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# Anti-synchronization of complex delayed dynamical networks through feedback control

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**To reveal the dynamical mechanism of anti-synchronization in complex networks with time delays, a general complex dynamical network with delayed nodes is introduced. Based on the Lyapunov stability theory, this paper presents the feedback controllers for anti-synchronization of complex delayed dynamical networks. Several sufficient conditions are drawn for the stability of the error dynamics and the design of the feedback controllers. Numerical simulations are performed to verify and illustrate the analytical results.**

**Key words:** complex networks, time delay, anti-synchronization, feedback control.

## INTRODUCTION

Over the past ten years, complex networks have attracted increasing interest in both theory and applications (Strogatz, 2001; Watts and Strogatz, 1998; Barabasi and Albert, 1999; Newman, 2003). A complex network is a large set of interconnected nodes, where the nodes and connections can be any element, such as World Wide Web, food web, neural networks etc (Wang and Chen, 2003).

Recently, synchronization of complex dynamical networks has been a focus in various fields of social, biological and engineering science (Li et al., 2006; Fan et al., 2005; Zhang et al., 2006; Liang et al., 2008). Wang and Chen (2002a, b) presented a uniform model and investigated its synchronization criteria in small-world and scale-free networks. Wu (2006) investigated the synchronization of random directed networks. Slotine et al. (2004) further discussed the synchronization of nonlinearly coupled continuous and hybrid oscillators networks by using the contraction analysis approach. Zhou et al. (2006) studied the adaptive synchronization of an uncertain complex dynamical network. Sorrentino et al. (2007) investigated the controllability of complex networks with pinning controllers. Xiao et al. (2010)

proposed a simple adaptive feedback controller to synchronize the dynamical network with unknown generally time-delayed coupling functions. However, most of the above studies have neglected the effects of coupling delays, while time delays commonly exist in the real world. This paper will further investigate the feedback anti-synchronization of complex dynamical networks with delayed nodes. Based on Lyapunov stability theory, we design the feedback controllers to anti-synchronize complex dynamical networks.

## THE COMPLEX DYNAMICAL NETWORK MODEL

We consider a general complex network consisting of  $N$  delayed dynamical nodes. Each node of the network is an  $n$ -dimensional non-autonomous dynamical system with time delay, which is described by

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t), t) + \sum_{j=1}^N c_{ij} Bx_j(t - \tau) + u_i, \quad (1)$$

where  $i = 1, 2, \dots, N$ ,  $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in R^n$  are the state variables of node  $i$ ,  $A \in R^{n \times n}$  is a constant matrix and  $\tau > 0$  is the constant time delay. The

matrix  $B = (b_{ij})_{n \times n} \in R^{n \times n}$  is the inner connecting matrix of each node, where  $B > 0$  is a positive definite matrix, and the matrix  $C = (c_{ij})_{N \times N} \in R^{N \times N}$  is the diffusely coupled matrix of the network. That is,

$$c_{ii} = - \sum_{j=1, j \neq i}^N c_{ij}, \quad (2)$$

where  $c_{ij} = c_{ji}$  is the coupling element, if there is a connection from nodes  $i$  to  $j$  ( $j \neq i$ ), then  $c_{ij} = c_{ji} > 0$ , else  $c_{ij} = c_{ji} = 0$ . Moreover,  $u_i \in R^n$  are the controllers designed for the network (1).

### Remark 1

Duan et al. (2008) and Li et al. (2009) discussed the synchronization for a class of complex dynamical networks without delays. In this paper, we have extended their model to the delay situation, which have more practicability in the real world.

## ANTI-SYNCHRONIZATION OF COMPLEX DELAYED DYNAMICAL NETWORKS

Let  $s(t)$  be a solution of the isolate node of the network (1), which is assumed to exist and is unique, then  $s(t)$  is a synchronous solution of the controlled complex dynamical network (1) because it is a diffusive coupling network, satisfying

$$\dot{s}(t) = As(t) + f(s(t), t), \quad (3)$$

where  $s(t)$  can be an equilibrium point, a nontrivial periodic orbit, or even a chaotic attractor. Before starting the main results, the following Definition and Assumption are given.

### Definition 1

For complex dynamical networks (1), it is said that they are anti-synchronization, if  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ , where  $e_i(t) = x_i(t) + s(t)$ . That is  $\lim_{t \rightarrow \infty} \|x_i(t) + s(t)\| = 0$ ,  $i = 1, 2, \dots, N$ .

