

## *Full Length Research Paper*

# **Use of emperical approaches and numerical analyses in design of buried flexible pipes**

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**In this article, vertical and horizontal deflections in 18 thermoplastics pipes buried under high fills within the scope of Ohio University project were analyzed by using the Iowa equation which is a traditional approach in design of buried flexible pipes, approaches derived from this equation and the method developed by German Standards. During the investigation, the results obtained through these methods were evaluated by taking pipe and backfill material into account. Furthermore, the behavior of thermoplastic pipes and burial medium were 2D modeled and analyzed with finite element method. The results determined through analyses conducted with the finite elements method as well as empirical approaches were compared with field measurements and evaluated.**

**Key words:** Flexible pipe, deflection, modified Iowa equation, German method, ATV, PLAXIS analysis.

## **INTRODUCTION**

In general, empirical approaches, field and laboratory experiments and numerical analyses can be made use of in the design of buried flexible pipes. As a traditional approach, modified Iowa equation (Watkins, 1958) is commonly used in the design of buried flexible pipes. In the last two decades, the studies have been conducted for considering the pipe-soil system interaction in the design which means interference on the aspects of modified Iowa Equation which has been considered to be insufficient.

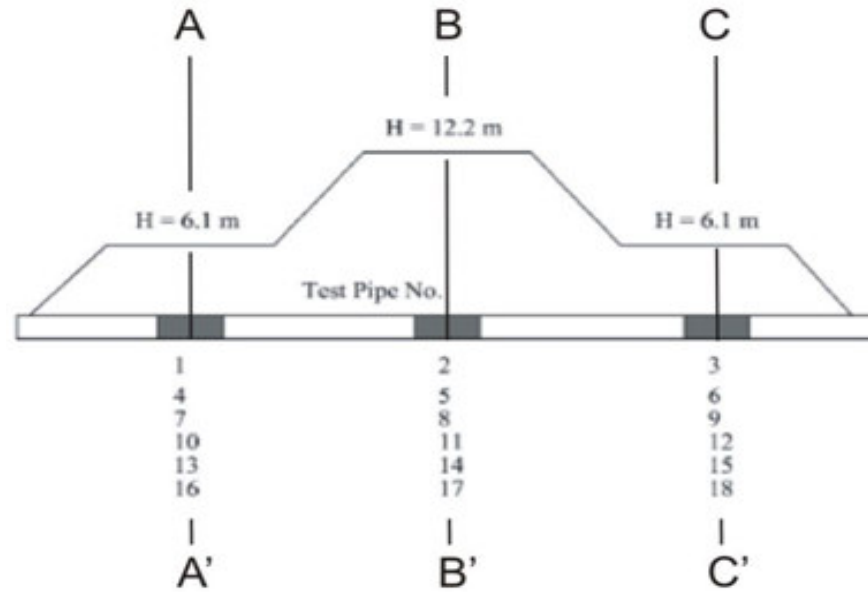
A number of limit states are important in the design of buried thermoplastic pipes. Pipe deflections have traditionally been the prime focus of attention in design to maintain the serviceability of a culvert structure, with deflection limits between 5 and 7.5% specified in various codes of practice (Dhar et al., 2004). On a flexible pipe, horizontal deflection under vertical load results in a passive resistance of sidefill and this resistance maintains sidefill to bear some part of the load on the pipe. Vertical soil load on pipe crown depends on the relative stiffness of pipe and sidefills. Relative

displacement on flexible pipe crown maintains the so-called arching phenomenon to occur in the soil. In case arching occurs at the desired ratio, load on the flexible pipe reduces significantly. So that, it is understood that flexible pipe and surrounding soils behave like elements of the same system. Therefore, while identifying the behavior of flexible pipes, not only the technical properties of the pipe but also properties of the soils must be known in detail and analyses must be conducted accordingly.

ORITE (Ohio Research Institute for Transportation and the Environment) and ODOT (Ohio Department of Transportation) conducted a comprehensive soil experimentation program in order to identify the behaviors of PVC and HDPE pipes buried under high fills (Sargand et al., 2002). In this article, horizontal and vertical deflections of thermoplastic pipes are estimated by using the Modified Iowa Equation (Watkins and Spangler, 1958) as well as equations derived from this equation (Greenwood and Lang, 1990; McGrath, 1998; Sargand et al., 2002)) and the method suggested by German Standards (ATV, 2000). In addition, numerical analyses have been made in 2D considering 3 cross-sections by using PLAXIS software utilizing the method of finite elements analysis. The results derived from empirical approaches and numerical analyses are

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**Figure 1.** Pipe installation plan and directions of the cross sections (Sargand et al., 2008).

compared with the *in-situ* measurements.

## MATERIALS AND METHODS

### Field tests

Sargand et al. (2002) conducted a comprehensive field study in order to analyze the behavior of buried thermoplastic pipes under high fills in short and long term. In these tests carried out by ORITE (Ohio Research Institute for Transportation and the Environment) and ODOT (Ohio Department of Transportation), behavior of a total of 18 thermoplastic pipes of which 6 is Polyvinyl chloride (PVC) and 12 is high-density polyethylene (HDPE) is analyzed. This field study data and evaluations base on this study are published in many periodicals (Sargand et al., 2002a,b, 2003a,b, 2004, 2005, 2006, 2008; Masada and Sargand, 2007).

Under the scope of testing program, pipes are installed as negative projection pipes in narrow and shallow trenches dug in native soil. As backfill material, sand and crushed materials are used at relative compaction of 86, 90 and 96%. Following the backfilling, embankment is constructed at two different heights (6.1 and 12.2 m) by using native soil. In Figure 1, pipe installation plan and directions of the cross sections are shown. In Table 1 test pipe installation conditions are summarized (Sargand et al., 2002). Compaction was not conducted on the mid 1/3 section of the bedding layer in order to maintain thermoplastic pipes to be seated in bedding layer. In addition, bedding material is prepared at different thicknesses (Sargand et al., 2002). Loss of stiffness depending on temperature and time in thermoplastic pipes are calculated by taking Sargand et al. (1998) equation into account. In this respect, reduction on ring stiffness is around 8% for PVC pipes and 60% for HDPE pipes. Letters A, B, C, D, E and F, listed under pipe type section in Table 1, indicate the pipe wall profile; more details about pipe wall profiles are provided in Sargand et al. (2002).

At this project, native soil is used as embankment fill material. The liquid limit of the soil is 27.2% and plastic limit is 16.5% and according to Unified Soil Classification System (USC), it is classified as low plasticity clay (CL). After embankment construction,

consolidated – undrained (CU) triaxial compression tests are conducted on soil samples taken and shear strength parameters are ( $c$  and  $\phi$ ) determined.

On sand and crushed limestone, used as backfill material, sieve analysis, compaction, one-dimensional compression and consolidated–drained (CD) triaxial compression tests are conducted. One-dimensional compression tests on the backfill soils were performed by compressing a soil sample placed inside a rigid mold for each backfill soil type at each degree of compaction achieved in the field. Each test yielded a nonlinear, concave-upward curve relating the axial stress to the axial strain. From the curve, constrained modulus  $M_s$  values were determined corresponding to different vertical stress levels (Table 2) and used as  $E'$  in the modified Iowa formula in an incremental manner. Hartley and Duncan (1987) have shown that  $E'$  is approximately equal to  $M_s$  for most flexible pipe installations (Sargand et al., 2005).

