Scientific Research and Essays Vol. 6(34), pp. 6798-6803, 30 December, 2011
Available online at http://www.academicjournals.org/SRE
DOI: 10.5897/SRE11.040
ISSN 1992-2248 ©2011 Academic Journals

Full Length Research Paper

Crack damage detection of reinforced concrete beams using local stiffness indicator

Ismail Z.[1]*, Ong A. Z. C.[2] and Rahman A. G. A.[3]

[1]Civil Engineering Department, University of Malaya, 50603 Kuala Lumpur, Malaysia.
[2]Mechanical Engineering Department, University of Malaya, 50603 Kuala Lumpur, Malaysia.
[3]Mechanical Engineering Department, Universiti Malaysia Pahang, Kuantan, Malaysia.

Accepted 31 October, 2011

This paper described the application of the generalized solution for transverse vibration to detect crack damages in reinforced concrete beams. A single crack was induced in a full-scale reinforced concrete beam by application of a point load. The load was increased in stages to obtain different crack heights to represent the extent and severity of the defect. Experimental modal analysis was performed on the beam prior to application of the load and after each load stage. The mode shape equation for the beams was obtained by using nonlinear regression. Global flexural stiffness *w*as derived by utilizing the regressed variable λ into the equation for transverse vibration of a Bernoulli-Euler prismatic beam. Local flexural stiffness at each coordinate point was derived by substituting the regressed data at that point and by using the centered-finite-divided-difference formula. The global stiffness decreased with increased severity of the crack in the beam. The results were compared with values computed using the secant modulus from the load-deflection plot obtained upon loading at each load stage and the trend was similar. The proposed algorithm could form the basis of a technique for structural health monitoring of load induced damaged reinforced concrete structures.

Key words: Crack damage, defect severity, local stiffness indicator, reinforced concrete beam, structural health monitoring.

INTRODUCTION

Periodic structural condition monitoring of reinforced concrete structures is necessary to ensure that they provide a continued safe service condition. Conventional assessment procedures usually rely on visual inspection and location-dependent methods. This study proposed the application of experimental modal analysis to locate crack damage in reinforced concrete beams. The presence of a crack can cause localized changes to flexural stiffness which is dependent on the severity of the crack. There have been several significant studies carried out to determine the existence and the severity of defects in structures using one or more of their modal properties. Examples are work by Kam and Lee (1992) and Lim (1991) on damage detection in structures using modal testing, Penny et al. (1993) on determination of damage location using vibration data, Sezer (2010) on non-linear analysis and Yildiz and Uğur (2009) on effect of corrosion on durability. Some research on beams was conducted by Dong et al. (1994), Maeck et al. (1999), Önal (2009), Yang et al. (2009), Yilmaz (2010), Zhang and Ma (2010), and Zhu and Law (2007). Specific researches on cracked beams were conducted by Chen et al. (2004), Law and Zhu (2006), Narkis (1994), and Nwosu et al. (1995). Some work on cantilevers was conducted by Rizos et al. (1990), and damage detection of strusses was done by Liu (1995). Dynamic characteristics of a plate with a growing surface crack were conducted by Chen and Swamidas (1994). Most of the dynamic tests conducted on actual structures utilized the fundamental natural frequencies which had been found to be the most convenient parameter to be studied (Javor, 1991; Konig and Giegerich, 1989). It was found that the most easily observable change was the reduction

*Corresponding author. E-mail: zubaidahjka@gmail.com. Tel: 60379675284. Fax: 60379675318.

in natural frequencies, and most investigators used this feature in one way or another (Cawley and Adams, 1979; Friswell et al., 1994; Gudmundson, 1982; Morassi and Rovere, 1997). Casas (1994) proposed a method of surveillance of concrete structures through monitoring the characteristics of the natural frequencies and mode shapes. Varying success has been reported where the change in modal damping have been utilized. Salawu and Williams (1994) used change of mode shape to detect damage. West (1994) presented a systematic use of mode shape data in the location of structural damage without the use of a prior finite element model. The modal assurance criteria (MAC) was used to determine the level of correlation between modes from the test of an undamaged body flap and the modes from the test of the flap after it had been exposed to loading. The mode shapes were partitioned using various schemes, and the change in MAC across the different partitioning techniques was used to locate the structural damage. It was shown that shape changes such as the MAC were relatively insensitive to damage in a beam with a saw cut. Graphical comparisons of relative changes in mode shapes are shown to be the best way of detecting the damage location when only resonant frequencies and mode shapes are examined. A method of scaling-up the relative changes in mode shape to improve the process of identifying the location of the damage have also been presented (Fox, 1992).

