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Full Length Research Paper

A research on the accuracy of landform volumes determined using different interpolation methods

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Mathematical formulation of unsmooth surfaces such as the physical earth poses several difficulties. To achieve an accurate formulation, each and every point of the surface must be defined, which is practically impossible. Therefore, in modelling such surfaces there is a need for an adequate number and distribution of reference points as well as an appropriate interpolation method. Working on computerised three-dimensional (3D) models, users can dynamically make various analyses on the surface, one of which is the volume calculation. The accuracy of the volume calculation depends on the appropriateness of the interpolation method used in creating 3D models. The present study, which aims to investigate the accuracy of the calculated volumes of objects with irregular structures, uses an artificial object with irregular form whose volume can be indirectly calculated. Using different interpolation methods on this artificial object through photogrammetrically measured reference points, 3D surface models were formed and volumes were calculated. A comparison between the measured and calculated volume figures revealed that the Inverse Distance Weighting (INDW), Nearest Neighbour (NENE) and Triangulation with Linear (TLIN) interpolation methods yielded more accurate results than the other methods. In order to investigate the volumetric accuracy, the same procedure was applied to a regular geometrically-shaped conical frustum and a pyramidal frustum with different numbers of faces, whose reference points on the surface were reproduced in accordance with mathematical rules. The volume of the pyramidal frustum which was obtained using different interpolation methods was then compared to that of the pyramidal frustum which was mathematically calculated and that of the conical frustum which was taken as the reference surface. The comparison demonstrated that the Data Metrics (DMET), INDW and Minimum Curvature (MCRV) interpolation methods yielded better results in terms of volumetric accuracy.

Keywords: Interpolation methods, DTM, landform volume, volumetric accuracy.

INTRODUCTION

Mathematical formulation of a surface which either exists in reality or is theoretically produced is referred to as surface modelling. This technology is used in various fields of science such as earth sciences, mathematics, statistics, biology and construction. Determining the shape and modelling the surface of any particular physical landform is among the tasks of several branches of earth sciences such as geology, geomorphology, geodesy and geophysics. The irregular geometric form of the physical earth prevents a formulation of a surface through mathematical functions (Torge, 1975; Heiskanen and Moritz, 2006; Huggett, 2002). The only method of modelling the surface is then to locate in the space an adequate number and distribution of earth points. Digitally reproducing surfaces defined with their 3D coordinates in the space and making various analyses on these surfaces through the modelling of location-dependent information is known as Digital Terrain Modelling (DTM) (Li et al., 2000; Peckham and Jordan, 2007).

DTM was first used as a concept in the late 1950s (Miller and Laflamme, 1958). Although at the outset it had a limited scope of application such as certain calculations and constructions regarding motorways, in parallel to developments in software and hardware technologies it is widely used in the present day for the purposes of 3D imaging, slope calculation, aspect calculation, traverse extraction, visibility analysis and volume calculations in various professional fields such as cartography, medicine, architecture, archaeology, hydrology, construction, agriculture, morphology, environment and mining (Li et al., 2000; El-Sheimy et al., 2005).

When forming a DTM, surfaces are modelled through sample points. Also named reference points, the sample points display different spatial distributions depending on the methods used to obtain and select the data. In order to obtain a high level of accuracy from the DTM, the sensitivity of the measurement method used to obtain the data, an adequate number and distribution of reference points and the selection of an appropriate interpolation method are of utmost importance. Several interpolation methods exist and thus in the selection of an interpolation method it is essential to pay attention to the intended purpose and accuracy (Skumar et al., 2001; Fencik et al., 2005; Carrara et al., 1997; Desmet, 1997).

As an important engineering application, volume calculation is used in various fields such as reserve estimation of mine sites and determination of the excavation and earth fill for sites such as roads, airports and tunnels. Three-dimensional modelling of the surface is a prerequisite of volume calculation. DTM particularly allows the calculation of the volumes of terrain models with irregular geometric shapes. Upon the construction of the 3D figure, by taking a particular enclosed area as a reference it is possible to make a dynamic and digital calculation of the volume of an enclosed shape which will be limited by a vertical distance to that surface. The accuracy of the 3D model improves the accuracy of the volume calculation (Yanalak and Baykal, 2003; Chen and Lin, 1992; Easa, 1992; Press et al., 1992; Kalmar et al., 1995; Yılmaz, 2007; Yakar, 2009).

The present study investigates the calculation of landform models constructed using different interpolation methods and volumetric accuracy. To this end, an artificial object surface allowing an indirect volume calculation was formed, then by determining the reference points on the surface using Photomodeler 5.0 software and based on the principles of photogrammetry, 3D models were formed according to different interpolation methods using Surfer 8.0 software and the volumes were calculated and compared. In addition, the volume of a conical frustum whose dimensions were previously known was used for volumetric accuracy. Transforming the surface of the conical frustum into a different number of smooth surfaces, multi-sided pyramidal frusta were obtained. In an upward direction from the base, several reference points at certain intervals were taken on the diagonal face of the pyramidal frustum, depending on which the volumes calculated on the basis of models constructed according to different interpolation methods were then compared to the volume of the conical frustum.

INTERPOLATION METHODS

A precise definition of any physical surface requires knowledge or measurement of the spatial information for every point of the surface. For such a procedure, which is practically impossible, a certain number of point sets are selected and then used to model the surface. The reference points used to represent the surface are obtained through various data collection methods. However, it is often impossible to collect data for all the needed points. In such cases, different interpolations are used with algorithms which yield the closest results to the reference points.

The number and distribution of reference points as well as the structure of the surface affect the degree of accuracy of the interpolation method. Hence while modeling a surface the intended accuracy and the objective are of crucial importance in selecting the interpolation method to be used (Mitas and Mitasova, 1999). The following paragraphs discuss some interpolation methods which are commonly used in geomorphologic and geodesic studies.

The Inverse Distance Weighting Method (INDW)

The INDW method is based on a quite simple algorithm. Therefore, it is extensively used in applications thanks to its technical appropriateness for programming. The INDW method is particularly used in defining continuously changing data on the same area.