### Assumption 1

For the vector function  $f(x_i(t), t)$ , suppose that the  
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uniform Lipschitz condition holds, that is, for any

$x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T$  and  $s(t) = (s_1(t), s_2(t), \dots, s_n(t))^T$ , then, there exists a positive constant  $L > 0$ , such that:

$$\|f(x_i(t), t) - f(s(t), t)\| \leq L \|x_i(t) - s(t)\|, \quad (4)$$

where  $i = 1, 2, \dots, N$ .

By Equations (1) and (3), we can get the error dynamical system

$$\dot{e}_i(t) = Ae_i(t) + f(x_i(t), t) + f(s(t), t) + \sum_{j=1}^N c_{ij} B e_j(t - \tau) + u_i. \quad (5)$$

Then the anti-synchronization problem of the dynamical network (1) is equivalent to the problem of global stabilization of the error dynamical system (5). In order to make dynamical network (5) controllable, the feedback controllers  $u_i$  will be appropriately chosen.

### Theorem 1

Based on Definition 1, the delayed complex dynamical network (1) is globally anti-synchronized under the following controllers

$$u_i = -f(x_i(t), t) - f(s(t), t) + (-\xi + c_{ii})e_i, \quad i = 1, 2, \dots, N, \quad (6)$$

and

$$A - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2 \text{ is negative definite matrix.}$$

### Proof

Construct the Lyapunov function candidate as follows:

$$V(t) = \frac{1}{2} \sum_{i=1}^N e_i^T e_i - \sum_{i=1}^N c_{ii} \int_{t-\tau}^t e_i^T(\theta) B^2 e_i(\theta) d\theta - \frac{1}{4} \sum_{i=1}^N c_{ii} \int_{t-\tau}^t e_i^T(\theta) B^2 e_i(\theta) d\theta \quad (7)$$

Then the time derivative of  $V(t)$  along the solution of the error system (5) is given as follows:

$$\dot{V}(t) = \sum_{i=1}^N e_i^T \dot{e}_i - \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \sum_{i=1}^N c_{ii} e_i^T (t - \tau) B^2 e_i(t - \tau).$$

$$-\frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \quad (8)$$

From Equation (5), it can be obtained that:

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^N e_i^T (Ae_i(t) + f(x_i(t), t) + f(s(t), t)) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) + \sum_{i=1}^N e_i^T u_i \\ & - \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \\ & - \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau). \end{aligned} \quad (9)$$

Now, Substituting Equation (6) into Equation (9), it follows that

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^N e_i^T A e_i(t) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) + \sum_{i=1}^N (-\xi + c_{ii}) e_i^T e_i - \sum_{i=1}^N c_{ii} e_i^T B^2 e_i \\ & + \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) - \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau). \end{aligned} \quad (10)$$

Hence we have

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^N e_i^T (A - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2) e_i(t) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) + \sum_{i=1}^N c_{ii} e_i^T e_i \\ & + \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) + \sum_{i=1}^N c_{ii} e_i^T e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \end{aligned} \quad (11)$$

By Equation (2), we can get

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^N e_i^T (A - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2) e_i(t) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) - \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_i^T e_j \\ & - \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_i^T (t-\tau) B^2 e_j(t-\tau) + \sum_{i=1}^N c_{ii} e_i^T e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau). \end{aligned} \quad (12)$$

From Section 2, apparently we have

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_i^T (t-\tau) B^2 e_j(t-\tau) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_j^T (t-\tau) B^2 e_j(t-\tau). \quad (13)$$

Therefore, it yields

$$\dot{V}(t) = \sum_{i=1}^N e_i^T (A - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2) e_i(t) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) - \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_i^T e_j$$

$$\begin{aligned} & - \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_j^T (t-\tau) B^2 e_j(t-\tau) + \sum_{i=1}^N c_{ii} e_i^T e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \\ & \leq \sum_{i=1}^N e_i^T (A - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2) e_i(t) \\ & + \sum_{i=1}^N c_{ii} e_i^T B e_i(t-\tau) + \sum_{i=1}^N c_{ii} e_i^T e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \\ & - \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} \left\| e_i^T - \frac{B}{2} e_j(t-\tau) \right\|^2 - \frac{3}{4} \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_j^T (t-\tau) B^2 e_j(t-\tau) \end{aligned}$$

Now it is readily seen that

$$-\sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} \left\| e_i^T - \frac{B}{2} e_j(t-\tau) \right\|^2 - \frac{3}{4} \sum_{i=1}^N \sum_{j=1, j \neq i}^N c_{ij} e_j^T (t-\tau) B^2 e_j(t-\tau) \leq 0 \quad (14)$$