A technique for locating damage in a beam that used a finite difference approximation of a Laplacian operator on mode shape data was presented by Ratcliffe (1997). In the case of a damage which is not so severe, further processing of the Laplacian output is necessary before damage location can be determined. The procedure is found to be best suited for the mode shape obtained from fundamental natural frequency. The mode shapes obtained from higher natural frequencies may be used to verify the damage location, but they are not as sensitive as the lower modes.

An alternative method to using mode shapes in obtaining spatial information about sources of vibration changes is by using the mode shape derivatives, such as curvature. It is noted that for beams, plates, and shells there is a direct relationship between curvature and bending strain. Pandey et al. (1991) demonstrated that absolute changes in mode shape curvature could be a good indicator of damage for the cantilever and simply supported analytical beam structures which they considered. The changes in the curvature increase with increase in damage. The curvature values are computed from the displacement mode shape using the central difference approximation. Stubbs et al. (1992) presented a method based on the decrease in the curvature of the measured mode shapes or the modal strain energy between two structural degrees of freedom. Topole and Stubbs (1995a, 1995b) further showed that using a limited set of modal parameters for structural damage detection was feasible. Stubbs and Kim (1996) also showed that localizing damage using this technique without baseline modal parameters is also possible. This approach was confirmed by Chance et al. (1994) who found that numerically calculating curvature from mode shapes resulted in unacceptable errors. As a consequence measured strains were instead used to measure curvature directly, and this improved results significantly. Shahrivar and Bouwkamp (1986) who presented the finite element and experimental data for a scale model of an offshore platform found that the fundamental mode shape is more sensitive to damage than the fundamental vibration frequency, further confirming the sensitivity of the mode-shape method. In this current study a local stiffness indicator was used to confirm the location of damage.

MATERIALS AND METHODS

Real structures are complex systems which are difficult to analyze and monitor. A beam with damage like a small crack represents a non-linear system. A simple but reliable algorithm which applies easily obtainable modal data would be a useful tool. The objective of this study is to develop such a tool. For simplification it is assumed that the beam under consideration satisfies the conditions for a Bernoulli-Euler type beam. The transverse forced vibration of such a beam can thus be simply derived from Newton's Law of Motion represented by the following equation:

$$\frac{\partial^2}{\partial x^2}\left(EI\,\frac{\partial^2 v}{\partial x^2}\right)+\rho A\frac{\partial^2 v}{\partial t^2}=p(x,t) \qquad (1)$$

Where, EI = flexural stiffness; ρ = density; A = area.

The beam is also relatively long and thin. For the case of free vibration this equation reduces to:

$$\left(EIv''\right)''+\rho A\ddot{v}=0 \qquad (2)$$

Assuming harmonic motion given by the equation:

$$v(x,t)=V(x)\cos(\omega t-\alpha) \qquad (3)$$

and substituting this into Equation 2 the eigenvalue equation is obtained:

$$\left(EIV''\right)''-\rho A\omega^2 V=0 \qquad (4)$$

For variable coefficients, closed-form solutions are not available for this equation. This restricts the case to only free vibration of uniform beams where the coefficients are constants, Equation 4 reduces to:

$$\frac{d^4V}{dx^2}-\lambda^4 V=0 \qquad (5)$$

Where;

$$\lambda^4 = \frac{(\rho A \omega^2)}{EI} \tag{6}$$

ω = frequency

The general solution of Equation 4 may be written in the form:

$$V(x) = A_1 e^{\lambda x} + A_2 e^{-\lambda x} + A_3 e^{i\lambda x} + A_4 e^{-i\lambda x} \tag{7a}$$

Two useful alternative forms are:

$$V(x) = B_1 e^{\lambda x} + B_2 e^{-\lambda x} + B_3 \sin \lambda x + B_4 \cos \lambda x \tag{7b}$$

And

$$V(x) = C_1 \sinh \lambda x + C_2 \cosh \lambda x + C_3 \sin \lambda x + C_4 \lambda x \tag{7c}$$

There are five constants in the general solution, namely, four amplitude constants and the eigenvalue,λ, which is the natural frequency. The boundary conditions are used in evaluating these constants. For free vibration of uniform beams, Equation 3 can be substituted into the end condition equations to give, for the case of a simply supported beam:

$$V = 0 \tag{8a}$$

and $$\frac{d^2V}{dx^2} = 0 \tag{8b}$$

Utilization of these boundary conditions the values of λ and the corresponding mode shapes of the beam can be evaluated.