The INDW method is a weighted average interpolator, which can be either exact or smoothing. With inverse distance to a power, data are weighted during interpolation such that the influence of one point relative to another declines with distance from the grid node (Yang et al., 2004). Force parameters indicate how the weight effect decreases as the distance increases from a grid corner. For a relatively smaller force the weights are more evenly distributed between reference points. While calculating a particular grid corner, the weight exerted on a reference point is proportional to the inverse of the distance to the specified force from the grid corner (Smith et al., 2007). The equation used for inverse distance to a power is

$$\hat{Z}_j = \frac{\sum_{i=1}^{n} \frac{Z_i}{h_{ij}^{\beta}}}{\sum_{i=1}^{n} \frac{1}{h_{ij}^{\beta}}} \quad (1)$$

$$h_{ij} = \sqrt{d_{ij}^2 + \delta^2} \quad (2)$$

where

h_{ij} is the effective separation distance between grid node j and the neighbouring point i,

$\hat{Z}_j$ is the interpolated value for grid node j,

Z_i are the neighbouring points,

d_{ij} is the distance between the grid node j and the neighbouring point i,

β is the weighting power (the power parameter), and

δ is the smoothing parameter (Yılmaz, 2007).

The Kriging Method (KRIG)

The KRIG interpolation method is a gridding method which has been extensively used in many fields such as mining, climatology and agriculture and has proved to be accurate in its fields of use. The method is named after D. G. Krige, a South African mining engineer who was the first to develop the technique. Making use of irregular reference points, the visual representation and contour lines of the surface are constructed. The KRIG interpolation method uses the distance or navigation between the reference points as a function that helps surface characterisation. Thus, in order to determine the output values for each location, it assigns a mathematical function to a certain number of points or all the points located within a certain area of effect. What uniquely distinguishes this method from other interpolation methods is that it is a variogram model. The KRIG interpolation method uses weighting which allows the closely located points to have a greater influence (Chaplot et al., 2006; Zimmerman et al., 1999; Inal et al., 2002). The semi-variogram provides a measure of spatial correlation by describing how spatial data are related to distance and direction and is defined as follows

$$g(h) = \frac{1}{2}E[(Z(x_1) - Z(x_2)^2] \ , \ \|h\| = \|x_1 - x_2\| \ , \ Z_P = \mu(P) + \varepsilon(P) \tag{3}$$

where; h is called the lag, μ is constant mean for the data, ε is random errors, Z_P is the variable of interest (Olgun and Erdogan, 2009).

The Minimum Curvature Method (MCRV)

The MCRV interpolation method is extensively used in earth sciences. A surface interpolated with minimum curvature could be compared to a linear, elastic and thin plane which passes through the reference points with a minimum amount of bending. Minimum curvature generates the smoothest possible surface while attempting to fit data as closely as possible (Nikolova and Vassilev, 2006). In order to characterize this surface, first, from the measurements used to formulate the grid data, the plane equation is taken as the trend and the residual values are calculated.

$$AX + BY + C = Z(X,Y) \tag{4}$$

The calculated residual values are taken as the measurement values to be used in the interpolation, which are then completed through the minimum curvature algorithm at each grid corner (Smith and Wessel, 1990). The following equation is used in the interpolation process with the minimum curvature algorithm:

$$(1 - T_i)\nabla^2(\nabla^2 Z) - (T_i)\nabla^2 Z = 0 \tag{5}$$

For the boundary values, the following equations apply:

$$(1 - T_b)\frac{\delta^2 Z}{\delta n^2} + (T_b)\frac{\delta Z}{\delta n} = 0 \quad \text{(on the edges)} \tag{6}$$

$$\frac{\delta(\nabla^2 Z)}{\delta n} = 0 \quad \text{(on the edges)} \tag{7}$$

$$\frac{\delta^2 Z}{\delta x \delta y} = 0 \quad \text{(at the corner)} \tag{8}$$

where

∇^2 is the Laplacian operator,
n is the boundary normal,
$\mathbf{T_i}$ is the internal tension, and
$\mathbf{T_b}$ is the boundary tension.

Although this method is more appropriate for areas with data values with lesser curvature, the data cannot thoroughly represent the surface as it is not an exact interpolator.

(3)

The Modified Shepard Method (MSHP)

The simplest form of inverse distance weighted interpolation is sometimes called "Shepard's method" (Shepard, 1968). The equation used is as follows:

$$F(x, y) = \sum_{i=1}^{n} w_i f_i \tag{9}$$

where

n is the number of scatter points in the set,
f_i are the prescribed function values at the scatter points, and
w_i are the weight functions assigned to each scatter point.

The effect of the weight function is that the surface interpolates each scatter point and is influenced most strongly between scatter points by the points closest to the point being interpolated. Although the weight function shown above is the classical form of the weight function in inverse distance weighted interpolation, the following equation is used:

$$w_i = \frac{\left[\dfrac{R-h_i}{Rh_i}\right]^2}{\sum_{j=1}^{n}\left[\dfrac{R-h_i}{Rh_i}\right]^2} \tag{10}$$

where

h_i is the distance from the interpolation point to scatter point **i**,

R is the distance from the interpolation point to the most distant scatter point and

n is the total number of scatter points.

This equation has been found to give superior results to the classical equation (Franke and Nielson, 1980). The weight function is a function of Euclidean distance and is radially symmetric about each scatter point. As a result, the interpolating surface is somewhat symmetric about each point and tends toward the mean value of the scatter points between the scatter points. Shepard's method has been used extensively because of its simplicity.

The Natural Neighbour Method (NANE)

The NANE interpolation method, which is based on the average mean, is similar to the INDW interpolation method. While investigating the points to be interpolated it uses the distance-dependent weights of reference points to the grid corner. With this method, the data on the reference points with irregular distribution are classified and the interpolation process is completed using the Triangular Irregular Network (TIN) functions without any need for custom-defined parameters. In the natural neighbour interpolation, first through the Delaunay traingulation, a triangulation is performed where each reference point constitutes the vertex of a triangle. Subsequently, the convex spaces are defined so that there is a minimum number of triangle sides for each point and the weight of each neighbouring point is assigned to these areas determined through the “Thiessen/Voronoi Technique” (Skumar et al., 2001; Yılmaz, 2007). The basic equation used in NANE interpolation is identical to the one used in inverse distance power interpolation,

$$G(x, y) = \sum_{i=1}^{n} w_i f(x_i, y_i), \tag{11}$$

where

$G(x,y)$ is the natural neighbour estimation at (x,y),

n is the number of nearest neighbours used for interpolation,

$f(x_i, y_i)$ is the observed value at (x_i, y_i), and

w_i is the weight associated with $f(x_i, y_i)$.

