A sensitive survey model for HIV seroprevalence research

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Sensitive questions like HIV status may cause biased estimation of unknown population parameters as well as increase in the variance of the estimates due to evasive responses. The randomized response techniques (RRT) can be used to avoid the concealment of information or evasive answers. The RRT guarantees the anonymity of respondents in surveys aimed at determining the frequency of stigmatic, embarrassing or criminal behaviour where direct techniques for data collection may induce respondents to refuse to answer or give false responses. Different randomized response models (RRMs) have been devised in the past decades for dealing with sensitive items; which usually involve the use of random devices, such as dice or cards to collect reliable data on sensitive issues. Most of these RRMs have been proposed without some specific applications to HIV seroprevalence surveys. The motivation was to improve upon the existing RRMs as well as to apply them to estimate HIV seroprevalence rates. The objectives were to use research frontier to devise a mixed-stratified RRM, use same to estimate HIV seroprevalence rates in a given population and compare results with the existing seroprevalence rates. Furthermore, the procedure of the field work and sampling design were well coordinated for the target population of 3,740 people aged 18 years and above using a sample size of 550. Furthermore, the model was used to estimate the HIV seroprevalence rate in a small population of adults attending a clinic in Kaduna, Nigeria. The model estimated the HIV seroprevalence rate as 8.74% with a standard error of 0.0134 and a 95% confidence interval of 6.1 and 11.4%, respectively. Accordingly, the sentinel projected seroprevalence rate, using the Epidemic Projection Package (EPP), for the next ten years (2013) was 9.7%; very consistent with the 95% confidence interval. Hence, the RRTs herein can serve as new viable methods for HIV seroprevalence surveys.

Key words: Randomized response techniques, randomized response models, seroprevalence rates, mixed-stratified, design parameter, efficiency, sentinel surveys, stratified random sampling.

INTRODUCTION

Sensitive questions like HIV status may cause biased estimation of unknown population parameters as well as increase in the variance of the estimates due to evasive responses. The randomized response techniques (RRTs)
were especially developed to improve the accuracy of answers to sensitive questions. Socially sensitive questions are thought to be threatening to respondents (Lee, 1993). When sensitive topics are studied, respondents often react in ways that negatively affect the validity of the data. Such a threat to the validity of the results is the respondents’ tendency to give socially desirable answers to avoid social embarrassment and to project a positive self-image (Rasinski, 1999). Warner (1965) reasoned that the reluctance of the respondents to reveal sensitive or probably harmful information would diminish when respondents could be convinced that their anonymity was guaranteed. Following this assumption, Warner (1965) designed the first randomized response model (RRM). The crux of his method and all other RRTs that followed is that the meaning of the respondents’ answers is hidden by a deliberate contamination of the data.

Studies with RRTs have been conducted in the areas of health care (Volicer and Volicer, 1982), on alcohol, drug abuse and sexual behaviour (Jarman, 1997), on child molestation (Fox and Tracy, 1986), on tax evasion (Houston and Tran, 2008), among others. Meta-analysis on 42 comparative studies showed that RRTs resulted in more valid population estimates than direct question-answer techniques (Lensvelt-Mulder et al., 2005). The advantage of using RRTs to question sensitive topics is that the results are less distorted than when direct question–answer designs are used, making the RRM more effective. A second advantage of using RRT when conducting sensitive research is that, the individual ‘yes’-answer becomes meaningless as it is only a ‘yes-answer’ to the random device (Van der Hout et al., 2002). However, the disadvantage of using RR methods is that they are less efficient than direct question designs. Since the RRTs work by adding random noise to the data, they all suffer from larger standard errors, leading to reduced power which makes it necessary to use larger samples than in question–answer designs. Unfortunately, larger samples are associated with prolonged completion time and higher research costs, making RRTs less attractive to applied researchers. This leads to the topic of efficiency versus effectiveness.

Brookmeyer and Gail (2004) defined HIV seroprevalence as the study of the number of cases where HIV is present in a specific population at a designated time. The presence of HIV in a specific individual is determined by the finding of HIV antibodies in the serum (HIV seropositivity). This study is set to develop an efficient mixed-stratified RRM particularly for HIV seroprevalence surveys and to use the new model for estimating the seroprevalence rate in a small population.

**METHODOLOGY**

The procedure of the field work and sampling design were well coordinated for the target population of 3,740 adults aged 18 years and above attending Gwanna Awan Hospital in Kaduna, Nigeria using a sample size of 550. Warner (1965) proposed the pioneering RRM for estimating the proportion of persons bearing a socially disapproved character. Quatember (2009) produced unified criteria for all RRTs; also, Kim and Warde (2005) proposed a stratified RRM and so many others. Furthermore, the model was used to estimate the HIV seroprevalence rate in the same population. Quatember (2009) both theoretically and empirically analyzed the effect of different design parameters on the performance of RRTs using different levels of privacy protection and thereafter concluded that 0.7 approximately works well for every mixed RRM where the questions are regarded as highly sensitive. Hence, 0.7 was adopted as the design parameter and deck of 50 cards as the random device throughout.