In the current study, natural frequencies and mode shapes are obtained from modal testing. The mode shape data derived have discrete values, and an equation is required to represent these discrete values. The data are thus curve-fitted into the generalized mode shape equation (Equation 7c) by the least-squares regression method. This produces estimates for the unknown coefficients. The aim of this approach is to derive a single curve that represents the general trend of the data because the data exhibit a significant degree of error or "noise". A nonlinear regression technique, namely the Marquardt-Levenberg algorithm is then used for the curve fitting. It is a weighted average of Newton's method and the Steepest Descent method for nonlinear systems of equations as defined in Transforms and Nonlinear Regression (1995) and SigmaStat (1995). An approximation is made which assumes that the generalized solution still applies for the beams with cracks which are not so severe. The next step is to derive the global flexural stiffness. This can be done by utilizing the estimated λ and rearranging Equation 6. To derive the local flexural stiffness at each coordinate point λ was derived by applying Equation 5 and the centered-finite-divided-difference formula, namely:

$$f^{(iv)} = \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4} \tag{8}$$

On the regressed data.

Employing Equation 6 and rearranging Equation 5 into:

$$\lambda^4 = \frac{\rho A \omega^2}{EI} = \frac{V^{(iv)}}{V} \tag{9}$$

Produce a shape indicator for the flexural stiffness, EI. Equation 5 is an egienvalue problem. The graph for Equation 9 would be a constant straight line. Deviation from this line indicates stiffness change. This may be the result damage.

For comparison, the RC beams are load-tested by applying a point load at point 0.5 L and 0.7 L where L is the length of the beam. The length to thickness ratio of the beam and the non-severity of the crack damage ensure that the approximation to a Bernoulli-Euler type beam still holds. In the load test, the applied load and deflection are measured during loading. The graphs of load versus deflection are plotted. The gradient of the linear portion of the graph gives the bending characteristic of the test beams. Subsequently, the flexural stiffness of the beam is calculated from the load-deflection graph.

For a Bernoulli-Euler type beam the theoretical mid-span deflections of a simply supported beam under point load, at mid span and a point $L - b$, where $b < 0.5L$, are given respectively by:

$$\delta = \frac{PL^3}{48EI} \tag{10a}$$

$$\delta = \frac{Pb(3L^2 - 4b^2)}{48EI} \tag{10b}$$

Where P = load, L = span (m). The bending stiffness of the beam, EI, can be calculated as;

$$EI = \left(\frac{L^3}{48}\right)\left(\frac{P}{\delta}\right) \text{ or } EI = \left(\frac{b(3L^2 - 4b^2)}{48}\right)\left(\frac{P}{\delta}\right)$$

Where (P/δ) is the gradient of the graph of load versus deflection.

RESULTS AND DISCUSSION

Crack at 0.5 L

Table 1 show that the results for flexural stiffness obtained from modal tests range from 3.4366 x 10^6 Nm^2 for an uncracked beam with zero load to 1.8997 x 10^6 Nm^2 for a beam with 1.31 mm crack width and 43 kN loading. Corresponding results from static load tests range from 4.0398 x 10^6 Nm^2 to 2.8825 x 10^6 Nm^2 respectively. This shows that the results obtained from the modal tests are comparable to those obtained from static load tests. In other words it shows that the algorithm proposed gives results which are in fair agreement with the results from static load tests. The results show that for a beam with an induced crack width of 1.31 mm. there is a corresponding loss of stiffness of up to 45% as indicated by the modal tests, and up to 29% as indicated by the load tests.

Figure 1 shows that the shape indicator plot is a straight line indicating no loss of flexural stiffness for the datum beam. For Load 1 the curve decreases slightly below the datum case; while for Load 2 the loss of flexural stiffness is more significant as evidenced by a

Table 1. Comparison of EI values with crack at 0.5 L.