### Empirical methods used in design of buried flexible pipes

The first and major study conducted on the behavior of flexible pipes is “Iowa Equation” developed by Spangler (1941). Spangler (1941) identified stress distribution around a flexible pipe on the basis of fill load hypothesis (Figure 2). Watkins and Spangler (1958) modified the equation by making some basic modifications on this equation (Modified Iowa Equation, 1958). In the last two decades, some researches thinking that Modified Iowa Equation is insufficient to determine the pipe behavior properly strived to improve the equation. Greenwood and Lang (1990) assigned coefficients to soil stiffness and pipe stiffness parameters which reflect the effect of soil conditions to the equation. McGrath (1998) and Sargand et al. (2005) reflected the arching effect to the prism load in their equations and developed an additional equation which calculates the circumferential shortening occurring on the thermoplastic pipes. In ATV (2000) a set of equations has been developed that can reflect soil conditions to analyses conducted to determine the deflections and loads on rigid and flexible pipes. One of the basic difference of this method from Iowa approaches is that the characteristic properties (Modulus of Elasticity and Rankine

**Table 1.** Test pipe installation conditions (Sargand et al., 2002).

Pipe No	Pipe material	Pipe diameter (mm)	Wall type <sup>(1)</sup>	Ring Stiffness <sup>(2)</sup> (kPa)	Ring Stiffness <sup>(3)</sup> (kPa)	Backfill		Final fill height (m)	Bedding thickness (mm)
						Type	RC (%)		
1	PVC	762	A	45.147	41.809	Sand	96	6.1	150
2	PVC	762	A	45.147	41.705	Cr. L.	96	12.2	150
3	PVC	762	A	45.147	41.511	Cr. L.	86	6.1	150
4	PVC	762	B	97.446	90.606	Sand	86	6.1	150
5	PVC	762	B	97.446	90.606	Cr. L.	96	12.2	150
6	PVC	762	B	97.446	89.996	Cr. L.	96	6.1	150
7	HDPE	762	C	73.308	33.599	Sand	96	6.1	150
8	HDPE	762	C	73.308	33.390	Sand	96	12.2	150
9	HDPE	762	C	73.308	32.973	Cr. L.	86	6.1	150
10	HDPE	762	D	82.099	33.386	Sand	86	6.1	150
11	HDPE	762	D	82.099	33.390	Cr. L.	96	12.2	0-300
12	HDPE	762	D	82.099	32.571	Cr. L.	96	6.1	80-380
13	HDPE	1067	E	61.686	24.242	Sand	90	6.1	0-300
14	HDPE	1067	E	61.686	24.242	Sand	96	12.2	80-230
15	HDPE	1067	E	61.686	24.033	Cr. L.	90	6.1	150-300
16	HDPE	1524	F	34.419	13.350	Cr. L.	90	6.1	80-230
17	HDPE	1524	F	34.419	13.454	Cr. L.	96	12.2	80-230
18	HDPE	1524	F	34.419	13.454	Sand	96	6.1	80-230

(1) Sargand et al. (2002) for details, (2) Initial values, (3) Values that adjusted for both time and temperature Cr. L. – Crushed limestone  
RC- Relative compaction.

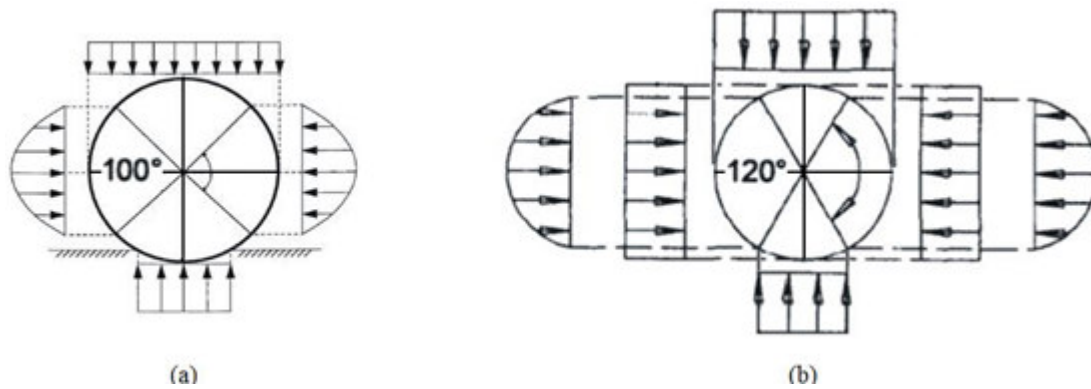
**Table 2.** Summary of one-dimensional compression test results (Sargand et al., 2002).

Backfill material type	Soil pressure (kPa)	M <sub>s</sub> values in (kPa) at relative compaction of:		
		86%	90%	96%
Sand (SP) w <sub>opt</sub> =%11.5 γ <sub>dmax</sub> =18.90 kN/m³	<34	8270	10480	13790
	34.5 – 68.9	9650	11380	14820
	69.0– 103.3	10340	11930	17580
	103.4 – 137.8	10690	12890	24130
	137.8<	-	-	31000
Crushed Limestone (GP) w <sub>opt</sub> =%7.63 γ <sub>dmax</sub> =22.0 kN/m³	<34.5	7580	13100	16890
	34.5 – 68.9	12070	15860	17440
	69.0 – 103.3	17580	19370	20550
	103.4 – 137.8	21370	22340	25510
	137.9 – 172.3	25860	26820	28610
	172.4 – 206.8	27580	28340	33100
	206.9 – 241.3	29300	31300	37230

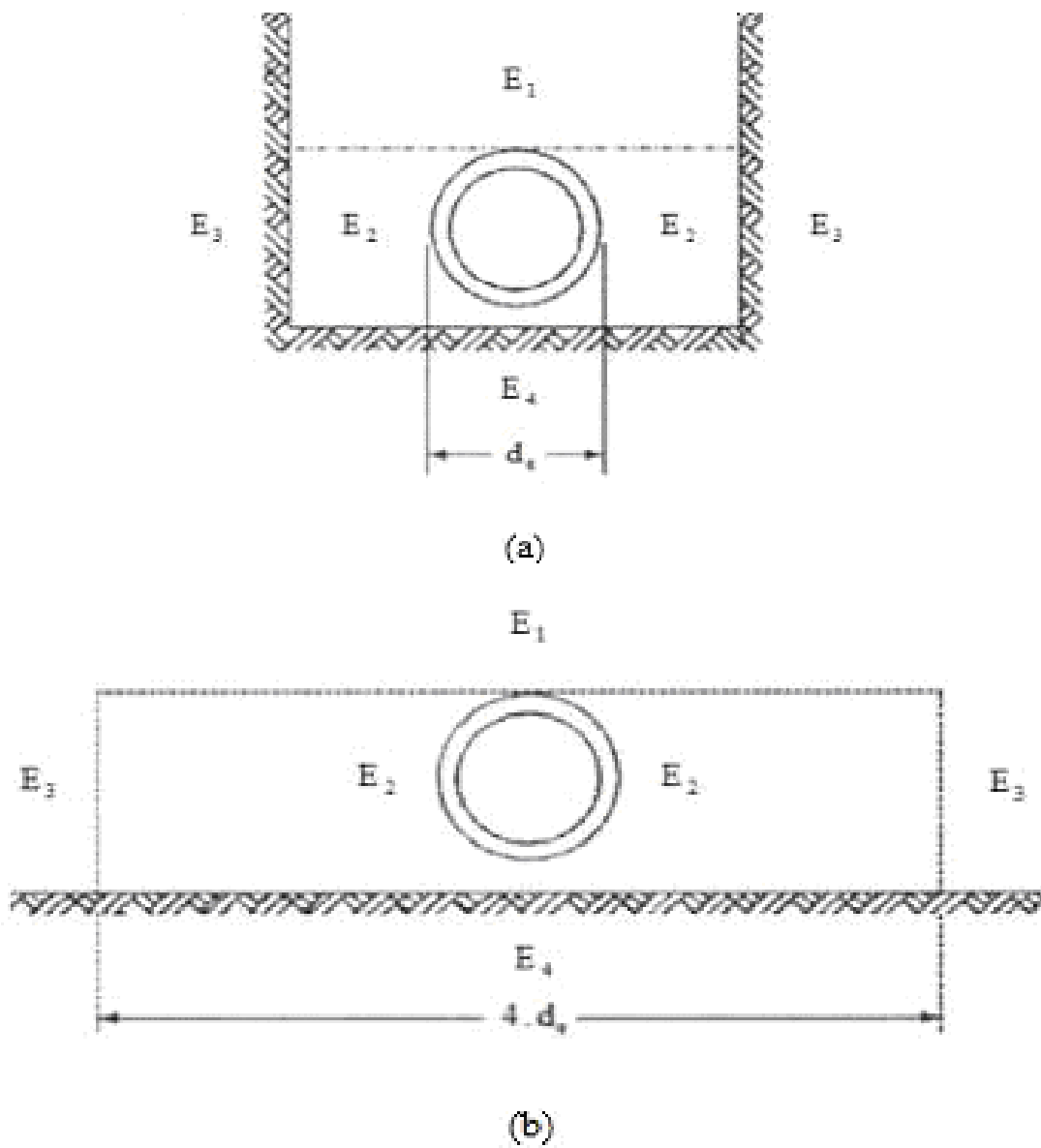
Coefficient) of surrounding soils in the trench is not same for entire surrounding and varies as per zones. In Figure 3, zones comprising the burial medium and soil load distribution are shown. Accordingly, modulus of elasticity is taken as E<sub>1</sub> for fill materials above the pipe crown level, E<sub>2</sub> for sidefills, E<sub>3</sub> for natural soil surrounding trench (Figure 3a) or embankment surrounding the limit in a trench where trench width quadruples pipe diameter (Figure 3b) and E<sub>4</sub> for natural soil under the pipe. In addition, the method suggests

