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Particle swarm optimisation based proportional integral and derivative (PID) controller design for linear discrete time system using reduced order model

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Design of proportional integral and derivative (PID) controller for a higher order discrete system is presented in this paper. The design and tuning of PID controller parameters proportional gain K_p, integral gain K_i and derivative gain K_d are complex and time consuming process for a higher order system. To overcome this, an approximate second order equivalent model is obtained for the higher order system which preserves the stability and retains the characteristics of original system. The proposed method of model reduction uses the advantages of Routh stability array and Particle swarm optimization (PSO). PID controller is designed using reduced order model and is cascaded with the original system. The performance specifications of the closed loop system with the designed controller are analysed. The proposed algorithm is illustrated through numerical examples.

Key words: Cumulative error index, model reduction, particle swarm optimisation, proportional integral and derivative (PID) controller, Routh array, stability.

INTRODUCTION

Proportional integral and Derivative (PID) control is one of the control methods for a plant. Many control systems, using conventional PID controllers are working well. So they still have wide range of applications in industrial control. Also these controllers are having simplicity and robust in nature. Most of the industrial processes are complex in nature and are defined by mathematical equations of higher order.

Design of PID controller for a complex higher order system is complicated in nature which requires model order reduction. Model order reduction is a technique that takes a system containing a large number of poles and reduces it to a smaller representation consisting of the dominant poles from the original linear system. The Routh Approximation method introduced by Hutton et al. (1975) has proven to be the most successful tools for getting the reduced order models due to its simplicity in computation and efficiency. Many researchers have been working with this method. Mixed methods for obtaining the reduced order models are given by Gutman et al. (1982), Chen et al. (1980) and Singm et al. (2004). In these, denominator is derived using stability criterion and numerator is obtained using some other methods.

The main problem associated with the PID controller tuning is overcome by using the advantage of Particle swarm Optimisation (PSO). In the recent years, evolutionary techniques are used in all the fields of research and Riccardo Poli (1978) presented a review article on the publications on the applications of Particle Swarm Optimisation. PSO is a computational method which is used to get optimised solution for the defined problem. It is a population based search algorithm in which each individual is referred to a particle and represents a solution. The position and velocity of each

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particle in the search space is dynamic and varying according to the individual and the neighbor particle. Each particle has a memory and hence it is capable of remembering the best position in the search space. The position corresponding to the best fitness is known as pbest and the overall best out of all the particles in the population is called gbest which was referred by Kennedy and Eberhart (1995). The equations defined in (1) and (2) are used to modify velocity and position of each particle in such a way that distance from the $pbest_{j,g}$ to $gbest_g$ is reached.

$$v_{j,g}^{(t+1)} = w * v_{j,g}^{(t)} + c_1 * r_1 * (pbest_{j,g} - x_{j,g}^{(t)}) + c_2 * r_2 * (gbest_{j,g} - x_{j,g}^{(t)}) \quad (1)$$

$$x_{j,g}^{(t+1)} = x_{j,g}^{(t)} + v_{j,g}^{(t+1)} \quad (2)$$

With j =1, 2,...,n and g = 1, 2,..., m
n = number of particles in the swarm
m = number of components for the vectors v_j and x_j
t = number of iterations (generations)
w = inertia weight factor
c_1, c_2 = cognitive and social acceleration factors
r_1, r_2 = random numbers distributed in the range (0, 1)
$v_{j,g}^{(t)}$ = The g-th component of the velocity of particle j at iteration t
$x_{j,g}^{(t)}$ = The g-th component of the position of particle j at iteration t.
$pbest_j$ = pbest of particle *j*
gbest = gbest of the group

Panda et al. (2009) presented an algorithm for the reduction of linear systems by conventional and evolutionary techniques. In this PSO method is based on the minimization of Integral Square Error (ISE) between the responses of original and reduced systems applied to step input. Model order reduction based on PSO was proposed by Gallehdari et al. (2009) applied to power systems and Mittal and Dinesh (2009) proposed a reduction method for discrete systems.

Hence to obtain reduced second order model, the mixed advantages of routh stability array and PSO are used.