Load	Ave. crack	EI X 10^6 (Nm^2)		Relative stiffness	
(kN)	Width (mm)	Modal test	Load test	Modal test	Load test
0	Uncracked	3.4366	4.0398	1	1
25	0.08	2.7009	3.4666	0.79	0.86
43	1.31	1.8997	2.8825	0.55	0.71

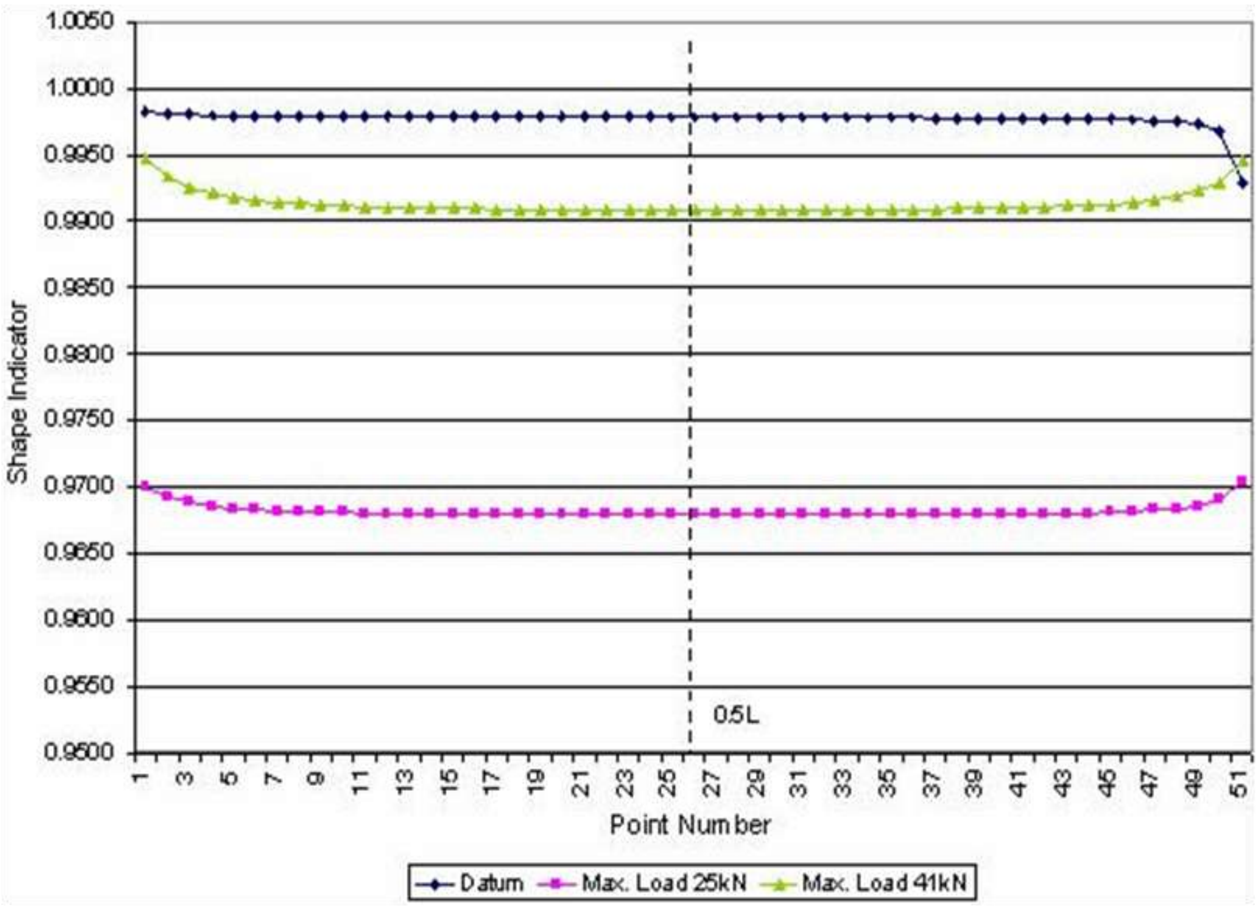

Figure 1. Normalized shape indicator for EI with crack at 0.5 L.

much lower decrease in the curve below the datum curve. This shows that the results for global flexural stiffness are consistent with the results of local flexural stiffness.