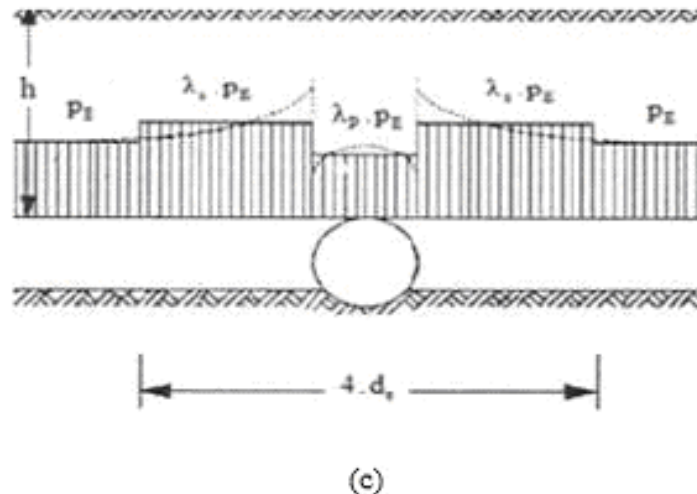
a correction on the sidefills in the modulus of elasticity by taking into account the non-application of compaction at desired quality, creep effect and presence of underground water in pipeline.

ATV (2000) method identifies stress distribution around a flexible pipe (Figure 2). In this method lateral stress on the pipe wall is composed of two components as lateral stress due to soil load and passive resistance of sidefill (stress due to bedding reaction). In addition, passive resistance of sidefills shows a parabolic



**Figure 2.** Stress distribution around buried flexible pipe a) Iowa Approach (Masada, 1996) b) German Method (ATV-DVWK-A 127-E, 2000).





**Figure 3.** a) Trench pipe (b) Embankment pipe (c) Stress distribution around buried flexible pipe (ATV-DVWK-A 127-E, 2000).

distribution over the 120 degree curves plotted from the center (ATV-DVWK-A 127-E, 2000).

In Iowa Approach a bedding constant is identified and assigned to vertical soil load parameter. The bedding constant accommodates the response of the buried flexible pipe to an opposite and equal reaction to the load force derived from the bedding under the pipe (Moser, 2008). In other words, the bedding constant represents the quality of pipe installation in the bottom half of the pipe cross section. Its value is defined by the angle called bedding angle over which the pipe is well supported by the bedding layer soil and can vary from 0.0843 at the bedding angle of  $180^\circ$  (full cradle support) to 0.110 at the bedding angle of  $0^\circ$  (no cradle support) (Masada, 2009). In the German Method, the deformation coefficients are identified and assigned to vertical soil load as well as lateral load due to soil, lateral load due to bedding reaction and load due to the water in which case the ground water is at the pipe line level and these coefficients are used for calculation of vertical and horizontal deflections. The empirical methods used under this study are presented in Table 3.

### Analysis with empirical approaches

Thermoplastic pipe deflections are computed with the commonly used empirical approach in the literature. In these analyses, vertical deflections of thermoplastic pipes occurring during backfilling process are not taken into account; only deflections due to the embankment fill load are considered.

Under the scope of research, the behavior of buried thermoplastic pipes under the high fills are analyzed by using Iowa equations developed by Watkins and Spangler (1958), Greenwood and Lang (1990), McGrath (1998) and Sargand et al. (2005) and the method suggested by German Standards (ATV-DVWK-A 127-E, 2000). In all the analyses conducted, loss of stiffness of pipe materials depending on temperature and time are taken into consideration. For this purpose, Modulus of Elasticity of pipe materials are adjusted on the basis of equations suggested by Sargand et al. (1998) (Table 3) (Sargand et al. 2005).

In the analyses conducted using Iowa equations, prism load theory ( $p=\gamma H$ ) is taken as a basis and furthermore in the analyses conducted with Iowa equations developed by Watkins and Spangler (1958), McGrath (1998) and Sargand et al. (2005) bedding angle is taken as  $90^\circ$  and for bedding coefficient  $K=0.096$  value is used. For

backfill materials, values of constrained modulus that are determined from laboratory tests are directly used as modulus of soil reaction (Table 2).

In the analyses conducted with Iowa Equation developed by Greenwood and Lang (1990) constrained modulus is directly used in place of modulus of soil reaction and soil stiffness factor is not multiplied with coefficient of 0.6 (Table 3). Bedding coefficient is determined according to Proctor relative compaction and the fines contents of backfill soil. For constrained modulus of native soil, values of Modulus of Elasticity suggested by Greenwood and Lang (1990) are used. Thermoplastic pipes are negative projection pipes, therefore reduction factor for trench load is taken as 1.0 (Negative projection pipe is defined as a pipe that is installed in a relatively narrow and shallow trench with its crown remaining below the level of the natural ground). Modulus of Elasticity of pipe materials are adjusted according to the equations suggested by Sargand et al. (1998), therefore no further creep factor is assigned to the ring stiffness parameter. Soil creep factor is taken as 1.0 for short term. Constants "a" and "b" used in the determination of pipe-soil interaction coefficient are determined according the relative compaction (Greenwood and Lang, 1990) (Table 3). In the analyses vertical deflections ( $\delta_{v0}$ ) occurring during backfilling process are not taken into consideration.

Iowa Equation by Sargand et al. (2005) is an approach developed for the calculation of the horizontal deflections in buried thermoplastic pipes. The first section of the equation comprising two sections is Iowa Equation in which constrained modulus of backfill material is used as modulus of soil reaction. Masada (2000) derived the relationship between vertical and horizontal deflections by applying numerical derivation on Modified Iowa Equation (Equation 1). With the assumption that circumferential shortening is uniform at every point of pipe section, total vertical deflection value is obtained by summing up the vertical deflection value with circumferential shortening value. Also, in the Iowa equations as developed by Watkins (1958) and Greenwood and Lang (1990), vertical deflection is calculated from equation (1).

$$\left| \frac{\Delta y}{\Delta x} \right| = 1 + \frac{0.0094E'}{PS} \quad (1)$$

In this equation  $|\Delta y/\Delta x|$  = Deflection ratio, PS = Pipe stiffness =  $F/\Delta y$ , F = Vertical load causing deflection on the flexible pipe in parallel plate loading test (ASTM D 2412, 2000).

Iowa Equation as modified by McGrath (1998) gives the vertical deflection. Horizontal deflection value is calculated by subtracting second term from first term.

According to German Method, vertical stress due to soil load at pipe crown level is calculated on the basis of Silo Theory (Janssen, 1895). In the analyses constrained modulus values provided in the Table 2 are used as  $E_2$ . Creep factor is taken as 1.0 for sand and crushed limestone. The pipeline level is above the ground water level. Backfill materials provided a 300 mm thick cover on pipe crown, therefore it is assumed as  $E_2=E_1$ . For the soil under the pipe, Modulus of Elasticity is determined with the assumption of  $E_4=10E_1$ , as suggested by the method. In the analyses, Leonhardt (1979) approach is adopted (ATV-DVWK-A 127-E, 2000).