The Nearest Neighbour Method (NENE)

The NENE method assigns the value of the nearest point to each grid node. This method is useful when data are already evenly spaced. Alternatively, in cases where the data are nearly on a grid with only a few missing values, this method is effective for filling in the holes in the data. The NENE method predicts the attributes of unsampled points based on those of the nearest reference point and is best for qualitative data, where other interpolation methods are not applicable (Burrough and McDonnell, 1998). Sometimes with nearly complete grids of data there are areas of missing data that one desires to exclude from the grid file. In this case, the search ellipse can be set to a value so the areas with no data are assigned the blanking value in the grid value. By setting the search ellipse radii to values less than the distance between data values in file, the blanking value is assigned at all grid nodes where data values do not exist (Yılmaz, 2007; Surfer 8 Software).

The Polynomial Regression Method (PREG)

Polynomial regression provides interpolation by approximating the source data points using a global polynomial expression. Polynomial regression is not really an interpolator because it does not attempt to predict unknown Z values (Brutman, 1997). There are several options that define the type of trend surface:

Simple planar surface: $Z(x, y) = A + Bx + Cy$ (12)

Bi-linear surface: $Z(x, y) = A + Bx + Cy + Dxy$ (13)

Quadratic surface:

$$Z(x, y) = A + Bx + Cy + Dx^2 + Exy + Ey^2 \tag{14}$$

Cubic surface:

$$Z(x, y) = A + Bx + Cy + Dx^2 + Exy + Fy^2 + Gx^3 + Hx^2y + Ixy^2 + Jy^3 \tag{15}$$

The Radial Basis Function Method (RBAF)

The RBAF interpolation method is the name given to a large family of exact interpolators. In many ways the methods applied are similar to those used in geostatistical interpolation, but without the benefit of prior analysis of variograms. On the other hand they do not make any assumptions regarding the input data points and provide excellent interpolators for a wide range of data (Smith et al., 2007).

For terrain modelling and earth sciences generally the so-called multi-quadric function has been found to be particularly effective, as have thin plate splines. The simplest variant of this method without smoothing can be viewed as a weighted linear function of the distance from grid point to data point, plus a “bias” factor, m. The model is in the form

$$Z_P = \sum_{i=1}^{n} w_i \phi(r_i) + m \quad (16)$$

or the equivalent model, using the untransformed data values and data weights

$$Z_P = \sum_{i=1}^{n} \lambda_i Z_i \quad (17)$$

where
Z_P is the estimated value for the surface at grid point P,
$\phi(r_i)$ is the radial basis function selected, with r_i being the radial distance from point P to the i^{th} data point, and
w_i, λ_i, and the bias value m are estimated from the data points (Smith et al., 2007).

The Triangulation with Linear Interpolation Method (TLIN)

The TLIN interpolation method uses the optimal Delaunay triangulation. The algorithm creates triangles by drawing lines between data points. This method uses the reference points as the vertices of non-overlapping triangles that cover the interpolation areas. The most common triangulation algorithms are optimal, greedy, and Delaunay triangulation. The optimal triangulation is defined as having a minimum sum of edge lengths. The Delaunay triangles define the nearest natural neighbours in the sense that the reference points at the vertices are closer to their mutual circumcenter than any reference point. It is well known that the Delaunay triangulation is a unique solution for triangulation because it does not depend on the starting point, while other methods depend on the starting point of the triangulation (Yanalak and Baykal, 2003; Lawson, 1977; Macedonio and Pareschi, 1991). The most common interpolation method on triangles is linear interpolation. A plane is defined in a rectangular coordinate system as

$$Z = a_{00} + a_{10}x + a_{01}y \quad (18)$$

The constants a_{00}, a_{10}, and a_{01} are calculated using the three corner points of the triangle. The interpolated height Z_0 is calculated by substituting (x_0,y_0) for (x,y) in the equation.

The Moving Average Method (MOAV)

This method of interpolation involves simple averaging using a moving window such as an ellipse or circle. For each interpolated grid point a circle of specified radius is placed with its centre at the grid point. The output grid node value is set equal to the arithmetic average of the identified neighbouring data. If there are fewer than the specified minimum numbers of data within the neighbourhood, the grid node is blanked (Yang et al., 2004; Smith et al, 2007).

The Data Metrics Method (DMET)

The collection of data metric interpolation methods creates grids of information about the data on a node-by-node basis. The DMET interpolation methods are not, in general, weighted average interpolators of the Z values (Yang et al., 2004).

Data metrics use the local data set including breaklines for a specific grid node for the selected data metric. The local data set is defined by the search parameters. These search parameters are applied to each grid node to determine the local data set. In the following descriptions, when computing the value of a grid node (r, c), the local data set S(r, c) consists of data within the specified search parameters centred at the specific grid node only (Yılmaz, 2007). The set of selected data at the current grid node (r, c) can be represented by S(r, c).

$$S(r,c) = \{(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots, (x_n, y_n, z_n),\} \quad (19)$$

where
n is the number of data points in the local data set.

The Local Polynomial Method (LPOL)

The global polynomial interpolation method creates a surface from a single polynomial formula. The LPOL interpolation creates a surface from many different formulae, each of which is optimised for a neighbourhood. The neighborhood shape, maximum and minimum number of points and sector configuration can be specified The neighbourhood shape, maximum and minimum number of points. In addition, as with inverse distance weighting, the reference points in a neighbourhood can be weighted by their distance from the prediction location. Thus, this method produces surfaces that better account for local variation. A first-order local polynomial fits a single plane through the data points in the search neighbourhood but

keeps only the fitted value at the prediction location. A second-order local polynomial fits a surface with a bend in it to each search neighbourhood; a third-order local polynomial fits a surface with two bends to each neighbourhood and so on. Local polynomials are more flexible than global ones.
The form of these polynomials (Kidner et al., 1997) is

Order 1: $F(x, y) = a + bx + cy$ (20)

Order 2: $F(x, y) = a + bx + cy + dxy + ex^2 + fy^2$ (21)

Order 3:
$F(x, y) = a + bx + cy + dxy + ex^2 + fy^2 + gx^2y + hxy^2 + ix^3 + jy^3$ (22)

The polynomial coefficients are calculated by using known coordinates of the reference points. The coefficients of the polynomial are calculated by least squares estimation (Petri and Kennie, 1987). Although it is more flexible than global polynomial interpolation, LPOL interpolation is not an exact interpolator like INDW.