Crack at 0.7L

Table 2 shows that the results for flexural stiffness obtained from modal tests range from 4.7537 × 10^6 Nm^2 for an uncracked beam with zero load to 4.5634 × 10^6 Nm^2 for a beam with 0.7 mm crack width and 43 kN loading. Corresponding results from static load tests range from 4.5367 × 10^6 to 4.2482 × 10^6 Nm^2 respectively. This shows that the results obtained from the modal tests are almost similar to those obtained from static load tests. This shows that the algorithm proposed gives results which are in good agreement with the results from static load tests. The results shows that for a beam with an applied load of 41 kN corresponding to a crack width of 0.38 mm there is a corresponding loss of stiffness of up to 4% as indicated by the modal tests, and up to 6% as indicated by the load tests. The loss in stiffness is apparently very small compared to the beam with a crack at 0.5 L.

Table 2. Comparison of EI values with crack at 0.7L.

Load	Ave. crack	EI X 10^6 (Nm^2)		Relative stiffness	
(kN)	Width (mm)	Modal test	Load test	Modal Test	Load test
0	Uncracked	4.7537	4.5367	1	1
25	0.073	5.2884	4.4502	1.10	0.98
41	0.383	4.8520	4.5695	1.02	1.01
43	0.7	4.5634	4.2842	0.96	0.94

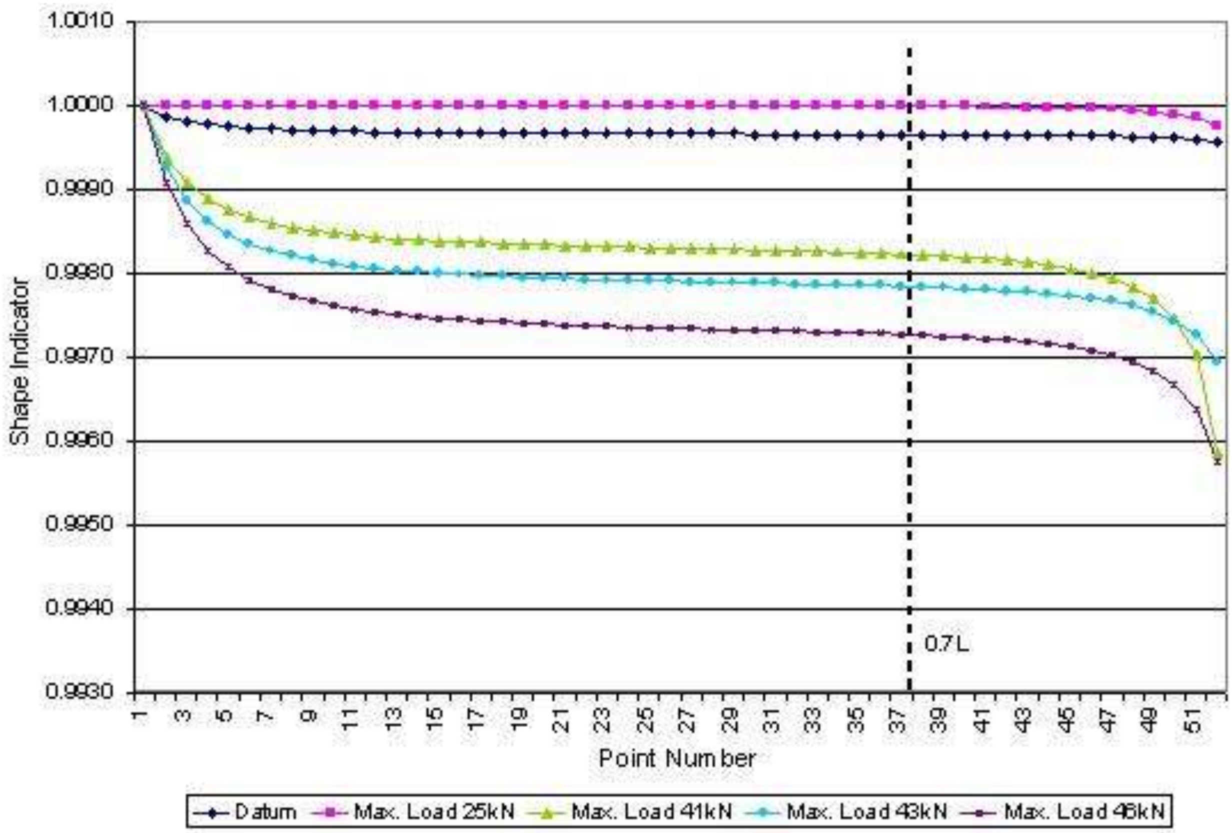

Figure 2. Normalized shape indicator for EI with crack at 0.7 L.

