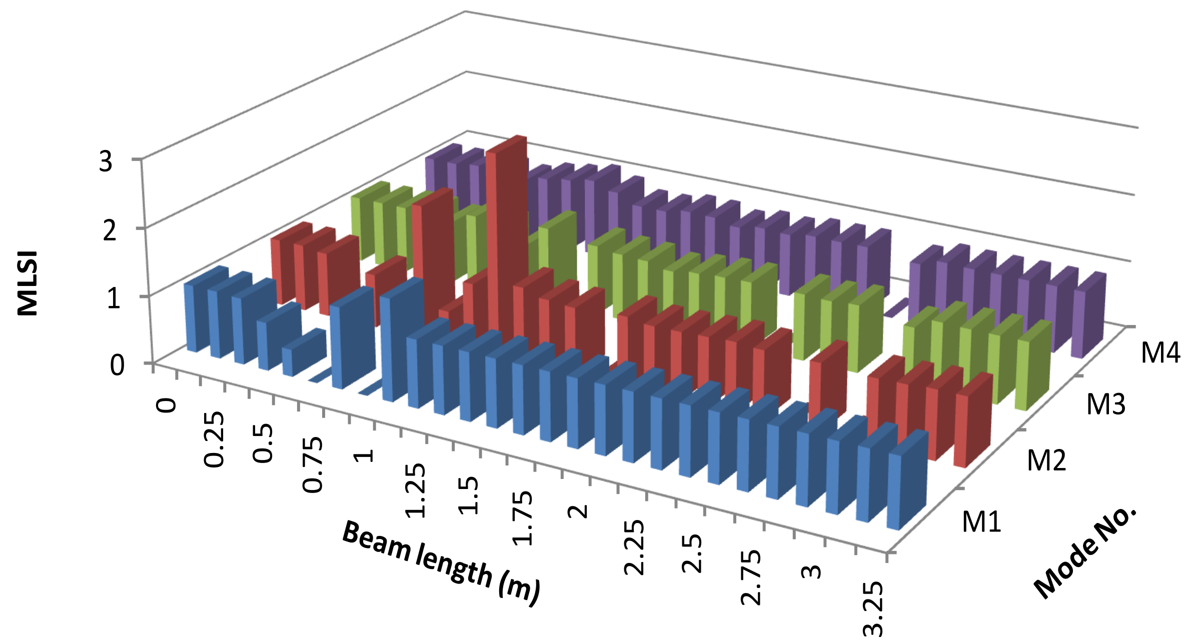


Figure 19. MLSI values for damages located at quarter-span.