To design PID controller, the reduced order model is used. Design of Controller using lower order models are discussed by Yadav et al. (2010) and Ekhlas et al. (2011). Hoang Bao et al. (2009) proposed a new semi analytical approach to design fixed PI controllers for higher order systems using order reduction technique. Design of PID controllers for discrete time linear system is presented in Ravichandran et al. (2007). It uses balanced approach for getting reduced order model. Ayyar et al. (2012) presented an application on design of speed control for DC motor drive using model order reduction technique and proved that MOR based controller enhances the performance and dynamics of DC motor in comparison to conventional PI controllers. A novel scheme is proposed to obtain a second order reduced model for stable LTICS by Manigandan et al. (2005) and Gomathi et al. (2009) which provides good approximation and preserves stability of the system.

PID controller is designed for a system to meet the desired performance specifications such as rise time, settling time and percentage overshoot. The performance specifications will normally selected for any process is given below:

Rise time ≤ 3 s
Settling time ≤ 3 s
Overshoot ≤ 2%

The transfer function of PID controller is defined as:

$$G_c(z) = K_p + K_i \frac{Tz}{z-1} + K_d \frac{z-1}{Tz} \quad (3)$$

Where,
K_p=Proportional gain constant
K_i=Integral gain constant
K_d=Derivative gain constant
T =Sampling time in seconds

The design of PID controller for a system involves the tuning of controller parameters such as proportional gain (K_p), Integral Gain (K_i) and Derivative gain (K_d) for satisfying the desired performance of the system. Tuning of PID controller involves the adjustment of parameters to meet the desired specification. The major problems in the manual tuning of PID controller are time taken for the tuning process and experience of the person doing the tuning. To reduce the complexity involved in manual tuning of the Controller parameters, many methods have been suggested by Wang et al. (2008), Girirajkumar et al. (2010), Rajinikanth and Latha (2011), Jalivand et al. (2011), Nagendra et al. (2012) and Rajinikanth and Latha (2012). It is found that Particle Swarm Optimisation is very simple to have the tuning of PID controller parameters.

Hence it is proposed to convert the given higher order discrete system from discrete to continuous domain using Bilinear transformation and reduced model is obtained. This reduced model is converted to discrete domain using inverse bilinear transformation. Using the reduced model in discrete domain, the PID controller is designed. The advantage of PSO is used in the paper for the tuning of parameters of discrete PID controller.

STATEMENT OF THE PROBLEM

Consider closed loop block diagram of a higher order

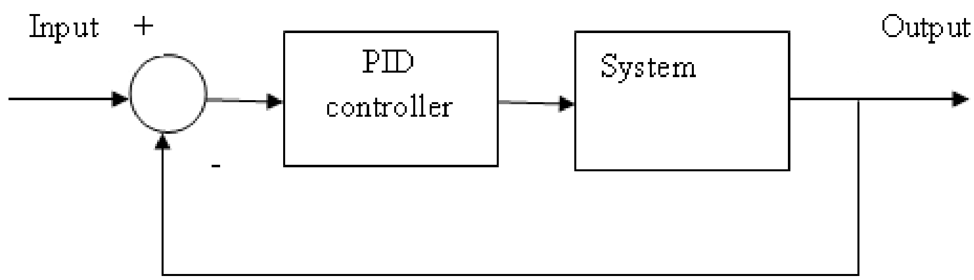

Figure 1. Block diagram for closed loop system.

system G(z) cascaded with PID controller $G_c(z)$ as shown in Figure 1.

The design of PID controller for a higher order system is difficult and so it is proposed to reduce the given higher order system into lower order model before the design of PID controller.

Consider G(z) as n^{th} order stable linear time invariant discrete system described by the transfer function

$$G(z)=\frac{N(z)}{D(z)}=\frac{B_0z^m+B_1z^{m-1}+B_2z^{m-2}+B_3z^{m-3}+.........+B_m}{A_0z^n+A_1z^{n-1}+A_2z^{n-2}+A_3z^{n-3}+.........+A_n} \quad (4)$$

where, A_i $(0 \le i \le n)$ and B_i $(0 \le i \le m)$ are scalar constants and $m \le n$.