Figure 2 also shows that the shape indicator plot is a straight line indicating no loss of flexural stiffness for the datum beam. For Load 1 the curve rises slightly above the datum case. The increase, however, is not significant, indicating that there is very little change in the flexural stiffness due to the damage. For Load 2, Load 3 and Load 4 there are indications of increasing losses of local flexural stiffness with increasing loads and crack widths. The global flexural stiffness results obtained above show very little change from the datum for Load 1, Load 2 and Load 3; although the plot of local flexural stiffness did show a decrease in stiffness with increasing loads and crack widths. The overall effect on the stiffness remains insignificant. There are also inflexions shown on the curves followed by further loss of stiffness.

From the values of relative stiffness obtained, it is observed that for the same level of severity induced the crack in the middle causes a greater loss of stiffness compared to a crack on the right side of mid-span. The algorithm has proven to be a simple and reliable method to detect crack damage in a RC beam.

Conclusion

The procedure of applying curve fitting using the generalized solution model and fourth order centered finite-divided-difference to the modal data produces the global flexural stiffness for the control and defect reinforced concrete beams. The results using this method are in fairly good agreement with the results obtained from static load tests. The procedure is capable of indicating the presence of damage by showing a loss in the flexural stiffness. The loss of flexural stiffness is more pronounced in the case of damage in the middle than for similar extents of damage located outside the mid-span region.

REFERENCES

Casas JR (1994). An Experimental Study on the use of Dynamic Tests for Surveillance of Concrete Structures. Mat. Struct., 27: 588-595.

Cawley F, Adams RD (1979). The Locations of Defects in Structures from Measurements of Natural Frequencies. J. Strain Anal., 14(2): 49-57.

Chance J, Tomlinson GR, Worden K (1994). Simplified Approach to the Numerical and Experimental Modeling of the Dynamics of a Cracked Beam. Proc. 12thInt. Modal Anal. Conf., p. 778.

Chen G, Mu H, Pommerenke D, Drewniak JL (2004). Damage Detection of Reinforced Concrete Beams with

Novel Distributed Crack/Strain Sensors. Struct. Health Monit., 3(3): 225-243.

Chen Y, Swamidas ASJ (1994). Dynamic Characteristics and Modal Parameters of a Plate with a small Growing Surface Crack. Proc. 12th Int. Modal Anal. Conf., pp.1155-1161.

Dong C, Zhang PQ, Feng WQ, Huang TC (1994). The Sensitivity Study of the Modal Parameters of a Cracked Beam. Proc. 12th Int.Modal Anal. Conf., pp. 98-104.

Fox CHJ (1992). The Location of Defects in Structures: A Comparison of the Use of Natural Frequency and Mode Shape Data. Proc. 10th Int. Modal Anal. Conf., pp. 522-528.

Friswell MI, Penny JET, Wilson DAL (1994). Using Vibration Data Statistical Measures to Locate Damage in Structures. Int. J. Anal. Expt. Modal Anal., 9(4): 239-254.

Gudmundson P (1982). Eigenfrequency Changes of Structures due to Cracks, Notches, or other Geometrical Changes. J.Mech. Phy. Solids., 30(5): 339-353.

Javor T (1991). Damage Classification of Concrete Structures. The State of the Art Report of RILEM Technical Committee 104-DCC. Mat. Struct., 24: 253-259.

Kam TY, Lee TY (1992). Detection of Cracks in Structures using Modal Test Data. Eng. Fract. Mech., 42(2): 381-387.

Konig G, Giegerich G (1989). Identification of Structural Properties using Dynamic Tests. Proc. IABSE Sym.Durability Struct., Lisbon., pp. 835-840.

Law SS, Zhu XQ (2006). Vibration of a beam with a breathing crack subject to moving mass. Comp. Meth. 1963-1968, DOI: 10.1007/978-1-4020-3953-9_141

Lim TW (1991). Structural Damage Detection using Modal Test Data. AIAA J., 29(12): 2271-2274.

