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Determination of economic thickness of insulation of local hemispherical clay pot

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Undue bulkiness, which characterizes most design in the past, is gradually giving way to economic approach to design, which gives optimal solutions to engineering problems through optimum design techniques, leading to material and cost economy. This paper formulates a mathematical problem and presents a computer–aided solution for determination of economic thickness of local fire-clay cooking pot for functional performance in heat retention at minimum expense on heating and material. Cost analysis of heating and pot material was integrated into heat transfer phenomenon in spherical shells. The thickness of the pot at which the total cost function goes through a minimum, found to be 10.02 mm for hemispherical pot of 3.3523 × 10^{-8} capacity was taken as the economic solution.

Key words: Hemispherical pot, clay, heat transfer, economic thickness.

INTRODUCTION

Until a few decades ago, components were generally over designed. They were either too bulky or were made with unwarranted precision for the intended use (Stephen and Raymond, 1980). Good engineering practice, however, requires that projects, products and planning be approached in a cost effective manner (Udomon, 2001). Today, the concept of optimum design makes possible economy of resources. An optimum economic design could either be based on conditions giving maximum profit per unit of production or minimum cost per unit of time (Robert et al., 1997). For the later, the total cost may go through a minimum at one value of a particular design variable. Such a value is ultimately taken as the optimum. This technique is herein adopted for determination of economic thickness of the local clay pot for cooking of food items and or boiling of fluids. Clay is one of the cheapest refractory raw materials in existence. This class of material is inorganic and non-metallic in nature, and may be described as those which will retain the original physical shape and chemical stability when subjected to high temperatures. Thus, they are characterized by the ability not only to withstand the heat but also to chemically attack, abrasion, impact and shock caused by thermal stresses (Borode et al., 2000). The thermal conductivity of clay is generally below 0.2 $Wm^{-1} K^{-1}$ at an operational temperature of 200°C (Warmer et al., 1993), though (Folaranmi, 2009) asserted that thermal conductivity increases with moisture content of clay while (Manukaji, 2013) reported that it decreases with the addition of sawdust. Borode et al. (2000) and Hassan and Adewara (1994) have confirmed the suitability of most types of clay, especially Nigerian clays; for high temperature applications. This justifies the practice, from the olden days to the present, of the use of clay pots for cooking. The subsequent retention of heat by the pot for a considerable period of time affords thermal

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conditioning of food items. This property prevents food items such as stew from early loss of taste. In order that the heat loss through the clay pot be reduced, increase in the thickness of the clay can be considered a practical solution.

However, increase in the thickness of the clay layer leads to an increased operational cost of the clay pot. A minimum thickness must therefore exist which ensures the required reduction of heat loss to the surrounding and minimizes heating and insulation cost. This work is aimed at determining this economic thickness of the clay pot. Local fire clay pots are modeled in a hemispherical configuration (Daniel, 1973); the heat transfer phenomenon associated with the clay pot is therefore analyzed using the thick spherical shell design approach (Kurmi, 1991).

MATERIALS AND METHODS

In this problem, it is assumed that conduction is the predominant mode of heat-transfer in the system, hence the emphasis in the use of Fourier's law. This is expressed as:

$$Q = kA\frac{dt}{dx} \quad \text{.............................} \quad (1)$$

where Q = quantity of heat transferred per second (W).
k = thermal conductivity of clay ($Wm^{-1}K^{-1}$).
A = surface area of the material (m^2).
dt = temperature difference on any two faces (K).,
dx = thickness of the body through which the heat flows (m).

Common clay pots are of hemi-spherical configuration, considering this shape (Figure 1); Equation (1) takes the form:

$$Q = kA\left(\frac{-dt}{dr}\right) \quad \text{....................} \quad (2)$$

where from Figure 1,
t_1 = temperature of the inside wall of the pot (K).
t_2 = outside temperature (ambient).
r_1 = inside radius of hemispherical pot (m).
r_2 = outside radius of the pot (m).
dt = temperature across the thickness (K).
dr = small element of thickness (m).