In the analyses conducted with German Method, two different cases are taken into consideration. In the first case (ATV (1)) stress distribution around a buried flexible pipe is considered to be affected from narrow trench conditions (ATV-DVWK-A 127-E, 2000). In the second case (ATV (2)) this effect is neglected. The differences between empirical approaches deployed in the analyses are provided in the Table 4.

### Numerical analysis with finite element method

Under the scope of the project the behavior of PVC and HDPE pipes buried under the high fills was numerically modeled previously using CANDE software (Sargand et al., 2002). However, modeling is made on the basis of individual pipes and by taking only trench medium into consideration. In the numerical analyses conducted under the scope of this study, material properties of native soil, burial medium, pipe and fill are all considered and analyses are conducted by modeling field embankment construction conditions.

In the numerical analyses, PLAXIS V9.02 software is used which performs analysis by means of finite element method. A-A', B-B' and C-C' cross-sections, shown in Figure 1 are analyzed by establishing their 2D numerical model. In Figure 4, numerical model and finite element mesh of A-A cross section is shown. The embankment height for the A-A and C-C cross-sections is 6.1 m and the embankment height on the pipes is 12.2 m in the B-B cross-section.

In the analyses soils are modeled by using "Hardening Soil Model" (HS) developed by Schanz et al. (1999) and Mohr–Coulomb Soil Model (MC).

In the geometry developed, thermoplastic pipes are modeled with the tunnel option. Material properties deployed in the analyses for six type of thermoplastic pipes with different wall profile characteristics are denoted as A (Pipes #1, 2 and 3), B (Pipes #4, 5 and 6), C (Pipes #7, 8 and 9), D (Pipes #10, 11 and 12), E (Pipes #13, 14 and 15) and F (Pipes #16, 17 and 18) as shown in Table 5.

Interface elements are generated between backfill materials and native soil, and as interface factor  $R_{int}=0.5$  and  $R_{int}=1.0$  (rigid) values are assigned. It is observed that additional deflections due to reduction of the interface factor are negligible (Akinay, 2010). Therefore it is accepted that interface between backfill material and native soil is rigid. Also, interface elements are generated between thermoplastic pipes and backfill materials and the reduction factor is assigned as  $R_{int}=0.67$  (Massicotte, 2000).

First layer of embankment fill was constructed with a thickness of 0.92 m and compacted with a sheep-foot roller. In following steps each fill layer was constructed with 0.61 m thick lifts and compacted with light-weight construction equipment. 6.1 m high embankment construction is completed in 32 days and 12.2 m high embankment construction in around 38 days (Sargand et al., 2002).

In order to examine the effect of native soil stiffness on the thermoplastic pipe behavior in numerical analyses, lower and upper limit stiffness values selected as 5000 kPa – 20000 kPa, respectively, in reference to Terzaghi-Peck (1968) is used.

Three different analyses are conducted with the finite elements method using three field cross-sections for the examination of thermoplastic pipe behavior. In the first two analyses, soils are modeled with HS model and analyses are conducted in the drained conditions in the cases where native soil is soft and stiff. The results of these analyses are presented as Numerical1 and 2. In the third analysis, soils are modeled with MC model and analyses are conducted considering the native soil behavior as undrained. The results of these analyses are presented as Numerical 3. The behavior of backfill materials are evaluated as drained in both models. Material properties used in analyses are given in Table 6.

In Sargand et al. (2002),  $E_{50}^{ref}$  in equation (2) used in the HS model depends on the values of  $E_{50}$  secant modulus determined from triaxial tests conducted on the backfill and fill materials.  $E_{50}^{ref}$  is the index which determines the value of Modulus of Elasticity and  $m$  stress dependency factor is defined according to reference pressure ( $p^{ref}$ ). In general, it can be taken as  $p^{ref}=100$  kPa and for non-cohesive soils as  $m=0.5$  and for cohesive soils as  $m=1$  and  $E_{ur}^{ref}=3E_{50}^{ref}$ . Value  $\sigma_3'$  in equation (2) is the cell pressure applied in triaxial test.

$$E_{50} = E_{50}^{ref} \left( \frac{c \cos \phi - \sigma_3' \sin \phi}{c \cos \phi + p^{ref} \sin \phi} \right)^m \quad (2)$$

## RESULTS

### Deflections derived from empirical approaches

Vertical and horizontal deflections under final embankment fill load calculated by using different empirical approaches, compared with field measurements are compared on the basis of types of pipe and backfill material. Because of heavy rains during the installation of Pipes #10, 11 and 17, burial medium could not be prepared at desired quality and major deflections were recorded on these pipes (Sargand et al., 2002). These pipes are not included in the evaluations.

In the Figure 5, computed and measured vertical and horizontal deflections on the PVC and HDPE pipes under final embankment load are shown. In the Figure 5a, computed vertical deflections on PVC pipes and in Figure 5b on HDPE pipes, and in Figure 5c computed horizontal deflections on PVC pipes and in Figure 5d on HDPE pipes are compared with field measurements.

From Figure 5, it is observed that vertical and horizontal deflections on the PVC pipes calculated with empirical approaches are more compatible with field measurements. It is also observed that ATV (1) gives the upper limit and ATV (2) gives the lower limit for pipe deflections.

The computed and measured vertical and horizontal deflections under final fill load for the cases in which crushed limestone and sand backfill material used are shown in Figure 6. Vertical deflections for crushed limestone and sand backfills are compared in Figures 6a and b whereas horizontal deflections for the same materials are compared in Figures 6c and d, respectively. In Figures 5 and 6, results of numerical analyses (to be explained later) are also shown.

**Table 3.** Empirical approaches used in the analyses.

Method	Equation	Symbol	Note
(1) (Watkins, 1958) (Modified Iowa Equation)	$\frac{\Delta x}{D} (\%) = \frac{D_L PK}{EI/r^3 + 0.061E'}$	$\Delta x/D$ = Horizontal deflection D = Pipe diameter r = Pipe radius $D_L$ = Deflection lag factor (taken as 1.5) P = Vertical geostatic stress K = Bedding constant (0.0843-0.11) E = Young's Modulus of pipe material I = Moment of inertia of pipe wall per unit length $E'$ = Modulus of soil reaction (Howard (1977, 2006))	1. With deflection lag factor parameter, deflections due to the creep factor in buried flexible pipe under the stable stress are considered. 2. Bedding constant is the indicator of the filling compaction quality in the half lower of the buried flexible pipe. It changes in a very narrow interval. 3. One of the most important factors that affect the pipe behavior is $E'$ parameter.
(2) Greenwood & Lang (1990)	$\frac{\Delta y}{D} (\%) = \frac{(\Delta y/\Delta x)K(C_L \gamma H)}{C_{TP} EI/r^3 + 0.061\zeta \cdot C_T 0.6E_2} \times 100 - \delta_{vo}$ $\zeta = \frac{1.662 + 0.639(B/D - 1)}{(B/D - 1) + [1.662 - 0.361(B/D - 1)]E_2/E_3}$ $C_L = a \left( \frac{EI}{1250D^3} \right)^b \quad C_L = \frac{1 - e^{-2K_a \mu' \frac{H}{B}}}{2K_a \mu' \frac{H}{B}}$	$\Delta y/D$ = Vertical deflection $\zeta$ = Leonhardt factor (1979) B = Trench width H=h = Fill height $\gamma$ = Unit weight of backfill soil K = Coefficient for horizontal stress determined according to Proctor density percentage and fines contents of backfill soil (Greenwood and Lang (1990)) $\mu'$ = Coefficient of friction between backfill soil and native soil $C_{TP}$ = Creep modulus for pipe material $C_T$ = Creep modulus for backfill soil $C_L$ = Coefficient for pipe – soil interaction a, b = Coefficients for $C_L$ , determined according to Proctor density percentage of backfill soil (Greenwood and Lang (1990)) $C_L$ = Retention factor for trench load (Short term) $E_2$ = Constrained modulus of backfill soil $E_3$ = Constrained modulus of native soil $\delta_{vo}$ = Vertical elongation	1. Deflection lag factor is omitted from Modified Iowa equation and for long term analysis reduction in pipe and soil stiffness is suggested. 2. Coefficients that reflect the effect of pipe-soil interaction and natural soil stiffness (Leonhardt Factor) to the soil stiffness. 3. In the soil stiffness factor, instead of soil reaction modulus, secant modulus derived from one-dimensional compression test is used. 4. Leonhardt (1979) suggested the reduction of secant modulus with 0.6 coefficient to have it reflect the trench conditions. 5. In the cases where trench width is equal or 5 times than the pipe diameter, it is presumed that sidefills properties do not get affected from natural soil conditions. 6. Vertical elongation in the flexible pipe during backfilling can be taken into consideration.
(3) McGrath (1998)	$\frac{\Delta y}{D} (\%) = \frac{P_c}{EA/r + 0.57M_s} + \frac{D_L KP_c}{EI/r^3 + 0.061M_s}$	$P_c$ = Vertical geostatic stress A = Cross-sectional area of pipe wall per unit length $M_s$ = Constrained modulus of backfill soil	1. It set forth that in the calculation of vertical deflection in a buried thermoplastic pipe, deflection as well as circumferential shortening must be taken into account. 2. By presuming that deflections in the vertical and horizontal axis are equal to each other, the first part of