Liu PL (1995). Identification and Damage Detection of Strusses using Modal Data. J. Struct.Eng., 121(4): 599-608.

Maeck J, Wahab MA, de Roec G (1999). Damage Localization in Reinforced Concrete Beams by Dynamic Stiffness Determination. Proc. 17th Int. Modal Anal. Conf., pp. 1289-1295.

Morassi A, Rovere N (1997). Localizing a Notch in a Steel Frame from Frequency Measurements. J. Eng. Mech.-ASCE., 123(5): 422-432.

Narkis Y (1994). Identification of Crack Location in Vibrating Simply Supported Beams. JSV, 172(4): 549-558.

Nwosu DI, Swamidas ASJ, Guigne JY, Olowokere DO (1995). Studies on Influence of Cracks on the Dynamic Response of Beams. Proc. 13th Int. Modal Anal. Conf., pp. 1122-1128.

Önal MM (2009). Reinforcement of beams by using carbon fiber reinforced polymer in concrete buildings.

Sc. Res. Essays., 4(10): 1136-1145.

Pandey AK, Biswas M, Samman MM (1991). Damage Detection from Changes in Curvature Mode Shapes. JSV, 145(2): 321-332.

Penny JET, Wilson DAL, Friswell MI (1993). Damage Location in Structures using Vibration Data. Proc. 11th Int. Modal Anal. Conf., pp. 861-867.

Ratcliffe CP (1997). Damage Detection using a Modified Laplacian Operator on Mode Shape Data. JSV, 204(3): 505-517.

Rizos PF, Aspragathos N, Dimarogonas AD (1990). Identification of Crack Location and Magnitude in a Cantilever from the Vibration Modes. JSV, 138(3): 381-388.

Salawu OS, Williams C (1994). Damage Location using Vibration Mode Shapes. Proc. 12th Int. Modal Anal. Conf., 933-939.

Sezer R (2010). The geometrical nonlinear analysis of the prismatic plane frames with the stiffness matrix method. Sc. Res. Essays., 5(22): 3363-3370.

Shahrivar F, Bouwkamp JG (1986). Damage detection in offshore platforms using vibration information. J. Energy Res. Tech., 108: 97-106.

SigmaStat (1995). User's Manual, Jandel Scientific Software, United States.

Stubbs N, Kim JT (1996). Damage Localization in Structures without Baseline Modal Parameters. AIAA J., 34(8): 1644-1649.

Stubbs N, Kim JT, Topole KG (1992). An Efficient and Robust Algorithm for Damage Localization in Offshore Platforms. Proc. ASCE 10th Struct. Cong., p. 543.

Topole KG, Stubbs N (1995a). Nondestructive Damage Evaluation of a Structure from Limited Modal Parameters. Earthquake Eng. Struct. Dyn., 24(11): 1427-1436.

Topole KG, Stubbs N (1995b). Nondestructive Damage Evaluation in Complex Structures from a Minimum of Modal Parameters. Int. J. Anal. Expt. Modal Anal., 10(2): 95-103.

Transforms and Nonlinear Regression (1995). User's Manual, Jandel Scientific Software, United States.

West WM (1984). Illustration of the Use of Modal Assurance Criterion to Detect Structural Changes in an Orbiter Test Specimen. Proc. Air Force Conf. Aircraft Struct. Integrity. 1.

Yang DS, Park SK, Neale KW (2009). Flexural behaviour of reinforced concrete beams strengthened with prestressed carbon composites. Comp. Struct., 88: 497-508.

Yıldız K, Uğur LO (2009). Examinatıon of durablity of high performance concrete (hpc) that has been subjected to $MgSO_4$ and NaCl corrosıon against freezing and thawing. Sci. Res.Essays., 4(9): 929-935.

Yilmaz US, Saritas I, Kamanli M, Kaltakci MY (2010). An experimental study of steel fiber reinforced concrete columns under axial load and modeling by ANN. Sci. Res.Essays., 5(1): 81-92.

Zhang SJ, Ma ZJ (2010). Damage detection of simply-supported beams using mid-span displacement indicator. Appl. Mech. Mater., pp. 29-32: 532-536.

Zhu XQ, Law SS (2007). Nonlinear Characteristics of Damaged Reinforced Concrete Beam from Hilbert-Huang Transform. J. Struct. Eng., 133(8): 1186-1191.