In stratified sampling, the population of $N$ units is first divided into subpopulations (strata) of $N_1, N_2, \ldots, N_L$ units, respectively. These subpopulations are non-overlapping and together they comprise the whole of the population so that $N_1 + N_2 + \ldots + N_L = N$. The sample sizes within the strata are denoted by $n_1, n_2, \ldots, n_L$, respectively. If a simple random sample is taken in each stratum, the whole procedure is described as stratified random sampling. The marital status is used to form three strata for this study.

**The proposed RRT Model**

The HIV seroprevalence survey model requires that a sample respondent in stratum $h$ answers an innocuous direct question and asked to use the random device $R_{h1}$ if his/her answer to direct question is “yes”. If answer to the direct question is “no”, he/she is requested to use another random device $R_{h2}$. The random device $R_{h1}$ consists of two statements (i) “I am HIV positive” and (ii) “I am HIV negative”, presented with probabilities $P_{h1}$ and $(1 - P_{h1})$ respectively. Similarly, the random device $R_{h2}$ consists of the two statements (i) “I am HIV positive” and (ii) “I am HIV negative”, presented with probabilities and $P_{h2}$ and $(1 - P_{h2})$ respectively. The probabilities of a ‘yes’ response from the respondents using $R_{h1}$ and $R_{h2}$ are respectively given by:

$$\lambda_{h1} = P_{h1}\tau_h + (1 - P_{h1})\tau_y = P_{h1}\tau_h + (1 - P_{h1})$$  \hspace{1cm} (1)

and

$$\lambda_{h2} = P_{h2}\tau_h + (1 - P_{h2})$$  \hspace{1cm} (2)

On the other hand, the probabilities of a ‘no’ response from the respondents using $R_{h1}$ and $R_{h2}$ are respectively given by:

$$\lambda_{h1}' = P_{h1}(1 - \tau_h) + (1 - P_{h1})(1 - \tau_y) = P_{h1}(1 - \tau_h)$$  \hspace{1cm} (3)

and

$$\lambda_{h2}' = P_{h2}(1 - \tau_h)$$  \hspace{1cm} (4)

Since the respondent using $R_{h1}$ has already answered yes to the direct question, $\tau_y = 1$. 


Among those that answered ‘yes’ to the innocuous questions in stratum \( h \), suppose that \( n_{1h} \) report ‘yes’ and \( (n_h - n_{1h}) \) report ‘no’, the likelihood of the sample in the same stratum is as follows:

\[
\xi = \left[ P_{h1} \pi_h + (1 - P_{h1}) \right]^{n_{1h}} \times \left[ P_{h1} (1 - \pi_h) \right]^{n_h - n_{1h}}
\]

(5)

The natural log of the likelihood is given below:

\[
\log \xi = n_{1h} \log \left[ P_{h1} \pi_h + (1 - P_{h1}) \right] + (n_h - n_{1h}) \log \left[ P_{h1} (1 - \pi_h) \right]
\]

(6)

To obtain the value of \( \pi_h \), differentiate \( \log \xi \) w.r.t \( \pi_h \) and equate to zero as follows:

\[
\frac{\partial \log \xi}{\partial \pi_h} = \frac{n_{1h} P_{h1}}{P_{h1} \pi_h + (1 - P_{h1})} - \frac{(n_h - n_{1h}) P_{h1}}{P_{h1} (1 - \pi_h)} = 0
\]

\[
P_{h1} \pi_h + (1 - P_{h1}) = \frac{(n_h - n_{1h}) P_{h1}}{n_{1h} P_{h1}}
\]

(7)

To obtain the unbiased estimator of \( \pi_h \), we substitute \( \hat{\pi}_h \) in \( \pi_h \) in the equation above to get:

\[
\hat{\pi}_h = \frac{n_{1h} P_{h1}}{n_h P_{h1}}
\]

Hence, the unbiased estimators in terms of the responses of the respondents using \( R_{h1} \) is given by:

\[
\hat{\pi}_{h1} = \frac{\hat{\pi}_h - (1 - P_{h1})}{P_{h1}}
\]

(8)

Where the proportion of ‘yes’ answers from \( R_{h1} \) in the sample is \( \hat{\lambda}_{h1} = n_{1h} / n_h \). The variance of \( \hat{\pi}_{h1} \) is obtained as follows:

\[
\text{Var}(\hat{\pi}_{h1}) = \frac{1}{P_{h1}^2} \left\{ \frac{\hat{\pi}_h (1 - \hat{\pi}_h)}{n_{1h}} \right\}
\]

(9)

Similarly, the unbiased estimators in terms of the responses of the respondents using \( R_{h2} \) is given by:

\[
\hat{\pi}_{h2} = \hat{\lambda}_{h2} = \frac{n_{1h}}{n_h}
\]

(10)

Where the proportion of ‘yes’ answers from \( R_{h2} \) in the sample is \( \hat{\lambda}_{h2} = n_{1h} / n_h \). The variance of \( \hat{\pi}_{h2} \) is obtained as follows:

\[
\text{Var}(\hat{\pi}_{h2}) = \frac{1 - \pi_h}{n_h^2 P_{h1}^2}
\]