MATERIALS AND METHODS

Artificial terrain

This study uses a hill-shaped object made of plaster as the testing surface. The diameter of the base of the object is 22 cm, its upper diameter approximately 10 cm, and its average height 25 cm. As it would make contact with water during the calculation of its volume, the object was first varnished with shellac to eliminate the effect of water on the volume of the object. The object was then submerged in a container full of water and the overflowing water was measured in a beaker and found to measure 2453 cm^3. The surface of the object was marked to reflect its characteristics. Then, in order to model the surface of the object, photographs were taken with a digital camera from different perspectives in a testing area. The camera used for photographing was calibrated. By evaluating the calibration values, photographs and reference points with Photomodeler 5.0 software, the coordinates of 777 points on the object's surface were found (Figure 1).

For X, Y, and Z directions, the quadratic means of the reference points are 0.0651 cm, 0.0740 cm and 0.1200 cm respectively.

Three-Dimensional Models

Using the point coordinates determined through photogrammetric evaluation, a 3D model of the artificial surface was reconstructed with Surfer 8.0 software. Surfer 8.0 software offers twelve different interpolation methods, whose functions and parameters were discussed above (Yang et al., 2004). The parameters of the interpolation method used in this study are provided in Table 1.

The 3D models of the artificial surface reconstructed according to these methods are provided in Figure 2.

Volume calculations

The volume of the artificial 3D object which was reconstructed according to the interpolation methods used was calculated by Surfer 8.0 software in this study. Surfer 8.0 software offers users three types of volume calculation methods: trapezoidal rule, Simpson's rule and Simpson's 3/8 rule. The formulae for these methods are, respectively:

$$V_T = \frac{h}{2}\left[A_0 + 2A_1 + 2A_2 + \cdots + 2A_{n-1} + A_n\right] + \frac{h_n A_n}{2} \quad (23)$$

$$V_S = \frac{h}{2}\left[A_0 + 4A_1 + 2A_2 + \cdots + 2A_{n-2} + 4A_{n-1} + A_n\right] + \frac{h_n A_n}{2} \quad (24)$$

$$V_{S(3/8)} = \frac{3}{8}\frac{h}{2}\left[A_0 + 3A_1 + 3A_2 + \cdots + 3A_{n-2} + 3A_{n-1} + A_n\right] + \frac{h_n A_n}{2} \quad (25)$$

where
A is the cross-section area, and
h is the distances between cross sections (Burington, 1973; James et al., 1985; Yılmaz and Yakar, 2008).

Table 2 provides the calculated volumes, volumetric differences and root mean square errors for the artificial object. Determination of volumetric differences was tested by using least- mean-difference method. Least difference value was found as 234.880 cm^3. According to least difference values, significance of volumetric differences was given in Table 2.

Figure 3 graphically presents the measured and calculated volumetric values for the artificial object.

Volumetric accuracy

In order to examine the effect of interpolation methods on the volume calculation, the shapes of the conical frustum and pyramidal frustum were used. Taken as a reference, the surface of the conical frustum was transformed into a multi-sided pyramidal frustum (Figure 4). Using the reference points taken on the edges of the sides the pyramidal frustum, a 3D model of the pyramidal frustum was obtained and its volume was calculated through the above-mentioned interpolation methods.
The formula for the volume of the conical frustum is

$$V = \frac{1}{3}\pi h\left(R_1^2 + R_2^2 + R_1 R_2\right) \quad (26)$$

The formula for the volume of the frustum (n-gon) is

$$V_P = \frac{1}{3}h\left(A_b + A_t + \sqrt{A_b A_t}\right) \quad (27)$$

where
R_1 is the radius of the bottom base,
R_2 is the radius of the top base,
A_b is the area of bottom base,
A_t is the area of top base, and h is the height from the top base to bottom base.

The volume of a conical frustum with R_1 = 25 cm, R_2 = 8 cm, h = 30 cm, and real and interpolated calculated volumes of 8, 16, 32, 64 and 128 sided conical frusta are given in Table 3, and their graphical representation in Figure 5.
3D model examples of 8, 16, 32, 64, and 128 sided conical frusta are given in Figure 6.

Figure 7 presents the approach rates for the interpolated calculated volumes of 8, 16, 32, 64, and 128 sided pyramidal frusta towards the volume of the conical frustum.