The higher order system is converted from discrete to continuous domain using Bilinear transformation $z=\frac{1+s}{1-s}$ as in Equation (5).

$$G(s)=\frac{N(s)}{D(s)}=\frac{b_0s^m+b_1s^{m-1}+b_2s^{m-2}+b_3s^{m-3}+.........+b_m}{a_0s^n+a_1s^{n-1}+a_2s^{n-2}+a_3s^{n-3}+.........+a_n} \quad (5)$$

The corresponding reduced second order model is of the form obtained by using the proposed method

$$R_2(s)=\frac{d_1s+d_0}{e_2s^2+e_1s+e_0} \quad (6)$$

The reduced model defined in Equation (6) is converted to discrete domain using Inverse bilinear transformation and is defined in (7).

$$R_2(z)=\frac{D_1z+D_0}{E_2z^2+E_1z+E_0} \quad (7)$$

Problem is to find a reduced order model in the form of Equation (7) for the original system described by Equation (4), such that the reduced order model preserves stability and the input output behavior of the original system. By using reduced model defined in (7) PID controller is designed for the higher order system.

PROPOSED ALGORITHM

The proposed algorithm for the design of PID controller is given in two stages.

(i) Obtaining reduced second order model for the given higher order system.
(ii) Design of PID controller to meet the desired performance specifications.

Procedure to obtain reduced second order model for the given higher order system

The procedure to obtain reduced second order model of the system defined in Equation (5) is explained as follows:

Step 1: Obtain the higher order discrete system transfer function.
Step 2: Obtain the response of the given system and check for the designer's specifications.
Step 3: If the specifications are not achieved, convert the given higher order discrete system to continuous domain using Bilinear transformation

$$z=\frac{1+s}{1-s}.$$

Step 4: Select the values for d_0 and e_0 from the continuous time transfer function G(s) as in Equation (5) d_0= b_m= Constant of Numerator of the original system e_0= a_n= Constant of denominator of the original system.
Step 5: Consider the numerator polynomial

N(s)= $b_0s^m+b_1s^{m-1}+b_2s^{m-2}+b_3s^{m-3}+.........+b_m$

of the given higher order system defined in Equation (4) and form the Routh stability array

Step 6: Obtain first order polynomial from the Routh stability array (8) and consider it as the numerator of the reduced second order transfer function,

$$Nr(s) = d_1 s + d_0 \quad (8)$$

Step 7: By equating and cross multiplying the transfer functions (5) and (6), obtain set of (n+2) equations in terms of d_1, d_0, e_2, e_1 and e_0.
Step 8: Compute the value e_2 and e_1, by solving (n+2) equations by substituting the values of d_1, d_0 and e_0.
Step 9: Obtain the reduced second order Equation $R_2(s)$ by substituting the values of e_2, e_1, e_0, d_1 and d_0 in (6).
Step 10: The values of e_2, e_1 and d_1 can be optimized using (1) and (2) of PSO in such a way that the cumulative error index for the step responses of original and reduced order models is minimized. For comparing the proposed model with various reduced order models, an error criterion called cumulative error index 'J' is used as given below.

$$J= [Y(ti)-Yk(ti)]^2 \quad (9)$$

Where, Y(ti) and Yk(ti) are the outputs of the original and reduced order systems respectively at the i^{th} instant.
Step 11: Obtain the optimum values for the co efficient e_2, e_1 and d_1.
Step 12: Obtain the optimized reduced order model which produces minimum cumulative error.
Step 13: Convert the reduced second order model from continuous domain to discrete domain using inverse bilinear transformation as defined in Equation (7).

Procedure for the design of proportional integral and derivative (PID) controller

Step 1: Get the reduced second order model for the given discrete system using the proposed method.
Step 2: Calculate the initial values of K_p, K_i and K_d using pole zero cancellation applied to reduced order model.
Step 3: Using PSO tune the values of K_p, K_i and K_d, so that the desired performance specifications are achieved.
Step 4: Construct the PID controller using the tuned values as in (3) and cascade it with the original and reduced order systems. Obtain the closed loop response by applying unit step input.
Step 5: Verify the desired specifications

Numerical example 1

Consider the eighth order system transfer function from Ravichandran et al. (2007),

$$G(z) = \frac{1.682z^7 + 1.116z^6 - 0.21z^5 + 0.152z^4 - 0.516z^3 - 0.262z^2 + 0.044z - 0.018}{8z^8 - 5.046z^7 - 3.348z^6 + 0.63z^5 - 0.456z^4 + 1.548z^3 + 0.786z^2 - 0.132z + 0.018} \quad (10)$$

Obtaining reduced order model

The eighth order original system given in Equation (10) is transformed into G(s) as in Equation (11) by using Bilinear transformation, $z = \frac{1+s}{1-s}$.