A thick sphere may be imagined to consist of a large number of thin concentric spheres of increasing radii (Kurmi, 1991). Considering any thin imaginary sphere of thickness *dr* at a distance *r* from the center of the sphere;
Surface area of spherical object is given as:

$$A = 4\pi r^2 \quad \text{.......................} \quad (3)$$

and the volume of the hemispherical pot is given as

$$V = 4/6\,(\pi r^3) \quad \text{..........................} \quad (4)$$

Substituting Equation (3) in (2):

$$Q = -4\pi k r^2 (dt/dr) \quad \text{..............} \quad (5a)$$

Solving Equations (5) using separation of variables

$$\frac{dr}{r^2} = \left(-\frac{4\pi k}{Q}\right)dt \quad \text{.............................} \quad (5b)$$

from which

$$Q = 4\pi k r_1 r_2[(t_1-t_2)/(r_2-r_1)] \quad \text{.................} \quad (6)$$

Let the clay pot thickness be X such that

$$X = r_2 - r_1 \quad \text{.............................} \quad (7)$$

Let the heat transfer coefficient from the item in the pot to the inner layer of the pot be denoted as U. Hence

$$Q = U(t_{item}-t_1) \quad \text{................................} \quad (8)$$

where:
t_{item} = the temperature of the item being cooked (K).

Combining Equations (6), (7), and (8)

$$4\pi k r_1 r_2(t_1-t_2)/X = U(t_{item}-t_1) \quad \text{.................} \quad (9)$$

From Equation (9)

$$X = 4\pi k r_1 r_2(t_1-t_2) \;/\; U(t_{item}-t_1) \quad \text{............} \quad (10)$$

Let the cost of heat transfer from the item (heated up through wood burning in a local stove) through the clay pot to the surrounding be C_{heat}

$$C_{heat} = 3600\, Q\, C_w T \quad \text{..........................} \quad (11)$$

where
C_w = dry wood cost; equivalent to 1.0 joule of heat. That is, cost of 0.0556 kg of dry wood (Folaranmi, 2009).
T = time of operation of the pot per year.

Cost associated with the insulating property of the pot can be expressed as

$$C_{pot} = C_c X / S_l \quad \text{..........................} \quad (12)$$

Where
C_c = Cost per unit volume of clay (₦m^{-3}).
S_l = Service life of the pot (years).

The cost per unit volume of the clay has been put at the cost of 304.34 kg of fine-clay.
Therefore the total cost of heating is given by Equation (11) and (12) as

$$C_{HT} = C_c X / S_l + 3600\, Q\, C_w T \quad \text{.........} \quad (13)$$

Substituting for Q from Equation (8) and X from Equation (10)

$$C_{HT} = [4C_c\pi k r_1 r_2(t_1-t_2) / [U(t_{item}-t_1) S_l]] + 3600\, U C_w T (t_{item}-t_1) \quad \text{...} \quad (14)$$

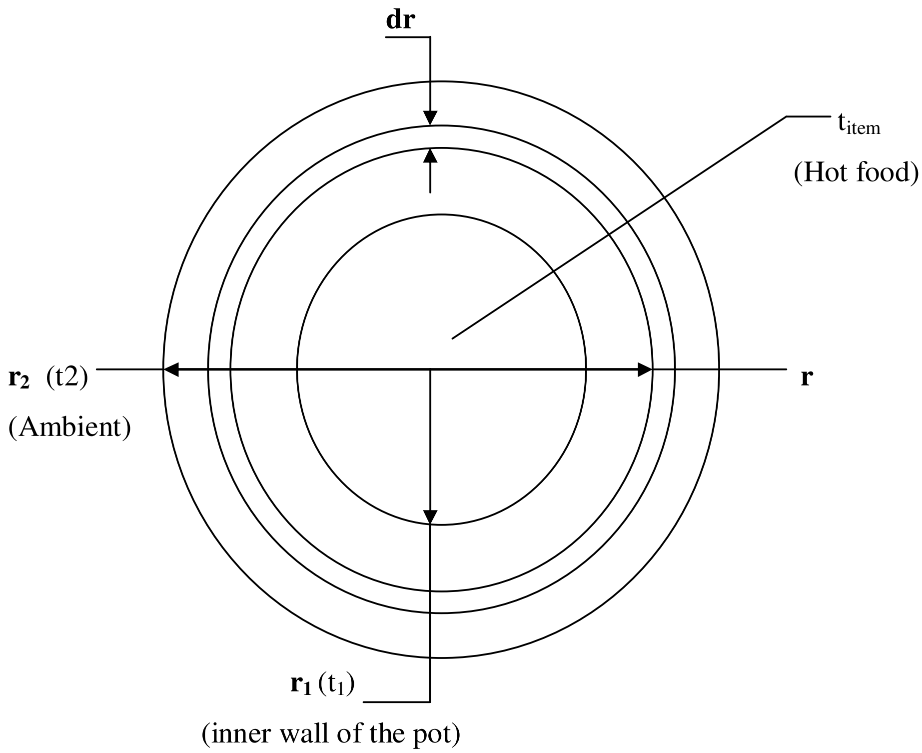

Figure 1. A typical hemispherical cross-section of the clay pot.