Conclusion

Forming a 3D model of the physical earth and an object

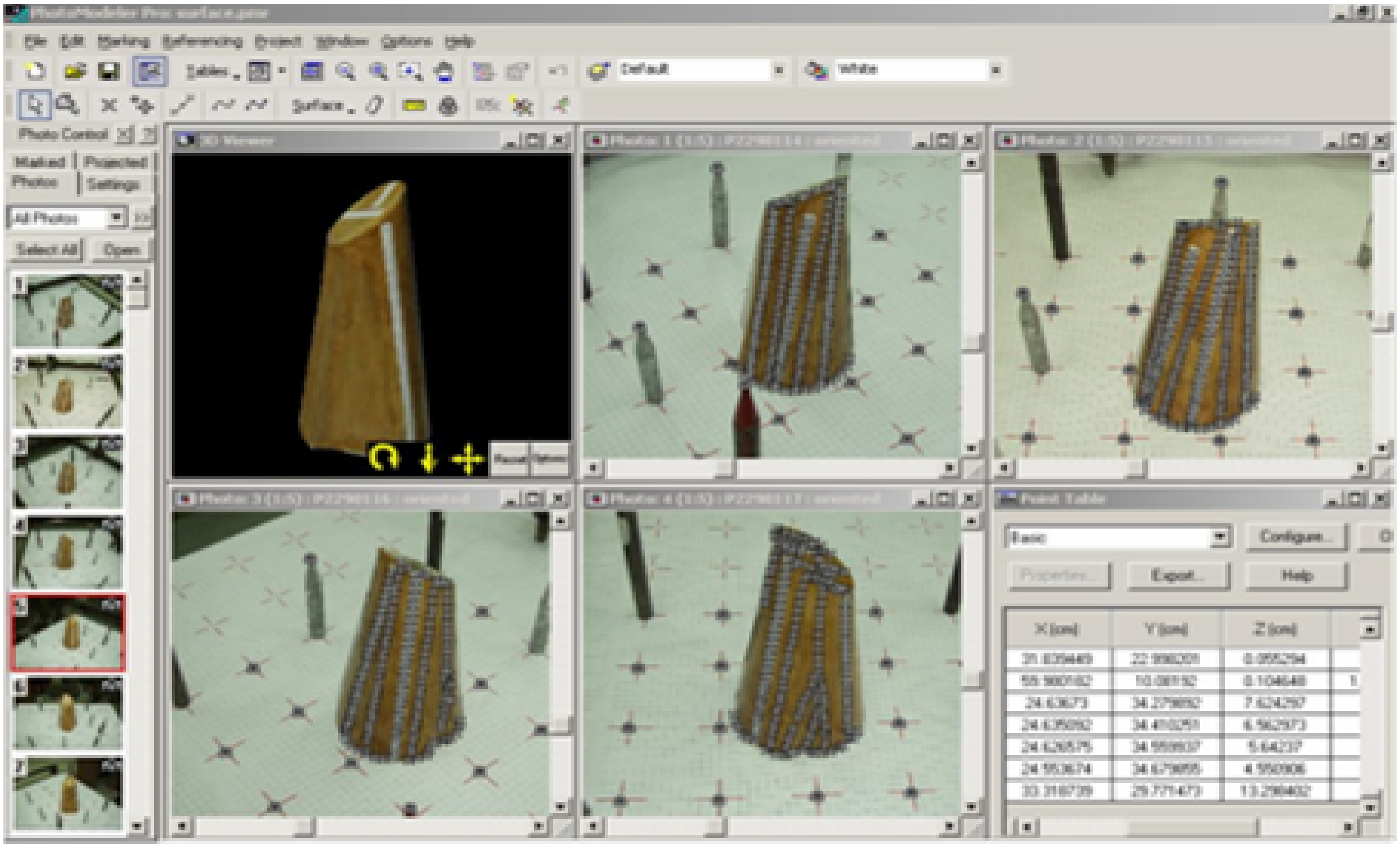

Figure 1. Photogrammetric evaluation of the artificial surface.

Table 1. Interpolation methods and parameters.

Method	Speed	Type	Parameters
INDW	Fast	Exact, unless smoothing factor specifieed	Power: 2; Smoothing: 0; Anisotropy ratio: 1; Anisotropy angle: 0.
KRIG	Slow Medium	Exact if no nugget	Variogram Slope: 1; Type: Point; Drift type: Linear.
MCRV	Medium	Exact / Smoothing	Max. Residual: 0.029; Max. iteration: 100000; Relaxation factor: 1, Anisotropy ratio: 1.
MSHP	Fast	Exact, unless smoothing factor specifieed	Quadratic neighbour: 13; Weighting neighbour: 19; Smoothing factor: 0.
NANE	Fast	Exact	Anisotropy ratio: 1; Anisotropy angle: 0.
NENE	Fast	Exact	Range 1: 20.4; Range 2: 20.4; Angle: 0.
PREG	Fast	Smoothing	Simple planar surface: $Z(x,y) = A + Bx + Cy$
RBAF	Slow / Medium	Exact if no smoothing value specified	Multiquadratic; R^2 parameter: 0.033; Anisotropy ratio: 1; Anisotropy angle: 0.
TLIN	Fast	Exact	Anisotropy ratio: 1; Anisotropy angle: 0.
MOAV	Fast	Smoothing	Search elipse; Radius 1: 10.2; Radius 2: 10.2.
DMET	Fast	Exact	Data location statistic: count; Number of sector to search: 4.
LPOL	Fast	Smoothing	$F(x,y) = a + bx + Cy$.