$$G(S) = \frac{-0.0015s^8 - 1.196s^7 - 11.66s^6 - 25.6s^5 - 17.78s^4 + 66.37s^3 + 223.4s^2 + 257.5s + 63.62}{s^8 + 19.66s^7 + 146.5s^6 + 526.8s^5 + 1168s^4 + 1601s^3 + 1114s^2 + 256s + 64} \quad (11)$$

Obtaining reduced order model

Order of the system n=8.
Select the values for d_0 and e_0 from the given transfer function G(s) as, d_0= 63.62; e_0= 64.
Consider the Numerator polynomial of the given transfer function and form the Routh stability array and get the value for d_1= 209.437.
Construct the numerator polynomial of the reduced transfer function as:

$N_2(s)$ = 209.437s+63.62.

Using step 7 of reduction algorithm, the following Equations (11 to 20) are obtained.

$$-0.0015e_1 - 1.196e_2 = d_1 \quad (12)$$

$$-0.0015e_0 - 1.196e_1 - 11.66e_2 = 19.66d_1 + d_0 \quad (13)$$

$$-1.196e_0 - 11.66e_1 - 25.6e_2 = 146.5d_1 + 19.66d_0 \quad (14)$$

$$-25.6e_0 - 17.78e_1 + 66.37e_2 = 1168d_1 + 526.8d_0 \quad (15)$$

$$-11.66e_0 - 25.6e_1 - 17.78e_2 = 526.8d_1 + 146.5d_0 \quad (16)$$

$$-17.78e_0 + 66.37e_1 + 223.4e_2 = 1601d_1 + 1168d_0 \quad (17)$$

$$66.37e_0 + 223.4e_1 + 257.5e_2 = 1114d_1 + 1601d_0 \quad (18)$$

$$223.4e_0 + 257.5e_1 + 63.62e_2 = 256d_1 + 1114d_0 \quad (19)$$

$$257.5e_0 + 63.62e_1 = 64d_1 + 256d_0 \quad (20)$$

$$63.62e_0 = 64d_0 \quad (21)$$

By substituting the values of d_1 and d_0 in (19) and (20), the values of e_2 and e_1 are calculated as:

e_2 = 891.5645 and e_1 = 207.649

Hence the reduced order model is obtained using (6) as:

$$R_2(s) = \frac{209.437s + 63.62}{891.65s^2 + 207.649s + 64} \quad (22)$$

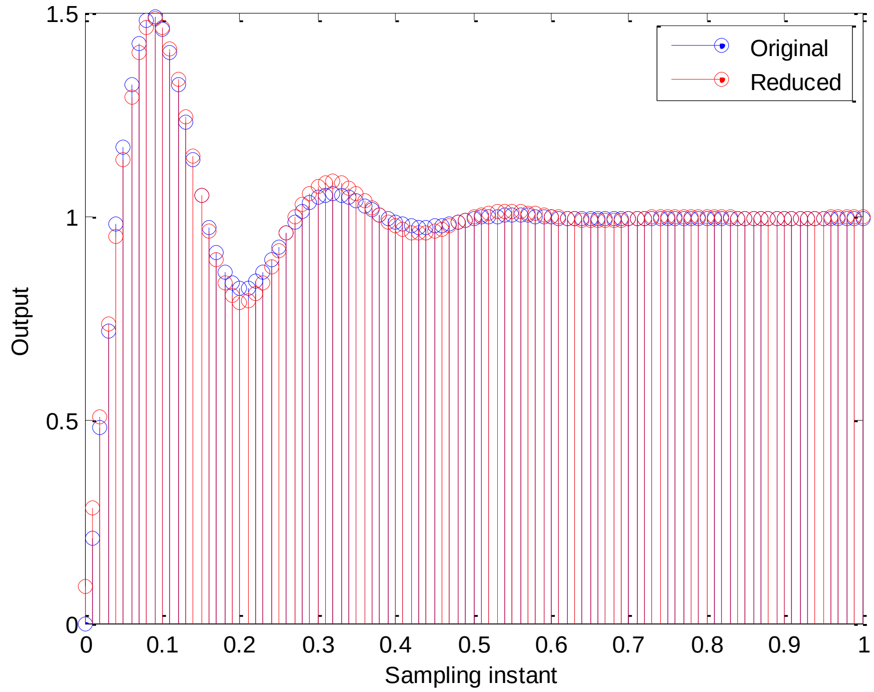

Figure 2. Comparison of step responses of original and reduced order system in example 1.