Similarly, we obtain

$$\sum_{i=1}^N c_{ii} e_i^T B e_i(t-\tau) + \sum_{i=1}^N c_{ii} e_i^T e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \leq \sum_{i=1}^N c_{ii} \left\| e_i^T + \frac{B}{2} e_i(t-\tau) \right\|^2 \leq 0$$

Thus we have

$$\dot{V}(t) \leq \sum_{i=1}^N e_i^T (A - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2) e_i(t) \leq 0. \quad (15)$$

It is obvious that  $\dot{V}(t) = 0$  if and only if  $e_i(t) = 0$  for all  $i = 1, 2, \dots, N$ . The orbits of the network (5) are globally asymptotically stable at  $e_i(t) = 0$ . That is, anti-synchronization of complex dynamical network (1) is achieved under the feedback controllers (6).

## Remark 2

It is obvious that there exists a sufficiently large positive constant  $\xi$  such that the matrix  $A - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2$  is negative definite, so the feedback controllers (6) hold.

## Theorem 2

Suppose Assumption 1 holds, the delayed complex dynamical network (1) is globally anti-synchronized under the following controllers

$$u_i = -f(-s(t), t) - f(s(t), t) + (-\xi + c_{ii}) e_i, \quad i = 1, 2, \dots, N, \quad (16)$$

and

$A + (L - \xi - c_{ii})I - \frac{5}{4}c_{ii}B^2$  is negative definite matrix.

### Proof

Construct the Lyapunov function candidate as follows:

$$V(t) = \frac{1}{2} \sum_{i=1}^N e_i^T e_i - \sum_{i=1}^N c_{ii} \int_{t-\tau}^t e_i^T(\theta) B^2 e_i(\theta) d\theta - \frac{1}{4} \sum_{i=1}^N c_{ii} \int_{t-\tau}^t e_i^T(\theta) B^2 e_i(\theta) d\theta. \quad (17)$$

Then the time derivative of  $V(t)$  along the solution of the error system (5) is given as follows

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N e_i^T \dot{e}_i - \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \\ &\quad - \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau). \end{aligned} \quad (18)$$

From Equation (5), it can be obtained that

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N e_i^T (A e_i(t) + f(x_i(t), t) + f(s(t), t)) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) + \sum_{i=1}^N e_i^T u_i \\ &\quad - \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \\ &\quad - \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau). \end{aligned} \quad (19)$$

Now, Substituting Equation (16) into Equation(19), it follows that

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N e_i^T A e_i(t) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) + \sum_{i=1}^N e_i^T (f(x_i(t), t) - f(-s(t), t)) \\ &\quad + \sum_{i=1}^N (-\xi + c_{ii}) e_i^T e_i - \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \\ &\quad - \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T B^2 e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau). \end{aligned} \quad (20)$$

Hence we have

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N e_i^T (A + LI - \xi I - c_{ii} I - \frac{5}{4} c_{ii} B^2) e_i(t) + \sum_{i=1}^N \sum_{j=1}^N c_{ij} e_i^T B e_j(t-\tau) \\ &\quad + \sum_{i=1}^N c_{ii} e_i^T e_i + \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) + \sum_{i=1}^N c_{ii} e_i^T e_i + \frac{1}{4} \sum_{i=1}^N c_{ii} e_i^T (t-\tau) B^2 e_i(t-\tau) \end{aligned} \quad (21)$$

The rest of proof is similar to Theorem 1 and omitted  
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here, therefore, this theorem has been proofed.

### Remark 3

In fact, as long as  $\partial f / \partial x_i$  ( $i=1, 2, \dots, N$ ) is bounded, Assumption 1 holds always (Li et al., 2009). And a variety of nonlinear chaotic systems satisfy Assumption 1, such as Lorenz systems, Qi systems (Qi et al., 2005), Chen systems and so on.

### NUMERICAL SIMULATIONS

In this section, to verify and demonstrate the effectiveness of the proposed methods, we consider two numerical examples, that is, the Chen chaotic system and the Qi chaotic system. It is well known that the Chen chaotic system is described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -x_1 x_3 \\ x_1 x_2 \end{pmatrix}, \quad (22)$$

where

$$A = \begin{pmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{pmatrix}, \quad (23)$$

while the Qi chaotic system is

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} x_2 x_3 \\ -x_1 x_3 \\ x_1 x_2 \end{pmatrix}, \quad (24)$$