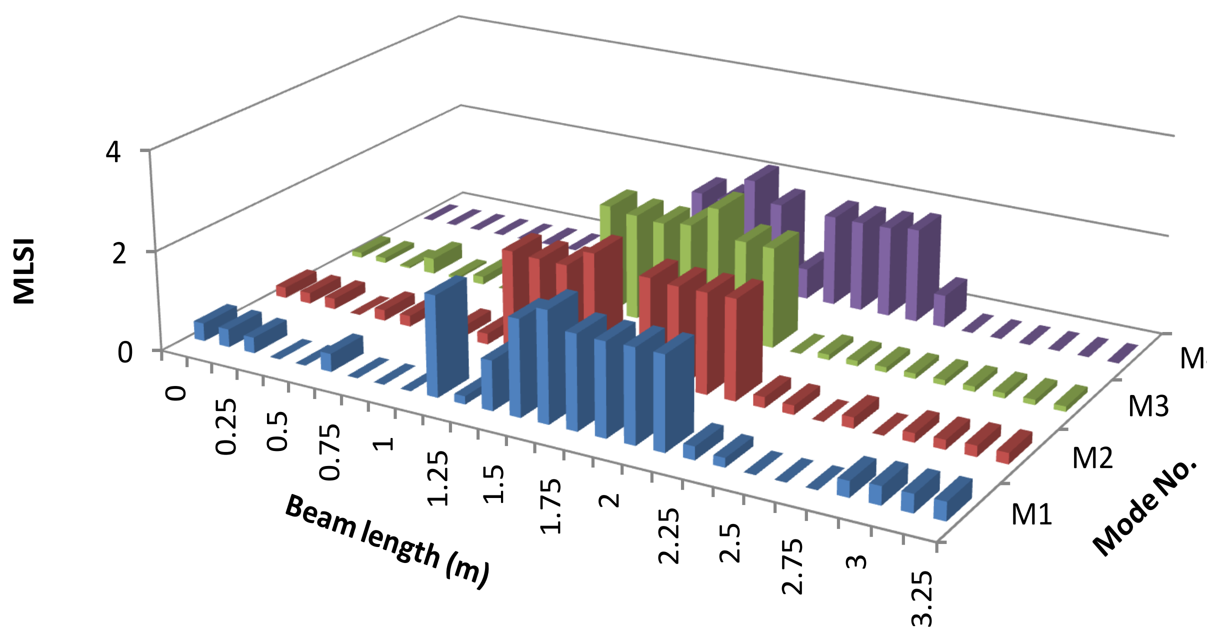


Figure 20. MLSI values for damage case C1 after applying SAE procedure.

The results showed that the modified LSI was more sensitive for detecting damage location compared to the original LSI in its original form. In addition, the peak values at the supports were absent, thus eliminating the anomalies which were obvious when using the unmodified form of the LSI. The results showed good ability of MLSI to detect the damage wherever it is located. However, MLSI values showed anomalies along the beam length for all the four modes. The SAE procedure was applied on the MLSI values to eliminate the anomalies and Figures 20 and 21 shows MLSI values after performing the SAE procedure. The results showed that the SAE had improved the sensitivity of MLSI byeliminating the anomalies along the beam length.

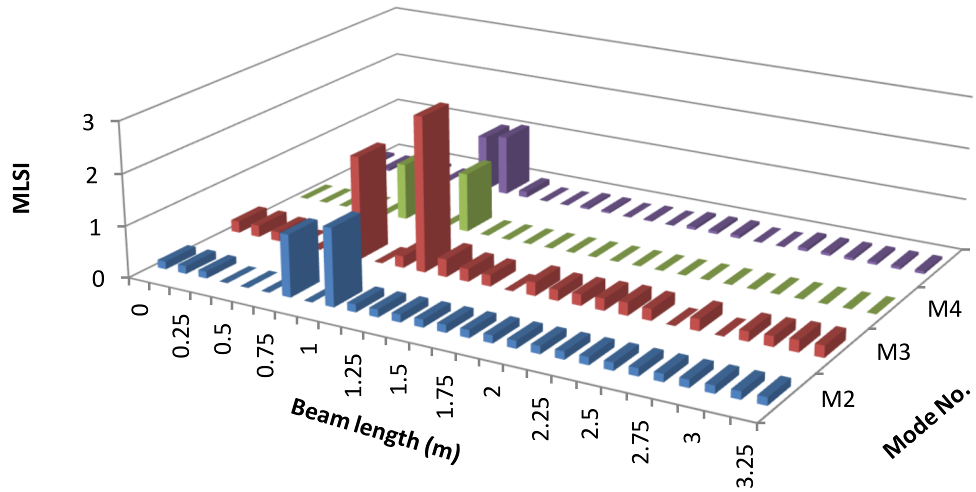


Figure 21. MLSI values for damage case C2 after applying SAE procedure.