Table 1 Comparison of cumulative error index for example 1.

Method of reduction	Reduced model	Cumulative error index 'J'
Ravichandran et al. (2007)	$R_2(z) = \dfrac{-0.02962z^2 + 0.2988z - 0.1953}{z^2 - 1.7593z + 0.8336}$	0.1500
Manigandan et al. (2005)	$R_2(z) = \dfrac{2.366z - 1.358}{11.26z^2 - 19.77z + 9.507}$	0.0839
Proposed method	$R_2(z) = \dfrac{0.08835z^2 + 0.03753z - 0.05081}{z^2 - 1.786z + 0.8614}$	0.0256

The coefficients d_1, e_2 and e_1 are optimized using Particle swarm optimization (PSO). Select the desired fitness function for the Particle swarm optimization as minimizing the cumulative error in step responses of G(s) and R_2(s). The following specifications are selected for the PSO to get better optimisation: Size of the swarm (number of birds) = 40; Number of iterations = 40; Inertia constant=0.8

The optimized values obtained are: d_1=117.9429, e_2 = 772.8146 and e_1 = 117.4470

The reduced order model for the given transfer function is as in (23),

$$R_2(s) = \frac{117.9429s + 63.62}{772.8146s^2 + 117.4470s + 64} \quad (23)$$

Using inverse bilinear transformation discrete time reduced model is obtained from (23) as in (24)

$$R_2(z) = \frac{0.08835z^2 + 0.03753z - 0.05081}{z^2 - 1.786z + 0.8614} \quad (24)$$

The comparison of step responses between original and reduced system is shown in Figure 2. Using (9) the cumulative error index is calculated for various methods and tabulated in Table 1.

Design of PID controller

Using pole zero cancellation method, the initial values of

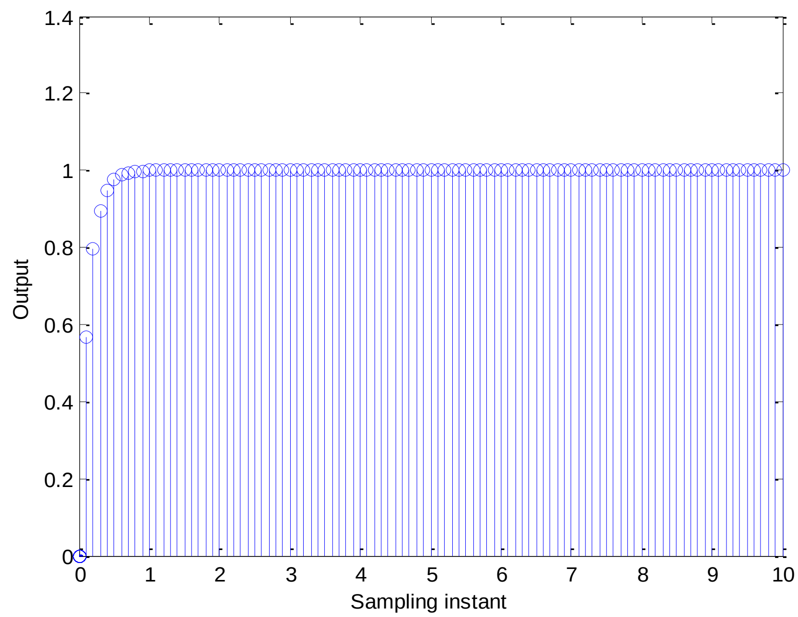

Figure 3. Step response of original system with PID controller of example 1.