The heat transfer coefficient, U is a function of t_1, hence

$U = J\,(t_1)$ ………………………………….. (15)

where J is a constant.

Substituting Equation (15) in Equation (9)

$4\pi k r_1 r_2 (t_1 - t_2) / X = J\, t_1\, (t_{item} - t_1)$ ………….. (16)

This reduces to a form

$J t_1^2 + ((4\pi k r_1 r_2 / X) - J t_{item})\, t_1 - (4\pi k r_1 r_2 t_2 / X) = 0$ …… (17)

The resulting quadratic equation can be solved for the practical value of the inner surface temperature of the pot. This is obtained as

$$t_1 = \frac{1}{2J}\left[\left(Jt_{item} - \frac{4\pi k r_1 r_2 t_2}{X}\right) \pm \sqrt{\left(\frac{4\lambda k r_1 r_2}{X} - Jt_{item}\right)^2 + \frac{16 J \pi k r_1 r_2 t_2}{X}}\right] \quad \text{……….} \quad (18)$$

Computer simulation

A non-differential numerical method is employed in determining the economic thickness of the clay pot. With a predetermined capacity of the pot intended, the hemispherical internal radius r_1, is determined from Equation (4).

A guess value of the outside radius r_2 of the pot is then chosen; this results in a guess value of the pot thickness through Equation (7). Other parameters needed to arrive at the pot inner surface temperature in Equation (18) are characteristic of the heating system and clay properties; values of which are known. Hence, the total cost of heating can be deduced from Equation (14). This process is then repeated several times; each time with an increment in the guess value for r_2.

A computer simulation of the above procedure was carried out. The program, written in C++ language and run on the Borland C++ version 5.02 compiler is presented with this paper. It carries out 50 iterations and "home in" on an optimum thickness value of the clay pot corresponding to the least cost of the pot material and the heating process (Algorithm 1).

The following are the data used for program assessment and validation.

k = 0.0865 $Wm^{-1}K^{-1}$, t_{item} = 100°C, U = 4.0, t_2 = 27.83°C, J = 0.25, C_c = ₦20.00, V = 3.3523×10^{-8} m^3, Outside diameter increment = 0.004 m, S_l = 5 yrs, T = 2180 h, C_w = ₦2.0×10^{-8}.

RESULTS AND DISCUSSION

The above data were fed into the program, and run using Borland C++ 5.02 compiler, the result of which is as presented in Table 1. It is observed at first that increase in the thickness of the pot results in a decrease in the total annual operational cost.

After some iteration, the total annual operational cost begins to rise. From a plot of the total cost versus pot

```
// Computer program
// Compiler  : BORLAND C++  Version  5.02    ^^^^^^^^^^^^^^^^^^^^^^^^^^
#include<iostream.h>
#include<fstream.h>
#include<math.h>
float a,b,c,Cpre;
char filename[20];
double k,t1,T,t2,Pi,Cc,Cht;
int main() {
cout<<"enter filename"<<endl;
cin>>filename;
ofstream fout(filename);
float U,J,R1,R2,titem,DD,b1,b2,d;
float Sl,Cw,X,D2,aa,bb;
float V,C1,C2,C3;
// INTERRACTIVE DATA ENTRY  ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
cout<<"Enter the capacity of the pot intended,(cubic metre).\n";
cin>>V;
cout<<"Enter cost of dry wood equivalent to\n";
cout<<"the consumption of 1.0Joule of heat,(=N=/J).\n";
cin>>Cw;
cout<<"Enter service life of the Clay Pot,(Yr.).\n";
cin>>Sl;
cout<<"Enter  operational time of the Pot,(hr./yr).\n";
cin >>T;
cout<<"Enter cost per cubic metre of clay,(=N=/cubic metre).\n";
cin>>Cc;
cout<<"Enter heat transfer coefficient from the inner surface\n";
cout<<"of the Pot to the item being cooked,(W/sq.metre/K).\n";
cin>>U;
cout<<"Enter the value of constant J."<<endl;
cin>> J;
cout<<"Enter a gues value for the outside diameter of the pot\n";
cout<<"intended,(m)."<<endl;
cin>>D2;
cout<<"Enter a small change in the value just entered."<<endl;
cin>>DD;
a=J;
```