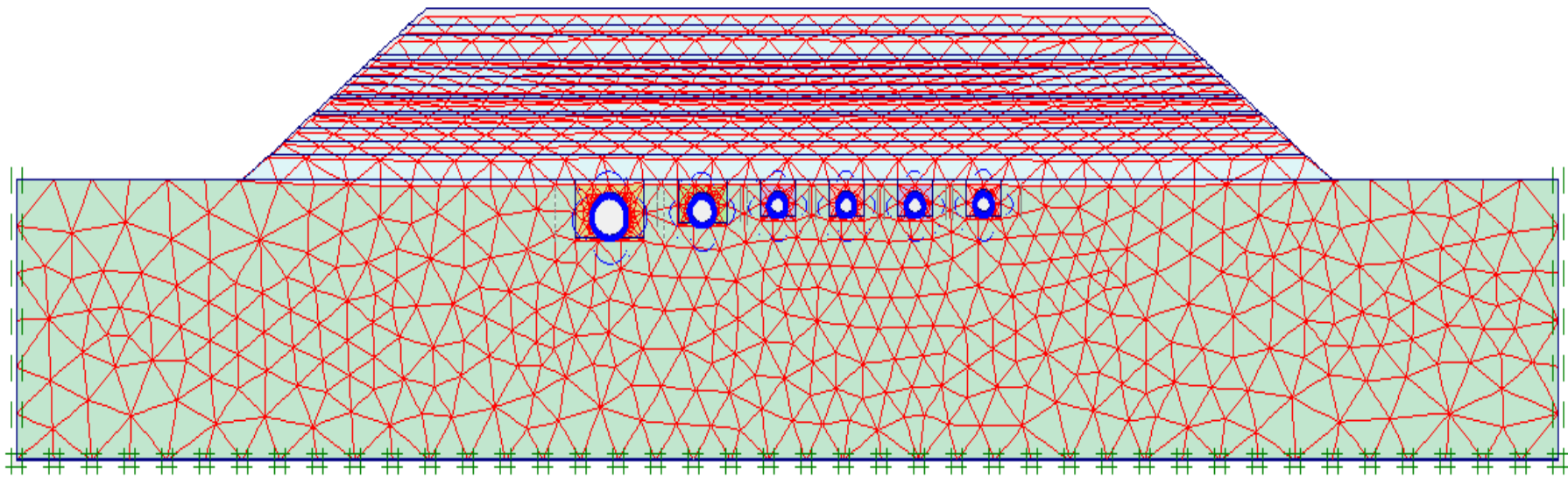
Table 3. Continued.

<p>(4)</p> <p>Sargand et al. (2005)</p>	$\frac{\Delta x}{d} (\%) = \frac{100D \cdot KP}{0.149 (PS) + 0.061E'}$ $- \frac{100P}{E'} \left( \frac{0.364S_H + 0.061S_B + 0.012S_H S_B}{2.571 + 0.572S_H + 0.163S_B + 0.039S_H S_B} \right)$ $VAF = 1 - 0.714 \left( \frac{S_H - 0.7}{S_H + 1.75} \right) + 0.29 \left( \frac{27.31 - S_B}{16.81 + S_B} \right)$ $P = (VAF) \gamma H \quad S_H = rM_s / EA \quad S_B = r^3 M_s / EI$ $E(T, t) = 0.97(532190 - 1787.1T)t^{-0.008334} \text{ (for PVC)}$ $E(T, t) = 0.85(257000 - 2150.5T + 4.8T^2)t^{-0.08257} \text{ (for HDPE)}$	<p><math>\Delta x/d</math> = Horizontal deflection  <math>P</math> = Vertical geostatic stress  <math>S_H</math> = Ring stiffness parameter  <math>S_B</math> = Bending stiffness parameter  <math>E(T, t)</math> = Young's modulus of pipe material that adjusted for both temperature and time (Sargand, vd. 1998)  <math>T</math> = Temperature (<math>F^\circ</math>),  <math>t</math> = Time (minutes)  <math>VAF</math> = Vertical arching factor  <math>PS</math> = Pipe Stiffness (ASTM D 2412, 2000)  <math>\gamma</math> = Unit weight of backfill soil  <math>I</math> = Moment of inertia of pipe wall per unit length</p>	<p>his equation is derived by using the elastic solution (Burns and Richard, 1964) and in the second section, modified Iowa equation is used where constrained modulus is used instead of soil reaction modulus.</p> <p>3. By subtracting the second part from first part in the equation, <math>\Delta x/D</math> is obtained.</p> <p>1. Sargand et al. (2005) considered deflection and circumferential shortening as in the McGrath (1998) approach in determination of horizontal deflection in a buried thermoplastic pipe. In the approach developed, while horizontal deflection is calculated by means of Iowa equation, circumferential shortening is derived by using elastic solution (Burns &amp; Richard, 1964).</p> <p>2. Prism load is allocated with a vertical arching factor (VAF) by considering pipe-soil stiffness interaction. Accordingly, vertical stress on the flexible pipe is identified with <math>P=(VAF)\gamma H</math> equation.</p> <p>3. In this approach, loss of rigidity of thermoplastic pipe depending on time and temperature is taken into consideration by using equations suggested by Sargand et. al (1998) and correction of pipe raw material in modulus of elasticity is suggested.</p>
<p>(5)</p> <p>German Method ATV-DVWK-A 127-E, 2000)</p>	$p_E = \kappa \cdot \gamma_s \cdot h$ $\kappa = \frac{1 - \exp\left(-2\frac{h}{b}K_t \tan \delta\right)}{2\frac{h}{b}K_t \tan \delta}$ $q_v = \lambda_{PG}(\kappa \cdot \gamma_s \cdot h + \kappa_o \cdot p)$ $q_h = K_2 \left( \lambda_s p_E + \gamma_s \frac{d_e}{2} \right)$ $q_h^* = \frac{c_{h,qv} \cdot q_v + c_{h,qh} \cdot q_h}{V_{PS} - c_{h,qh}}$ $\Delta d_v = \frac{2r}{8S_0} (c_{v,qv} \cdot q_v + c_{v,qh} \cdot q_h + c_{v,qh^*} \cdot q_h^*)$ $\Delta d_h = \frac{2r}{8S_0} (c_{h,qv} \cdot q_v + c_{h,qh} \cdot q_h + c_{h,qh^*} \cdot q_h^*)$ $S_0 = EI/r^3$	<p><math>p_E</math> = Vertical geostatic stress  <math>\kappa</math> = Reduction factor  <math>\delta</math> = Friction angle between backfill soil and native soil  <math>q_h</math> = Horizontal stress due to vertical soil load (Figure 2)  <math>q_h^*</math> = Horizontal stress due to bedding reaction (Figure 2)  <math>\Delta d_v</math> = Vertical deflection  <math>\Delta d_h</math> = Horizontal deflection  <math>\lambda_p p_E</math> = Vertical stress on pipe crown  <math>\lambda_s p_E</math> = Vertical stress on sidefills  <math>\lambda_p</math> = Concentration factor for pipe  <math>\lambda_{PG}</math> = Concentration factor for pipe (adjusted for narrow trench condition)  <math>\lambda_s</math> = Concentration factor for sidefills  <math>V_{PS}</math> = Stiffness of pipe – soil system  <math>c_v, q_v, c_v, q_h, c_v, q_h^*</math> = Deformation coefficients for bending moments (ATV-DVWK-A 127-E, 2000)  <math>K_1, K_2</math> = Coefficients for horizontal stress (Figure 1)</p>	<p>1. Soil load stress of a pipe laid in a trench at the crown level according to German Method is calculated according to the Silo Theory (Janssen, 1895). In this calculation, the friction angle between backfill soil and natural soil are reduced according to the compaction method of the backfill.</p> <p>2. Burial medium is divided into four zones (Figure 1a).</p> <p>3. This method advocates that elasticity modulus of pipe material should be reduced in the long-term analyses.</p> <p>4. According to this method, since pipe and sidefill soil has different deformation capacity, soil load stress is shared between pipe and sidefill soil through concentration factors (Figure 1c).</p> <p>5. According to German Method, the stress distribution around buried flexible pipe is shown in Figure 2b).</p> <p>6. In this method, by taking into consideration of four different situations for compaction of sidefill and overlay fill, the reduction of friction angle (between natural soil and backfill material) in the trench wall is suggested.</p>