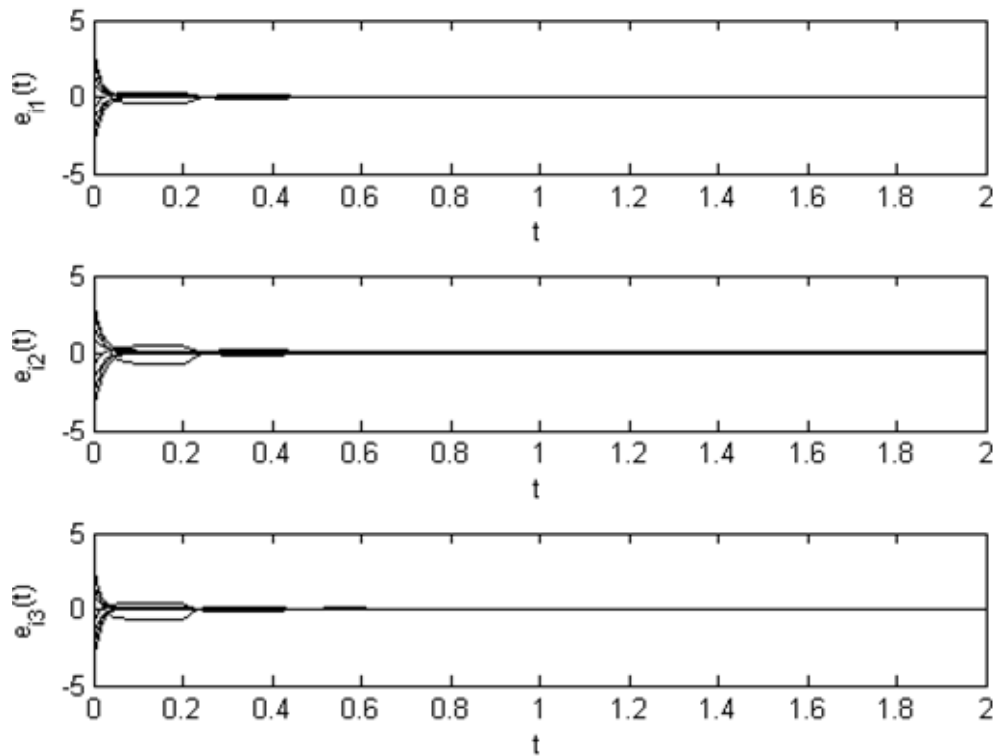
where

$$A = \begin{pmatrix} -35 & 35 & 0 \\ 80 & -1 & 0 \\ 0 & 0 & -8/3 \end{pmatrix}. \quad (25)$$

Then, we will investigate these two chaotic systems in detail to validate the effectiveness of Theorems 1 and 2.

### Example 1

To verify the effectiveness of Theorem 1 with the Chen system. Now, we consider a weighted linearly coupled complex dynamical network (26) with coupling delay



**Figure 1.** Anti-synchronization errors  $e_{i1}(t)$ ,  $e_{i2}(t)$ ,  $e_{i3}(t)$  of the network (26).

consisting of 8 identical Chen chaotic systems. Then, the network system is defined as

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = A \begin{pmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{pmatrix} + \begin{pmatrix} 0 \\ -x_{i1}x_{i3} \\ x_{i1}x_{i2} \end{pmatrix} + \sum_{j=1}^N c_{ij} Bx_j(t-\tau) + u_i, \quad (26)$$

Where

$$C = \begin{pmatrix} -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -3 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -3 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -5 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$i = 1, 2, \dots, 8. \quad (27)$$

According to Theorem 1 in Section 3, the following feedback controllers are chosen

$$u_i = -f(x_i(t), t) - f(s(t), t) + (-\xi + c_{ii})e_i, \quad i = 1, 2, \dots, 8. \quad (28)$$

By Equations (26) and (28), we can get the error

dynamical system

$$\dot{e}_i(t) = Ae_i(t) + \sum_{j=1}^N c_{ij} Be_j(t-\tau) + (-\xi + c_{ii})e_i. \quad (29)$$

Assume that time delay  $\tau = 0.2$ . In accordance to Theorem 1, we select control parameter  $\xi = 99$  which satisfies the stability conditions that is  $A - \xi I - c_{ii}I - \frac{5}{4}c_{ii}B^2$  is negative definite matrix.

Figure 1 shows the anti-synchronization errors of  $e_{i1}(t)$ ,  $e_{i2}(t)$ ,  $e_{i3}(t)$  ( $1 \leq i \leq 8$ ) under the feedback controllers (28). Clearly, all anti-synchronization errors are rapidly converging to zero.