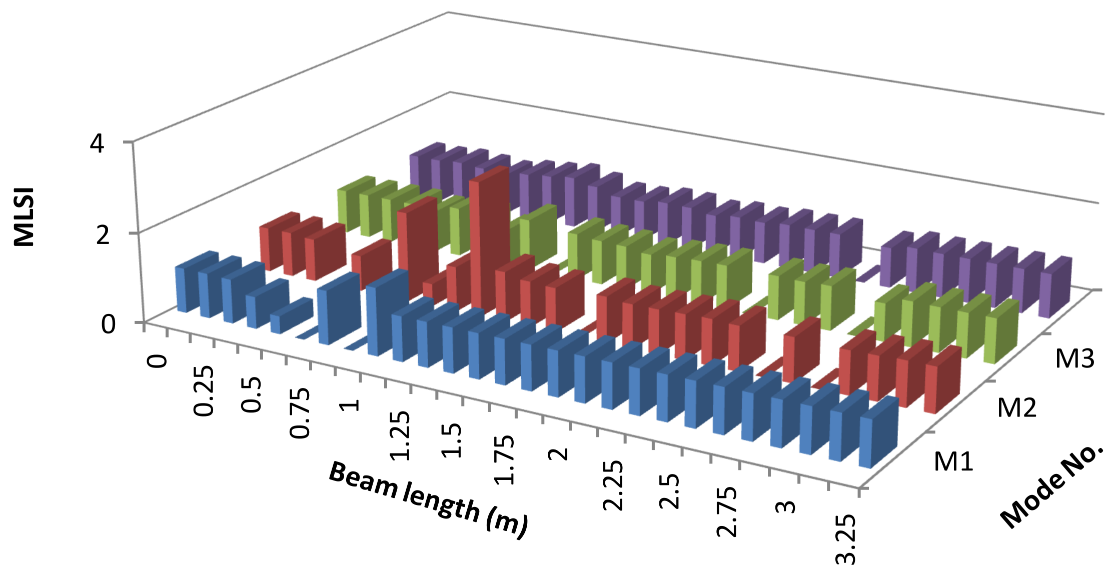


Figure 22. MLSI for multi-damages located at quarter and mid span.

damage along the distance from 1250 to 2000 mm from the left support which is matching the actual damage area although it is wider than the actual damage area. For the case when the damage located at quarter-span, MLSI had higher sensitivity where all the four modes detect a damage located at the area from 626 to 875 mm from the left support, which matches the actual damage area. MLSI was examined for the multi damage locations cases; and Figures 22 and 23 show the MLSI results for damage cases C3 and C4 respectively. For the case of two damage locations, MLSI detected a damage located between the mid-span and the quarter-span which is matching with the actual damage areas while it showed anomalies along the beam length. In the case of three damage locations, MLSI still had good sensitivity to detect the different damage locations, while it also showed anomalies along the beam length. The values of MLSI needs to be improved, so SAE procedure was applied and the results are shown in Figures 24 and 25 for MLSI values at damage cases C3 and C4 respectively. The results showed that the SAE has improved the sensitivity of MLSI by eliminating the anomalies along the beam length. For the case of two damage locations, mode one detected damage located at element 4 which is close to one of the actual damage locations; mode two detected a damage located at elements 2, 3 and 7 which matches the actual damage locations, modes three and four detected a damage

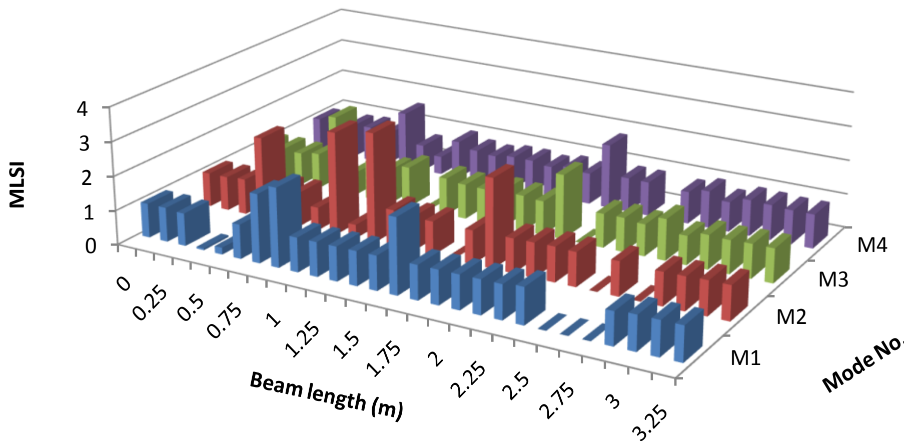


Figure 23. MLSI for multi damage case C4.