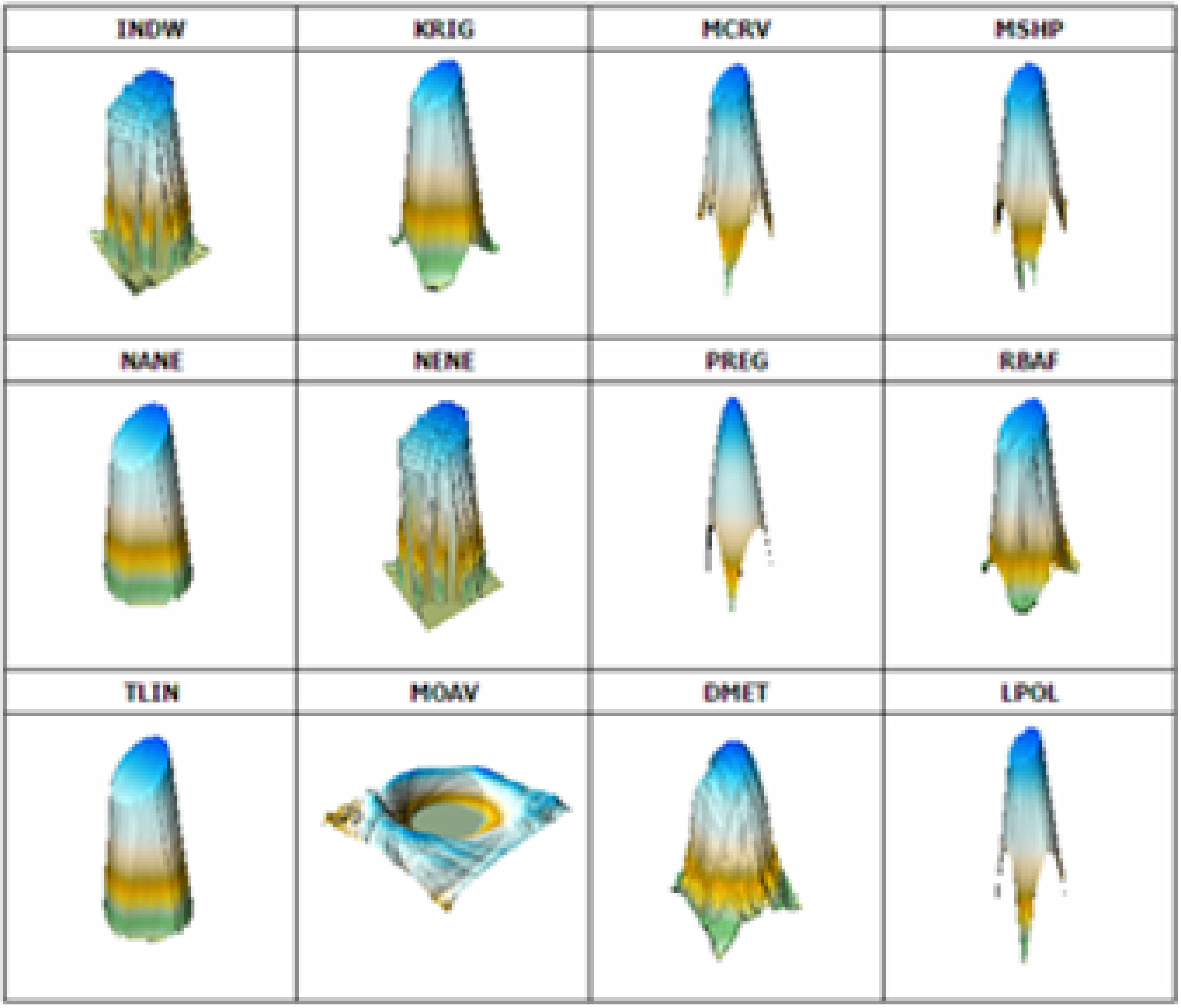

Figure 2. Three-dimensional models.

Table 2. Volumes according to the interpolation methods ($V_{object} = 2453\ cm^3$).

Method	Computing volume (cm^3)			Volume difference (cm^3)			RMSE (cm^3)			Test
	T	S	S (3/8)	T	S	S (3/8)	T	S	S(3/8)	
INDW	2453.722	2454.205	2453.623	-0.722	-1.205	-0.623	0.218	0.363	0.188	UNSIGNIFICANT
KRIG	2318.816	2318.981	2319.086	134.184	134.019	133.914	40.458	40.408	40.377	UNSIGNIFICANT
MCRV	2134.705	2135.279	2135.359	318.295	317.721	317.641	95.970	95.796	95.772	SIGNIFICANT
MSHP	2221.968	2221.435	2221.897	231.032	231.565	231.103	69.659	69.819	69.680	UNSIGNIFICANT
NANE	2421.463	2421.362	2421.512	31.537	31.638	31.488	9.509	9.539	9.494	UNSIGNIFICANT
NENE	2446.871	2447.047	2447.124	6.129	5.953	5.876	1.848	1.795	1.772	UNSIGNIFICANT
PREG	2158.965	2159.859	2159.863	294.035	293.141	293.137	88.655	88.385	88.384	SIGNIFICANT
RBAF	2289.066	2288.831	2289.588	163.934	164.169	163.412	49.428	49.499	49.271	UNSIGNIFICANT
TLIN	2428.352	2428.238	2428.457	24.648	24.762	24.543	7.432	7.466	7.400	UNSIGNIFICANT
MOAV	3028.831	3028.872	3028.869	-	-	-	173.620	173.632	173.631	SIGNIFICANT
DMET	2702.086	2700.936	2702.553	-	-	-	75.102	74.756	75.243	SIGNIFICANT
LPOL	1976.704	1977.260	1977.268	476.296	475.740	475.732	143.609	143.441	143.439	SIGNIFICANT

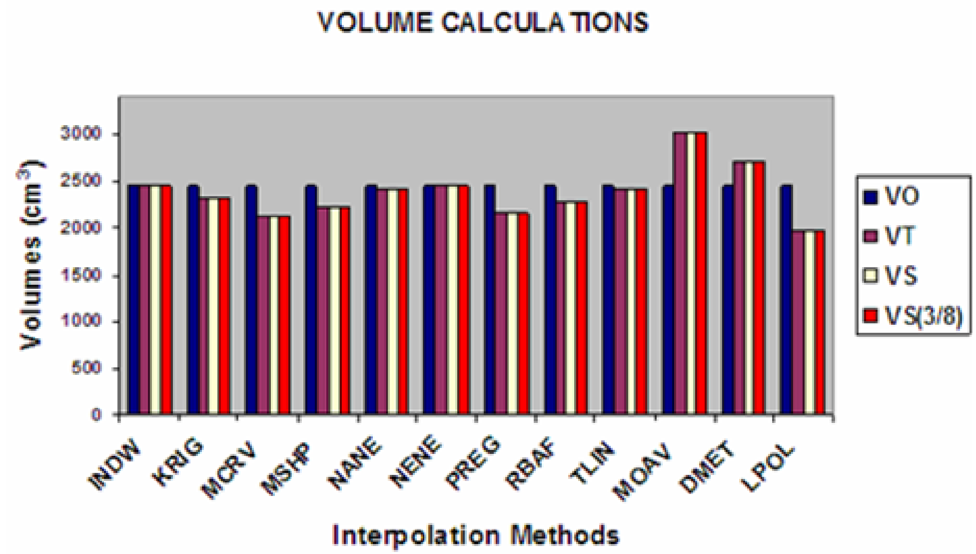

Figure 3. Volumes according to the interpolation methods.

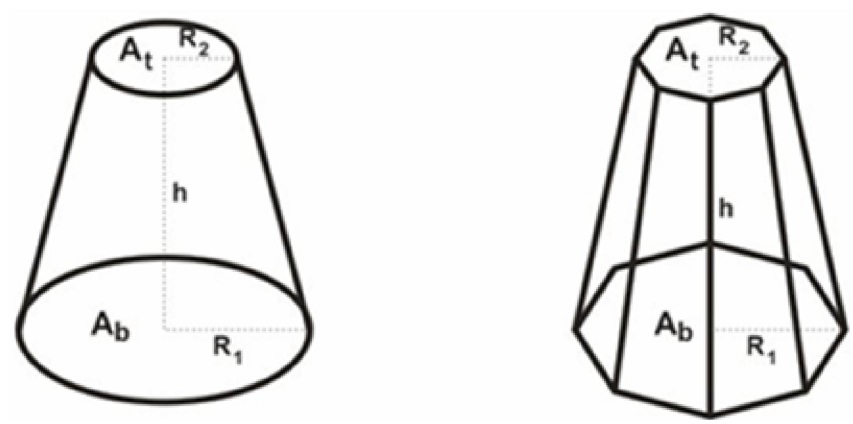

Figure 4. Conical frustum and pyramidal frustum.