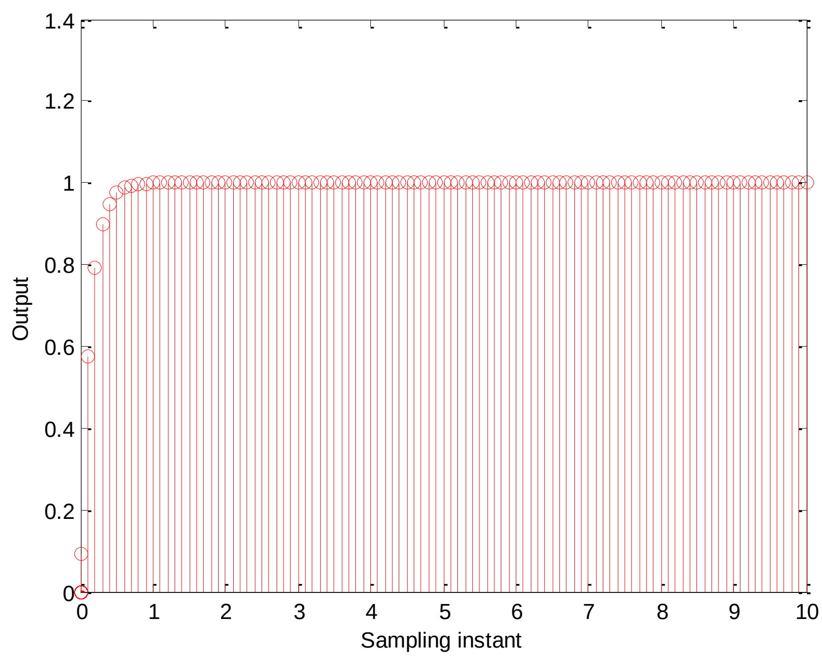

Figure 4. Step response of reduced system with PID controller of example 1.

the PID controller are obtained from (28) as: K_p=0.0632 K_i=0.0754 and K_d=0.8614.

The values of $K_{p,}$ K_i and K_d are tuned using PSO to achieve the designers specification. The tuned values using PSO are: K_p=0.076, K_i=0.0805 and K_d =0.9094. Using the tuned values of $K_{p,}$ K_i and K_d discrete PID controller is designed as in (3).The sampling time T is considered as 0.01 s.

For the verification, the designed PID controller with tuned values is cascaded with original system (10) and reduced system (24). The closed loop performances of original and reduced systems with PID controller are given in Figures 3 and 4.