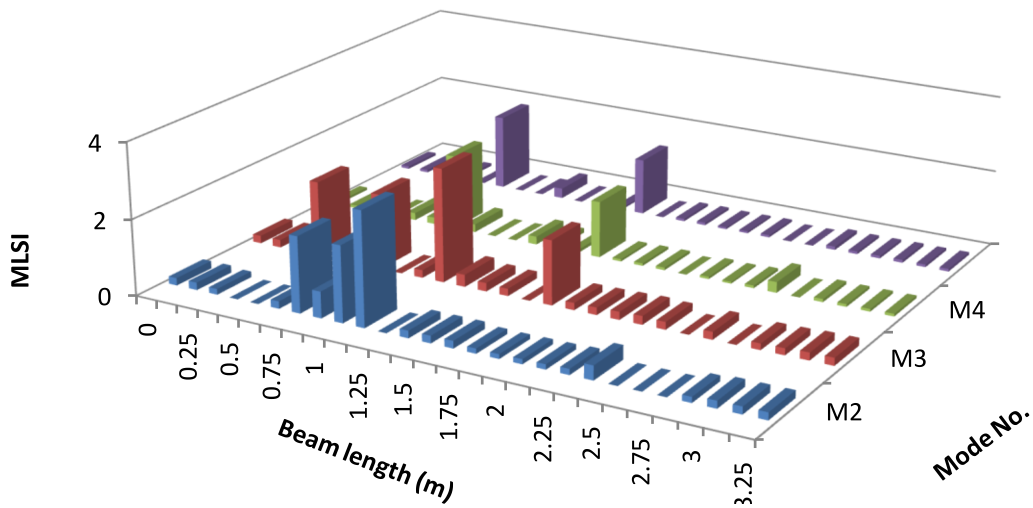


Figure 24. MLSI for damage case C3 after applying SAE procedure.

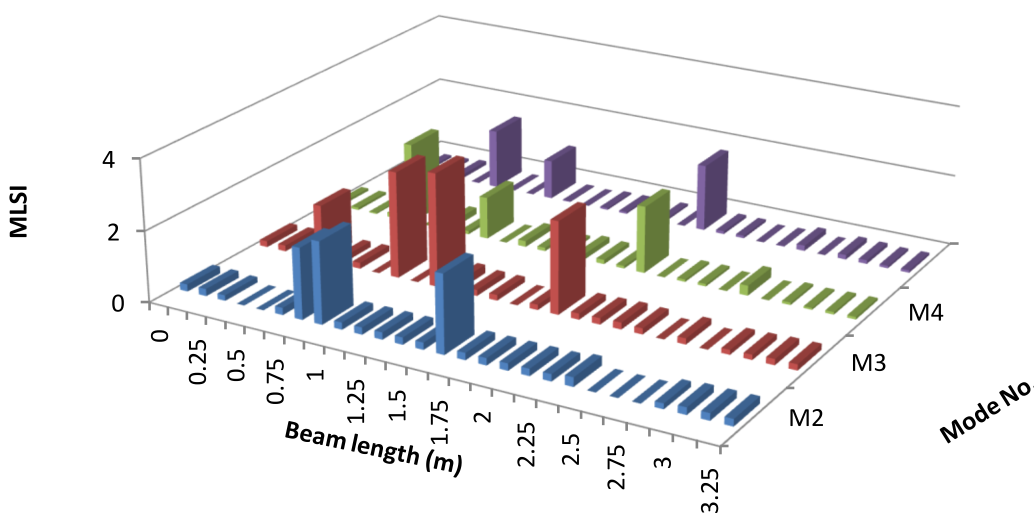


Figure 25. MLSI for damage case C4 after applying SAE procedure.

located at elements 3 and 7 which matches the actual damage locations. This proves that the MLSI detected the two damage location through the four modes.

For the case of three damage locations, mode one detected a damage located at elements 4 and 7 which is close to the actual damage locations at elements 5 and 8; mode two detected a damage located at 2, 5 and 8 which matches the actual damage locations, modes three and four detected a damage located at elements 2, 3 and 8 which matches the two actual damage locations. This indicates that MLSI was able to detect all the three damage locations through the four modes.

Conclusions

From this study, the following conclusions were drawn:

i) Both the change in curvature and the fourth derivative of mode shape are satisfactory indicators for location of singular cracks in the beam.
ii) Both CDF and LSI in their original form experience anomalies at the supports and along the beam length which affects their sensitivity to detect the damage location.
iii) The mathematical modifications on the original form of CDF and LSI by meaning of MCDF and MLSI have improved the sensitivity of both original algorithms.
iv) Applying SAE procedure has improved the sensitivity of MCDF and MLSI by suppressing the anomalies along the beam length.
v) MCDF and MLSI have shown good ability to detect the damage location for the cases of single crack location, two crack locations and three crack locations.
vi) MCDF and MLSI were validated as good damage localization algorithms, although for the cases of very small damage level which is 1% reduction in the stiffness.

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