Table 3. Real and calculated volumes of the conical frustum and pyramidal frustum.

CONE cm^3	METHODS	PYRAMID (8) cm^3	PYRAMID (16) cm^3	PYRAMID (32) cm^3	PYRAMID (64) cm^3	PYRAMID (128) cm^3
27 928.759	$VOLUME_C$	22 182.782	24 010.470	24 480.863	24 599.316	24 628.982
27 928.759	INDW	26 423.908	25 123.568	24 814.352	24 703.749	24 674.313
27 928.759	KRIG	24 513.334	24 002.421	23 797.498	24 707.906	24 687.423
27 928.759	MCRV	21 656.645	26 696.023	25 234.141	24 971.552	25 792.465
27 928.759	MSHP	24 385.041	24 660.541	24 221.085	23 716.878	23 204.608
27 928.759	NANE	22 167.785	23 999.908	24 586.034	24 606.805	24 617.702
27 928.759	NENE	26 022.850	24 982.144	24 775.622	24 667.189	24 644.873
27 928.759	PREG	24 983.990	24 984.008	24 983.448	24 983.473	24 983.449
27 928.759	RBAF	24 504.700	23 743.844	24 742.121	24 719.961	24 696.031
27 928.759	TLIN	22 182.650	24 010.454	24 605.529	24 620.785	24 630.619
27 928.759	MOAV	40 777.486	40 698.344	40 295.566	40 612.527	40 697.687
27 928.759	DMET	29 712.509	29 168.615	27 122.756	26 161.935	25 518.724
27 928.759	LPOL	22 228.903	24 038.044	24 488.430	24 513.046	24 546.414

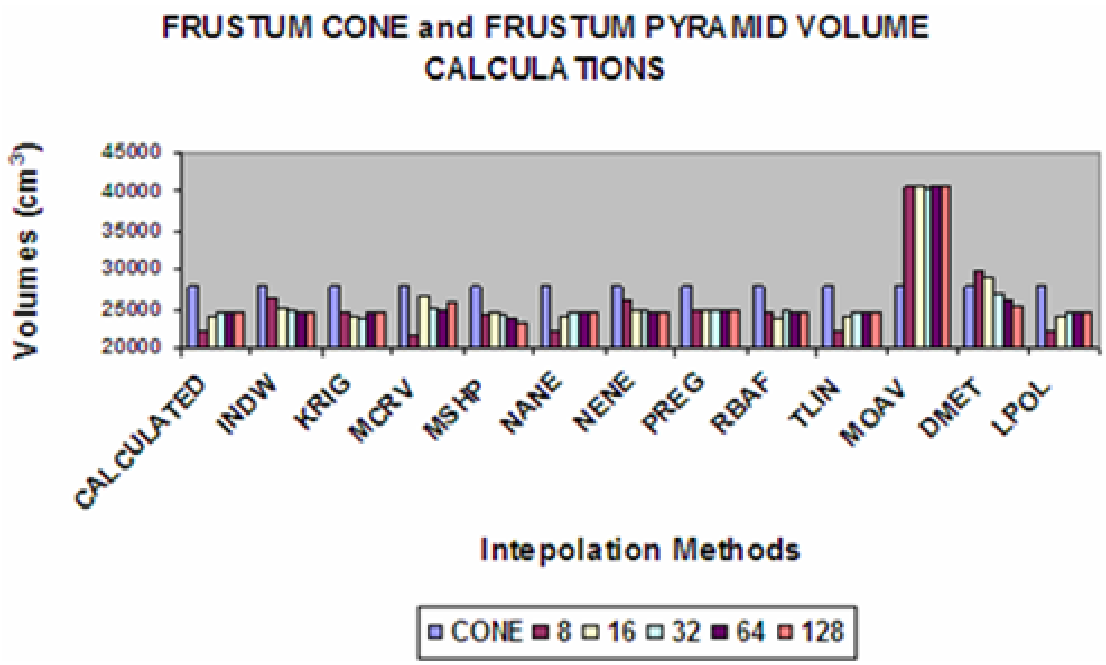

Figure 5. Volumes calculated and those obtained by interpolation methods.