Numerical example 2

Consider the sixth order system transfer function from

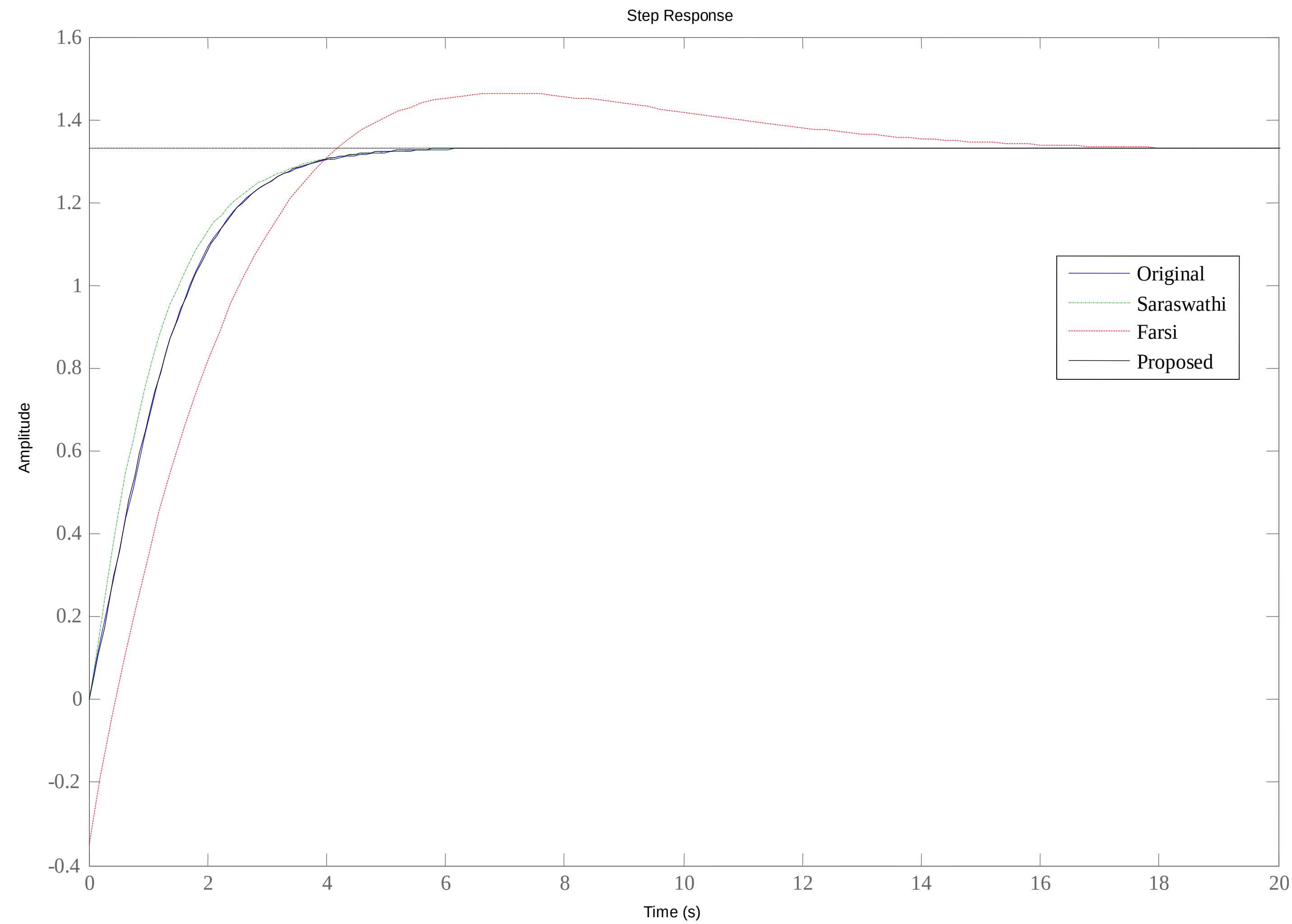

Figure 5. Comparison of step responses of original system and various reduced order systems of example 2.

Saraswathi (2011),

$$G(z)=\frac{0.3277z^6+0.9195^5+1.038z^4+0.5962z^3+0.1618z^2+0.006986z-0.005308}{z^6+1.129z^5+0.2889z^4-0.08251z^3-0.04444z^2-0.00476z} \quad (25)$$

Obtaining reduced order model

The sixth order original system given in equation (25) is transformed into G(s) as in equation (26) by using Bilinear transformation, $z=\frac{1+s}{1-s}$.

$$G(S)=\frac{s^5+15.6s^4+124.2s^3+510.3s^2+1166s+959.3}{s^6+21s^5+175s^4+735s^3+1624s^2+1764s+720} \quad (26)$$

Using the proposed algorithm of model order reduction, the reduced order model for the transfer function is given in (27)

$$R_2(z)=\frac{0.3228z^2+0.4409z+0.1181}{z^2-0.3693z+0.03116} \quad (27)$$

The step response of the proposed method is compared with existing methods given by Saraswathi (2011) in Figure 5. For comparing reduced model with the original model error criterion ‘J’ is calculated and it is tabulated for various methods in Table 2.

The values of K_p, K_i and K_d are obtained from Equation (23) and tuned using PSO to achieve the designers specification. The tuned values are:

Kp=0.8581; Ki=0.7938; Kd=0.0105

Using the tuned values of K_p, K_i and K_d discrete PID controller is designed as in (3).The sampling time T is considered as 0.01 s.

Table 2 Comparison of cumulative error index for example 2.

Method of reduction	Reduced model	Cumulative error index 'J'
Farsi et al. (1986)	$R_2(z)=\frac{0.589z-0.4495}{z^2-1.428z+0.5329}$	3988e-4
Saraswathi (2011)	$R_2(z)=\frac{0.3052z^2+0.5922z+0.2869}{z^2-0.1111}$	7.10e-04
Proposed method	$R_2(z)=\frac{0.3228z^2+0.4409z+0.1181}{z^2-0.3693z+0.03116}$	0.95e-04