**Table 4.** Differences between empirical approaches.

	Empirical method	Load theory	Stiffness of native soil	Circumferential shortening	Arching	Bedding constant
Iowa Approaches	Watkins (1958)	Prism load theory	Not considered	Not considered		0.096
	Greenwood and Lang (1990) (G and L)		Considered by employing Leonhardt factor (1979)			Greenwood and Lang (1990)
	McGrath (1998)		Not considered	Considered (Table 1)	Not considered	0.096
	Sargand et al. (2005) (S and M)					0.096
German Method	ATV (2000) ATV (1)	Silo theory; Stress distribution is affected by narrow trench condition (Table 1)	Considered by employing Leonhardt factor (1979)	Not considered	Considered (Table 1)	Deformation coefficients are determined. (ATV-DVWK-A 127-E, 2000)



**Figure 4.** Numerical model and finite element mesh of A-A cross section (Akinay, 2010).

**Table 5.** Pipe wall profile types and pipe material parameters.

Pipe wall profile type	A	B	C	D	E	F
Normal stiffness, EA (kN/m)	32620	35550	8335	9655	11960	18190
Flexural rigidity, EI (kNm <sup>2</sup> /m)	2.49	5.39	4.05	4.54	9.36	15.22
Equivalent thickness, d (m)	0.030	0.043	0.076	0.075	0.097	0.100
Poisson's Ratio, $\nu$	0.30	0.30	0.45	0.45	0.45	0.45

**Table 6.** Material parameters used in numerical analyses.

Soil type	R.C (%)	$\gamma_n$ (kN/m <sup>3</sup> )	$E_{50}^*$ (kPa)	$E_{50}^{ref}$ (kPa)	$c'$ (kPa)	$\phi'$ (°)	$\nu_{ur}$ or $\nu$	m
Sand	86	17.70-18.03	9700	9500	0.00	36.5	0.20-0.35	0.5
	90	18.90	20600	20300	0.00	41.0	0.20-0.35	0.5
	96	19.35-19.95	36000	35500	0.00	44.8	0.20-0.3	0.5
Crushed limestone	86	19.46-19.77	48500	48000	41.0	43.8	0.20-0.25	0.5
	90	20.28-20.71	70000	69500	55.0	42.3	0.20-0.25	0.5
	96	22.19-23.81	90000	89200	69.0	46.0	0.20-0.25	0.5
Fill material	-	20.40	-	5210	34.5	15.0	0.20-0.49	1.0
Bedding	Loose	16.00-18.00	-	6350-32000	0-20	33-40	0.20-0.35	0.5
Native soil	-	20.40	-	5000 <sup>(1)</sup>	0.00	24.0	0.20-0.49	1.0
				20000 <sup>(2)</sup>				
				20000 <sup>(3)</sup>				

Vertical and horizontal deflections developed at each load step up to the final load value are also analyzed. In Figure 7, the variations of deflections calculated through empirical approaches with the embankment fill height are compared with the field measurements. In Figure 7, positive values on the vertical axis indicate the horizontal deflections and negative values indicate the vertical deflections.

Analyses conducted with Iowa Equation developed by Spangler and Watkins (1958) and Greenwood and Lang (1990) gave the more conservative results. In these analyses, it is assumed that constrained modulus is equivalent to soil reaction modulus (Chambers et al., 1980; Hartley and Duncan, 1987; McGrath, 1998).

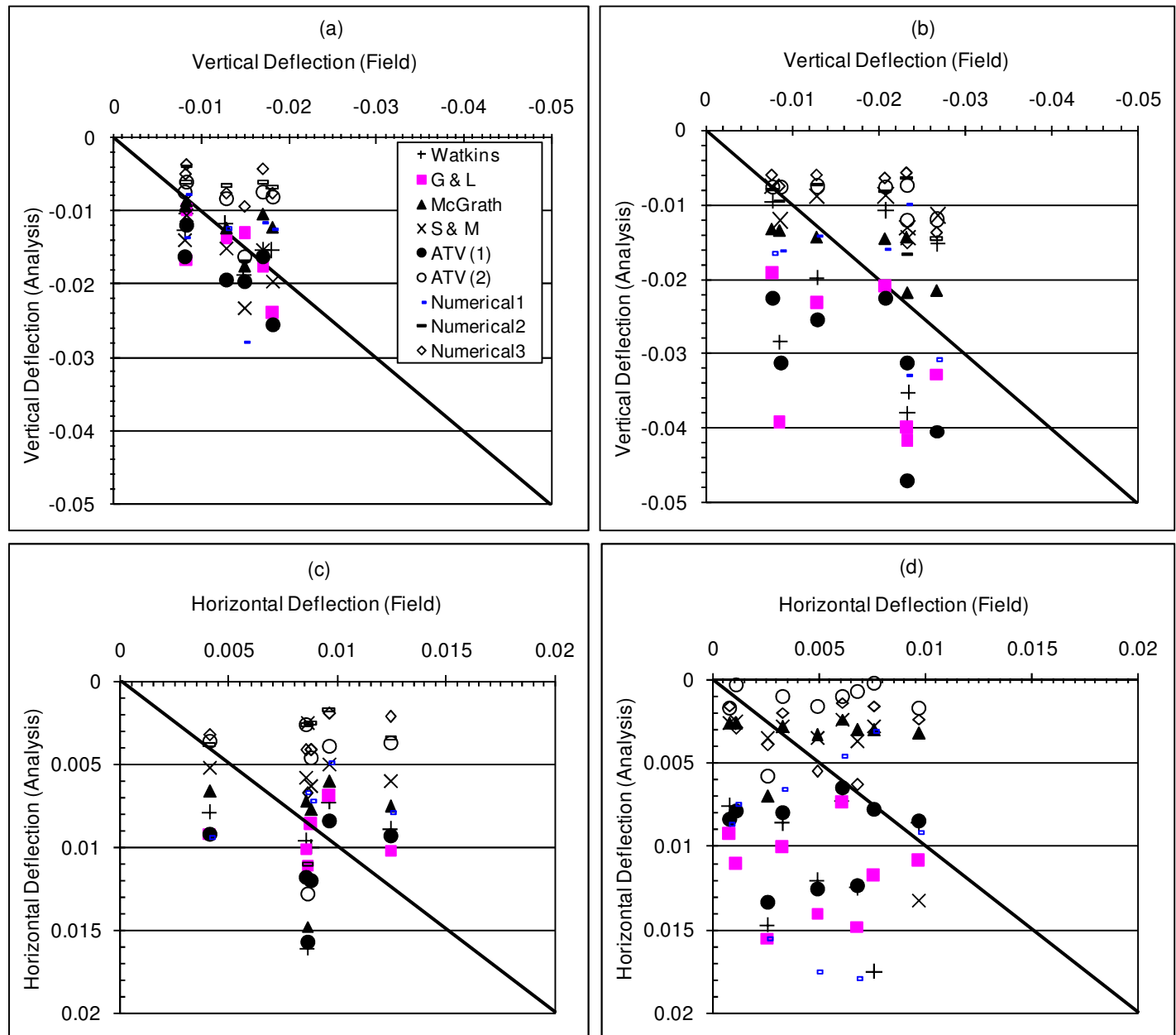
The deflections derived with the McGrath (1998) and Sargand et al. (2005) equations are observed to be generally more compatible with field measurements. Vertical and horizontal deflections calculated with the first case of the German Method (2000) (ATV(1)) are greater than the deflections calculated with its second case (ATV(2)). In general, deflections determined through the first and second cases of ATV method constitute upper and lower boundaries which also include the field measurement.