**Example 2**

To verify the effectiveness of Theorem 2 with the Qi chaotic system. We consider a weighted linearly coupled complex dynamical network (29) with coupling delay consisting of 8 identical Qi chaotic systems. Then, the network system is defined as

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = A \begin{pmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{pmatrix} + \begin{pmatrix} x_{i2}x_{i3} \\ -x_{i1}x_{i3} \\ x_{i1}x_{i2} \end{pmatrix} + \sum_{j=1}^N c_{ij} Bx_j(t-\tau) + u_i, \quad (30)$$

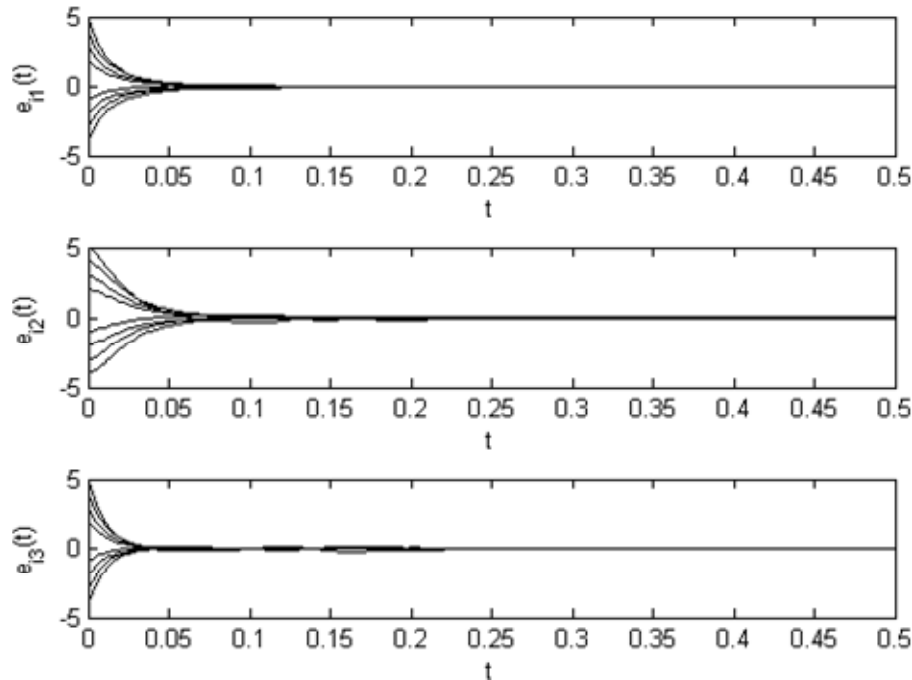


Figure 2. Anti-synchronization errors  $e_{i1}(t), e_{i2}(t), e_{i3}(t)$  of the network (30).

Where

$$C = \begin{pmatrix} -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -3 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -3 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -4 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -5 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$i = 1, 2, \dots, 8.$  (31)

According to Theorem 2 in Section 3, the following feedback controllers are chosen

$$u_i = -f(-s(t), t) - f(s(t), t) + (-\xi + c_{ii})e_i, \quad i = 1, 2, \dots, 8. \quad (32)$$

By Equations (30) and (32), we can get the error dynamical system

$$\dot{e}_i(t) = Ae_i(t) + \sum_{j=1}^N c_{ij}Be_j(t-\tau) + f(x_i(t), t) - f(-s(t), t) + (-\xi + c_{ii})e_i. \quad (33)$$

Assume that time delay  $\tau = 0.2$ . In accordance to

Theorem 2, we select control parameter  $\xi = 70$  which satisfies the stability conditions that is  $A + (L - \xi - c_{ii})I - \frac{5}{4}c_{ii}B^2$  is negative definite matrix. Then, the anti-synchronization errors  $e_{i1}(t), e_{i2}(t), e_{i3}(t)$  ( $1 \leq i \leq 8$ ) of the complex network (30) is shown in Figure 2. The numerical results show that feedback controllers for complex delayed dynamical networks (30) are effective in Theorem 2.

### CONCLUSIONS

A general complex dynamical network with delayed nodes has been studied in this paper. By constructing appropriate Lyapunov function, two feedback anti-synchronization criteria are derived. The criteria are very useful for understanding the mechanism of anti-synchronization in complex networks with time delayed nodes. Moreover, the feedback controllers for achieving network anti-synchronization are expressed in simple forms that can be readily applied in practical situations. Finally, numerical simulations have been presented to demonstrate the effectiveness of the proposed anti-synchronization criteria.

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