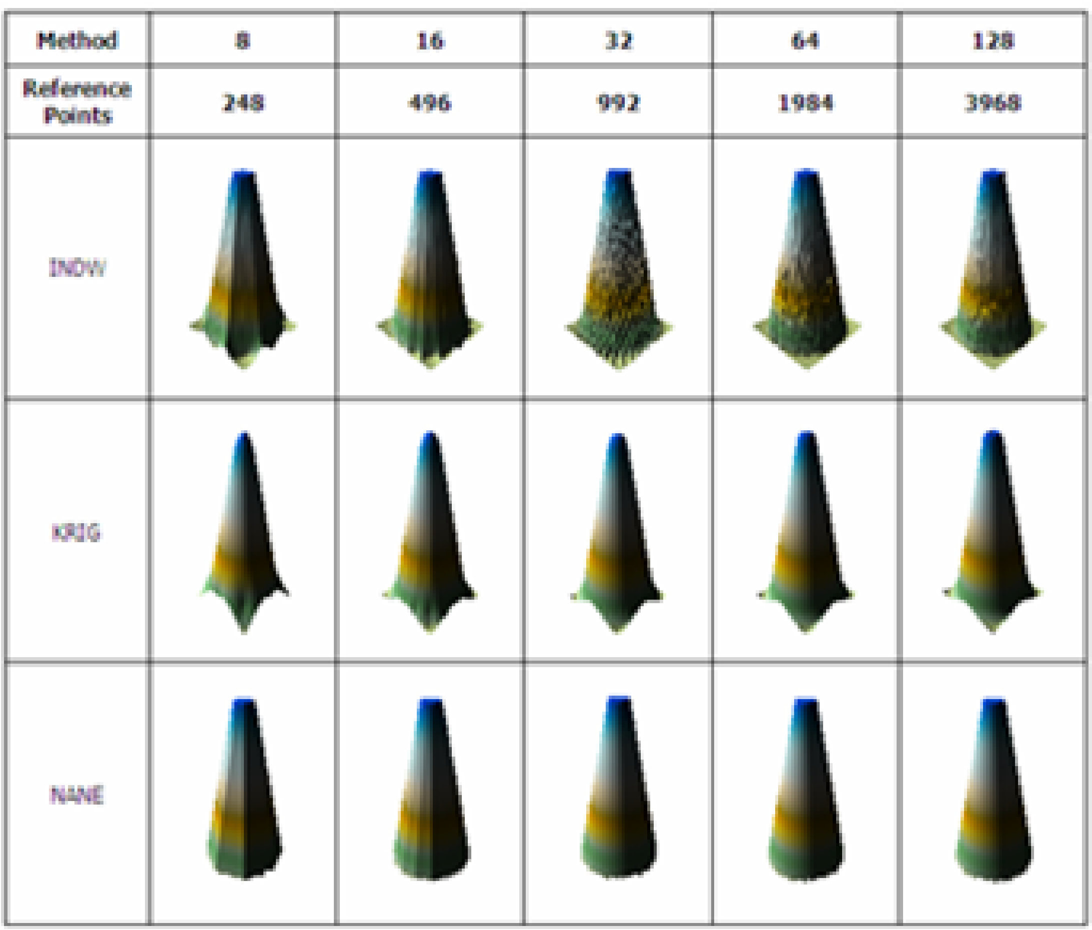

Figure 6. Three-dimensional models of the pyramidal frustum.

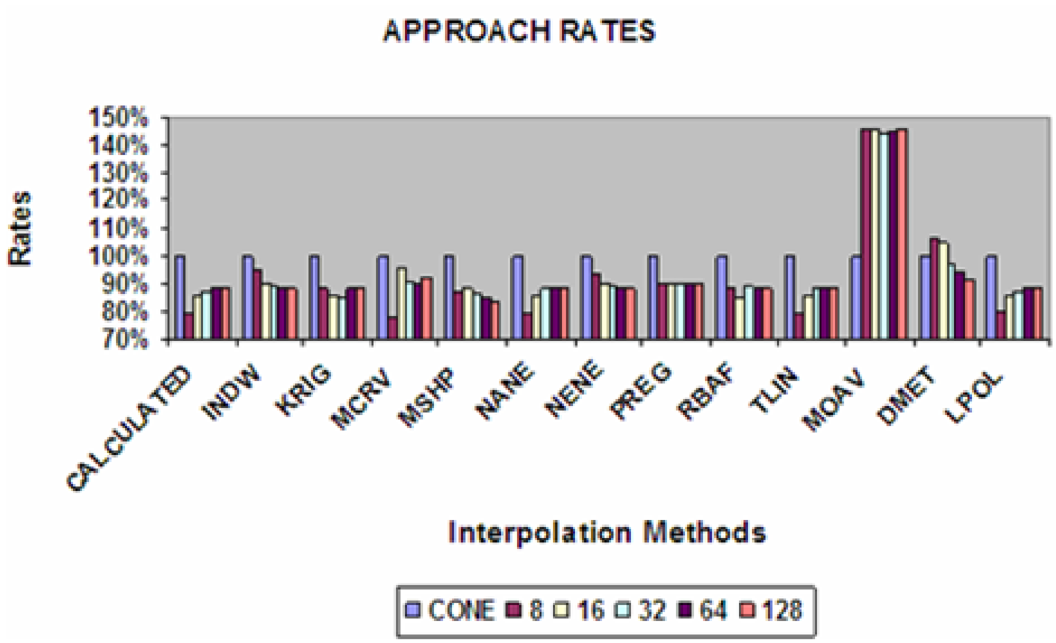

Figure 7. Approach rates in interpolation methods.

with satisfactory sensitivity requires an adequate number of reference points. Information about the reference points, those which are inappropriate are eliminated, is obtained by any measurement method and applying the interpolation process with an appropriate function the reference points on the surface are condensed and the model which best represents the real object is obtained. Users can perform every kind of mathematical and logical operation on the model, one of which is the volume calculation. Volume calculation is used in various fields of science and engineering.

This study investigated the calculability of land tracts or objects with irregular shapes, which do not allow direct volumetric calculation, through 3D models and also investigated the accuracy of the models. Table 2 presents the real and the different interpolated calculated volumetric values of an artificial object surface which was reconstructed to this end and allows an indirect and sensitive volume calculation. Examination of Table 2 reveals that the three values which approximate most closely to the real volume are the ones obtained through the INDW, NENE and TLIN methods. The greatest difference in volume was found when using the MOAV method. An examination of the 3D model presented in Figure 2, which was reconstructed through the method in question, also shows that the model does not represent the real shape. It could be suggested that the reason for such a result is that the mathematical function of the MOAV method is not appropriate for modelling such objects. Furthermore, if we examine Figure 2, we can observe that the models obtained using the KRIG, NANE and TLIN methods are visually more similar to the artificial object.

In order to investigate volumetric accuracy, the study used regular geometric shapes which allow mathematical volume calculation. The volumes for the models derived from the points located on pyramidal frusta with different numbers of sides were compared to the real volume of the pyramidal frustum as well as the volume of the conical frustum. An examination of Table 3 and Figure 5 reveals that the number of sides in the PREG method do not change the volume of the model, that the DMET, INDW and MCRV methods yield results which approximate more closely to the volume of the conical frustum and the real volume of the pyramidal frustum and that while an increase in the number of sides is supposed to result in a closer approximation to the real volume, in fact it differs greatly from the supposed value. From Figure 7, which graphically presents the approach rates of the calculated volumes towards the real volumes, it can be observed that, in parallel to an increase in the number of sides, the volume calculated with the LPOL method approaches more closely to the supposed volume of the pyramidal frustum as well as to the volume of the conical frustum which was taken as the reference. Each interpolation method has its own advantages and disadvantages and a particular interpolation method does not yield sound results from all aspects. Therefore, in accordance with the intended purpose of a study, it would be necessary to select an interpolation method yielding the object or landform models which best suit the original surface. If a sensitive calculation of the volume is desired, then the INDW method can be selected.

Further studies will investigate the effects of the quality of the data used in modelling and the surface characteristics on the calculated volume as well as the selection of an interpolation method which could yield better results in terms of volumetric accuracy.

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