### Deflections derived from numerical analysis

In analyses vertical deflections due to compaction are

neglected and only deflections due to embankment fill load are calculated. The results derived from numerical analyses are shown in Figure 8. There are 6 pipes in each of A-A, B-B and C-C cross-sections shown in the pipe installation plan in Figure 1. The variations of deflections with embankment fill height are shown in Figure 8. Pipe deflections derived from Numerical1 ( $E_{50}^{ref} = 5000$  kPa) are generally found out to be greater than field measurements (Figure 8a). Pipe deflections derived from Numerical2 ( $E_{50}^{ref} = 20000$  kPa) are lower (at half level) as compared to Numerical 1 (Figure 8b). The lowest deflections are derived from Numerical 3. It is observed that stiffness of native soil and material behavior found out to be greater than field measurements (Figure 8a). Pipe deflections derived from Numerical2 ( $E_{50}^{ref} = 20000$  kPa) are lower (at half level) as compared to Numerical 1 (Figure 8b). The lowest deflections are derived from Numerical 3. It is observed that stiffness of native soil and material behavior (drained-undrained) have considerable effect on the vertical and horizontal deflection behavior of thermoplastic pipes. It can be seen that deflection values derived from Numerical 1 is more compatible with field measurements.

Computed deflections under the final fill load through finite element analyses are also shown in Figures 5 and 6 for pipe together with values calculated using empirical equations.



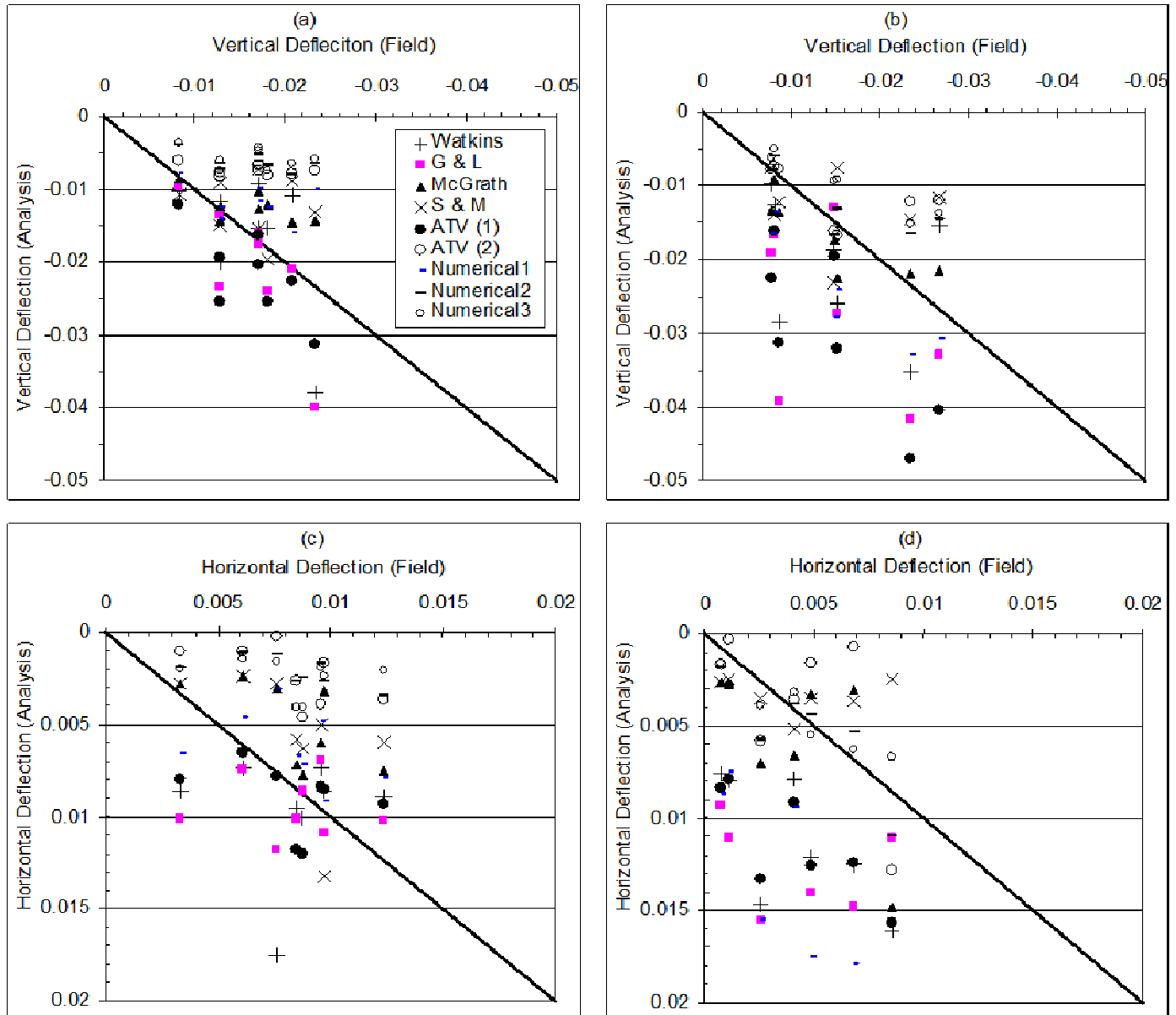
**Figure 5.** Comparison of computed vertical and horizontal deflections on the PVC and HDPE pipes calculated by empirical approaches and measured values in the field: (a) vertical deflections on PVC pipes (b) vertical deflections on HDPE pipes (c) horizontal deflections on PVC pipes (d) horizontal deflections on HDPE pipes.

## Conclusion

Empirical approaches and numerical analyses are commonly used in design of buried flexible pipes. In this study, behavior of thermoplastic pipes buried under high fills studied within the scope of Deep Burial Project (Sargand et al., 2002) are firstly analyzed using the equations derived from Iowa Approach and German Method. Furthermore, behavior of thermoplastic pipes in three different sections is modeled by using 2D finite

element analysis. Vertical and horizontal deflections derived from the empirical approaches and finite element method are compared with field measurements.

Deflections under final embankment fill load calculated using the empirical approaches by taking into consideration types of pipe and backfill material. Vertical and horizontal deflections calculated for PVC pipes are observed to be more compatible with the values measured in the field compared to the values calculated for HDPE pipes. As a result, it can be concluded that the



**Figure 6.** Comparison of vertical and horizontal deflections calculated with empirical approaches and measured in the field when crushed limestone and sand backfills (a) Vertical deflection when crushed limestone backfill is used. (b) Vertical deflections when sand backfill is used (c) Horizontal deflections when crushed limestone backfill is used (d) Horizontal deflections when sand backfill is used.