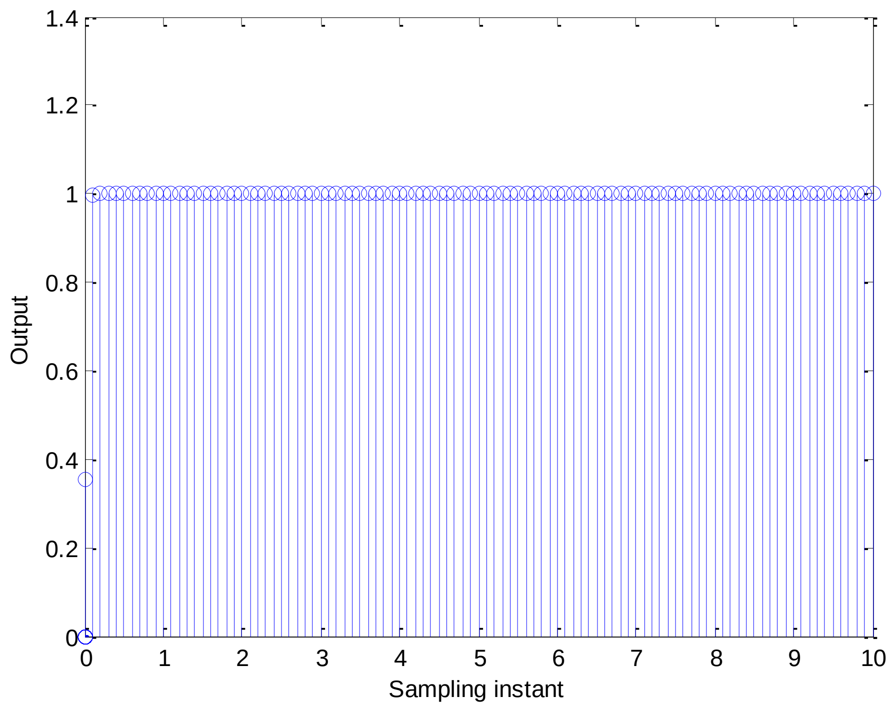

Figure 6. Step response of original system with PID controller of example 2.

The designed controller is cascaded with original system defined in (25) and reduced system defined in (27) to get the closed loop response supplied with unit step input. The responses are given in Figures 6 and 7. The performance specifications are verified.

CONCLUSION

In this paper, PID controller for a higher order discrete system was designed using reduced order model. The given higher order discrete system was converted into continuous domain using Bilinear transformation and reduced to equivalent second order model using Routh stability array and Particle Swarm Optimization. After model reduction the system was converted to discrete system using inverse bilinear transformation. The initial values of PID controller parameters (K_p, K_i and K_d) were identified using pole zero cancellation method applied to second order model. The tuning K_p, K_i and K_d values was done by using Particle swarm optimization. The algorithm was explained with numerical example and the results were compared. The simulation results proved that the proposed method improves the dynamic response of the system comparing with other existing methods and simplifies the process of PID controller tuning. Hence the proposed method reduces the complexity involved in the tuning of PID controller parameters for higher order systems and it can be used for the design of PID controller for complex systems.

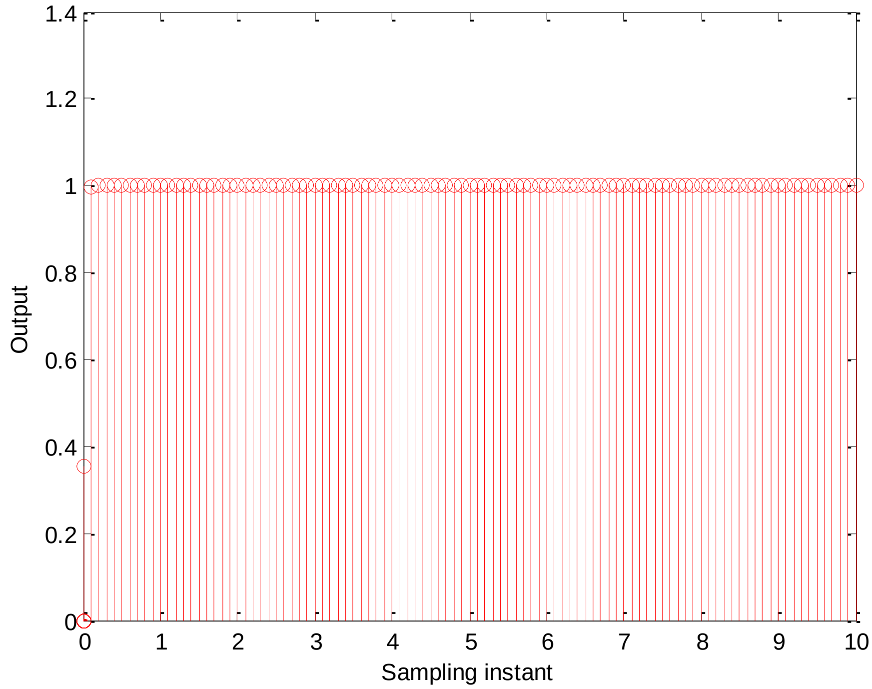

Figure 7. Step response of reduced system with PID controller of example 2.

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