Algorithm 1. Computer simulation written in C++ language and run on the Borland C++ version 5.02 compiler.

```
cout<<"Enter coeff of thermal conductivity of clay,(W/m/K)./n";
cin>>k;
cout<<"Enter flame temperature,or temperature of the hot item.\n";
cin>>titem;
cout<<"Enter the temperature of the surrounding\n";
cout<<"for which the pot is to be put,(K).\n";
cin>>t2;
//  OUTPUT HEADING   ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
cout<<"Clay thickness  Total Cost   Inner Surface Temp."<<endl;
fout<<"Clay thickness  Total Cost   Inner Surface Temp."<<endl;
cout<<"     (m)          (=N=)            (K)         "<<endl;
fout<<"     (m)          (=N=)            (K)         "<<endl;
//   ITERATION   ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
 for(int i=0;i<50;i++) {
  Pi = 22.0/7.0;   cout<<"Pi"<<Pi;
  aa = 1.0/3.0;   cout<<"aa"<<aa;
  bb = (3.0*V)/(4.0*Pi);   cout<<"bb"<<bb;
  R1 = pow(bb,aa);   cout<<"R1"<<R1;
  R2 = D2/2.0;   cout<<"R2"<<R2;
  X  = R2-R1;   cout<<"X"<<X;
  b1 = (4.0*Pi*k*R1*R2)/X;
  cout<<"b1"<<b1;
  b2 = J*titem;   cout<<"b2"<<b2;
  c  = ((4.0)*Pi*k*R1*R2*t2)/X;
  cout<<"c"<<c;
  b  = b1-b2;   cout<<"b"<<b;
  d  = (b*b-4*a*c);   cout<<"d"<<d;
  t1 = (-b+sqrt(d))/(2.0*a);   cout<<"t1"<<t1;
  C1 = 4.0*Pi*k*Cc*R1*R2*(t1-t2);
  cout<<"C1"<<C1;
  C2 = U*(titem-t1)*S1;   cout<<"C2"<<C2;
  C3 = 3600.0*Cw*T*U*(titem-t1);   cout<<"C3"<<C3;
  Cpre=(C1/C2)+C3;
  cout<<"Cpre"<<Cpre;
  Cht = Cpre;    D2+=DD;
//  WRITING RESULTS TO OUTPUT FILE  ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
  cout<< X <<"\t"<<"\t"<<Cht<<"\t"<<"\t"<<"\t"<<t1<< endl;
  fout<< X <<"\t"<<"\t"<<Cht<<"\t"<<"\t"<<"\t"<<t1<< endl;    }}
//   ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
```

Algorithm 1. Contd.

Table 1. Output of the Program.

Clay thickness (m)	Total cost (=N=)	Inner surface temperature (℃)
0.00500016	0.419450	99.4433
0.00520016	0.411501	99.4605
0.00540016	0.404352	99.4763
0.00560016	0.397914	99.4911
0.00580016	0.392116	99.5048
0.00600016	0.386891	99.5176
0.00620016	0.382186	99.5296
0.00640016	0.377952	99.5408
0.00660016	0.374145	99.5514
0.00680016	0.370729	99.5613
0.00700016	0.367670	99.5707
0.00720016	0.364936	99.5795
0.00740016	0.362503	99.5879
0.00760016	0.360347	99.5958
0.00780016	0.358446	99.6033
0.00800016	0.356781	99.6105
0.00820016	0.355333	99.6173
0.00840016	0.354090	99.6237
0.00860016	0.353036	99.6299
0.00880016	0.352157	99.6358
0.00900016	0.351444	99.6414
0.00920016	0.350885	99.6468
0.00940016	0.350467	99.6519
0.00960016	0.350187	99.6569
0.00980015	0.350031	99.6616
0.0100002	0.349996	99.6661
0.0102002	0.350072	99.6705
0.0104002	0.350253	99.6747
0.0106002	0.350534	99.6788
0.0108002	0.350910	99.6826
0.0110002	0.351373	99.6864
0.0112002	0.351921	99.6900
0.0114002	0.352549	99.6935
0.0116002	0.353252	99.6969
0.0118002	0.354027	99.7001
0.0120002	0.354870	99.7033
0.0122002	0.355778	99.7063
0.0124002	0.356747	99.7093
0.0126002	0.357776	99.7121
0.0128002	0.358860	99.7149
0.0130002	0.359997	99.7175
0.0132002	0.361186	99.7201
0.0134002	0.362423	99.7227
0.0136002	0.363707	99.7251
0.0138001	0.365035	99.7275
0.0140001	0.366405	99.7298
0.0142001	0.367816	99.7320
0.0144001	0.369267	99.7342
0.0146001	0.370755	99.7363
0.0148001	0.372279	99.7384