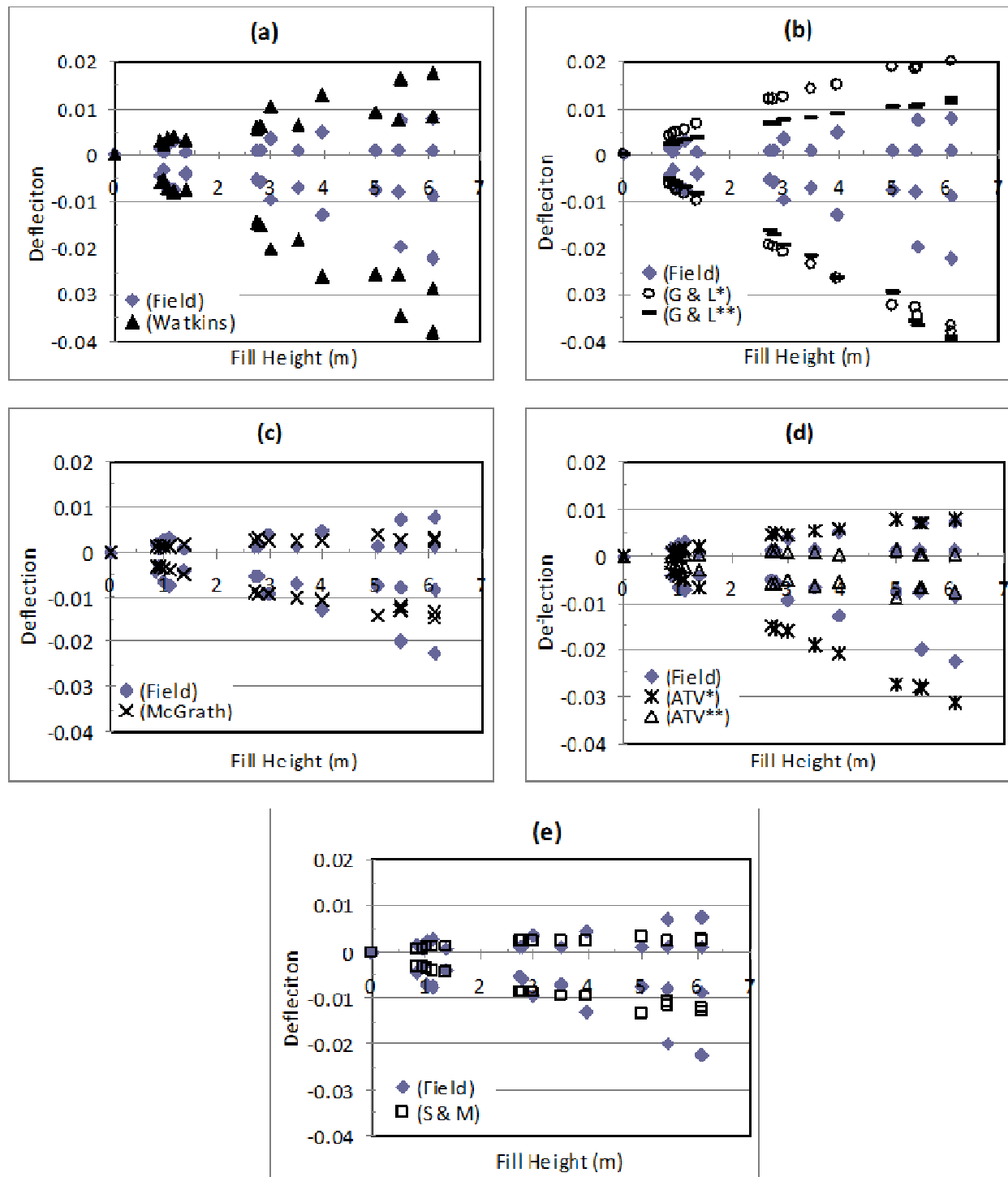
empirical approaches used in the literature are more reliable in the calculation of deflections that will occur on the PVC pipes.

Two of the most important parameters which affect the deflection behavior of buried flexible pipes are the stiffness of backfill and stiffness of native soil. The effect of native soil stiffness on the pipe stability is only considered in the Greenwood and Lang (1990) and ATV (2000). Due to lack of data concerning native soil stiffness, the values suggested by Greenwood and Lang (1990) are used in analyses, and therefore computed

deflections could not reflect field conditions properly.

In the analyses conducted with German Method, it is observed that first case (ATV (1)) yields the upper limit and second case (ATV (2)) yields the lower limit values for pipe deflections. Upper and lower limits of deflections constituted the boundaries of an interval and measured deflections generally fall into this interval.

In this study the behavior of thermoplastic pipes buried under high fills within the scope of deep burial project (Sargand et al., 2002) are also numerically analyzed. Deflections obtained from the finite element analysis

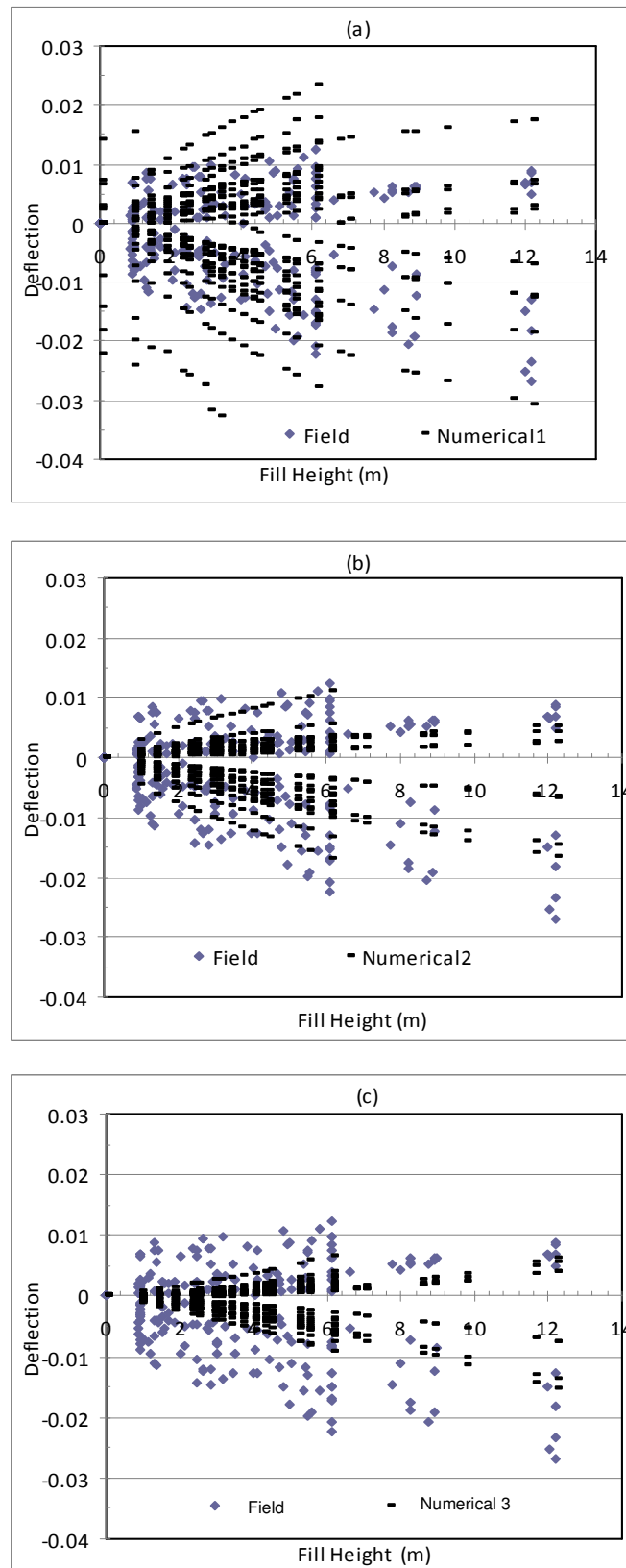


**Figure 7.** Variation of vertical and horizontal deflections calculated through empirical approaches with the fill height a) Watkins (1958) b) McGrath (1998) c) Greenwood and Lang (1990) d) ATV (2000) e) Sargand et al. (2005).

where the lower limit values for native soil stiffness are taken into account are observed to be closer to the field measurements. Based on the analysis results, it can be considered that native soil stiffness considerably affects

the pipe deflections, therefore native soil conditions must be taken into account in the design of pipes.

In design of buried flexible pipes, empirical approaches may be utilized provided that the field conditions are



**Figure 8.** The variation of thermoplastic pipe deflections during embankment construction and values derived from a) Numerical1 b) Numerical2 c) Numerical 3.

taken into consideration and/or numerical analysis can be employed by taking into account these field conditions into the finite element analyses.

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