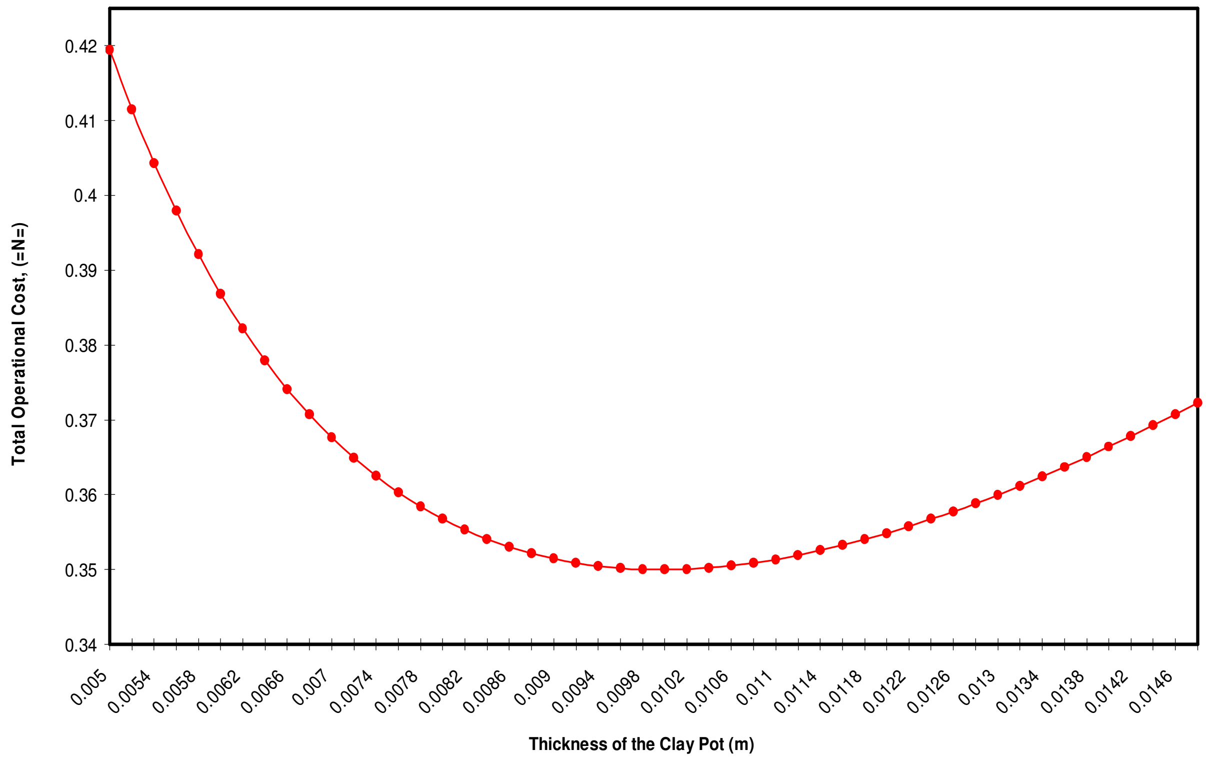

Figure 2. A plot of total cost against pot thickness.

thickness, presented in Figure 2, the turning point was found to correspond with the least value of the total cost function. This value is therefore taken as the minimum cost and the corresponding thickness (10.02 mm) adopted as the economic thickness of the clay pot.

Conclusion

The economic thickness of local fire clay cooking pots was found to be 0.01002 m for hemispherical pots of capacity 3.3523×10^{-8} m^3 for any food item with a maximum boiling temperature of 100℃.

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