In stratum \( h \), two randomization devices \( R_{h1} \) and \( R_{h2} \) are equally protective against the privacy of the respondents if \( P_{h1} = P_{h2} = P_h \). Under this setting, the variances of the two unbiased estimators \( \hat{\pi}_{h1} \) and \( \hat{\pi}_{h2} \) become the same. We can also propose an estimator based on all the information collected in stratum \( h \) which we can use to estimate seroprevalence rates in stratum \( h \) as follows:

\[
\hat{\pi}_h = \frac{n_{1h}}{n_h} \hat{\pi}_{h1} + \frac{n_{1h}}{n_h} \hat{\pi}_{h2}
\]

(11)

Its variance is given by:

\[
\text{Var}(\hat{\pi}_h) = \left( \frac{n_{1h}}{n_h} \right) \text{Var}(\hat{\pi}_{h1}) + \left( \frac{n_{1h}}{n_h} \right) \text{Var}(\hat{\pi}_{h2})
\]

(12)

If we decide that \( P_{h1} = P_{h2} = P_h \), we thus get:

\[
\text{Var}(\hat{\pi}_h) = \left( \frac{n_{1h}}{n_h} \right) \left[ \frac{(1 - \pi_h)(P_h \pi_h + 1 - P_h)}{P_h} \right]
\]

(13)

Hence

\[
\text{Var}(\hat{\pi}_h) = \frac{\pi_h (1 - \pi_h) + (1 - P_h)(1 - \pi_h)}{n_h P_h}
\]

An unbiased stratified seroprevalence rates estimator is given by:

\[
\hat{\pi}_{h\text{sero}} = \sum_{h=1}^{L} W_h \hat{\pi}_h
\]

(14)

where:

\[ W_h = N_h / N \] for \( h = 1, 2, \ldots, L \)

\( N_h \) is the total number of individuals in the stratum \( h \).

\( N \) is the total number of individuals in the population.

Obviously, \( \sum_{h=1}^{L} W_h = 1 \)
Its variance is given by:

\[ \text{Var}(\hat{\pi}_{s_{\text{sero}}}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left[ \pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{P_h} \right]^2 \]

(Cochran 1977) established that the sampling fraction \( n_h / n \) is ignorable, and \( \text{Var}(\hat{\pi}_{s_{\text{sero}}}) \) is minimized for a fixed total sample size \( n \) if:

\[ n_h = \frac{\sum_{h=1}^{L} W_h \left[ \pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{P_h} \right]^2}{\sum_{h=1}^{L} W_h} \]

(16)

Where \( n_h = n_{h1} + n_{h2} \)

\[ \sum_{h=1}^{L} n_h = n \]

Thus, substituting the optimum value of \( n_h \) in Equation 15 we get:

\[ \text{Var}(\hat{\pi}_{s_{\text{sero}}}) = \frac{1}{n} \left[ \sum_{h=1}^{L} W_h \left[ \pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{P_h} \right]^2 \right] \]

(17)

Relative efficiency of the RRT model

One of the most important ways of assessing any sample survey model is through its efficiency relative to the existing models. We hereby compare the relative efficiency of the proposed for HIV seroprevalence model with Kim and Warde (2005) stratified estimator. Hence, the proposed model is more efficient for a fixed sample size if and only if:

\[ \text{Var}(\hat{\pi}_{SK}) - \text{Var}(\hat{\pi}_{s_{\text{sero}}}) \geq 0 \]

(18)

\[ \frac{1}{n} \left[ \sum_{h=1}^{L} W_h \left\{ \pi_h(1-\pi_h) + \frac{(1-P_h)\lambda_{h}P_h(1-\pi_h) + 1 - \lambda_h}{P_h} \right\} \right]^2 \]

\[ - \frac{1}{n} \left[ \sum_{h=1}^{L} W_h \left[ \pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{P_h} \right]^2 \right] \geq 0 \]

The above inequality will be true if for each stratum \( h \), \( h = 1,2,\ldots,L \) resulting to:

\[ \left\{ \pi_h(1-\pi_h) + \frac{(1-P_h)\lambda_{h}P_h(1-\pi_h) + 1 - \lambda_h}{P_h} \right\} \geq 0 \]

\[ (1-P_h)\lambda_{h}P_h(1-\pi_h) + 1 - \lambda_h \geq 0 \]

\[ (1-P_h)(1-\pi_h) \geq 0 \]

(19)

The LHS of Equation 19 is always non-negative, hence the proposed model is more efficient than Kim and Warde (2005) stratified estimator.

RESULTS ANALYSIS

The analysis was manually computed to arrive at the following results (Tables 1, 2 and 3). Recall that the unbiased mixed-stratified seroprevalence model is given by:

\[ \hat{\pi}_{s_{\text{sero}}} = \sum_{h=1}^{L} W_h \hat{\pi}_h \]

where:

\[ W_h = n_{h1} / N \]

for \( h = 1,2,\ldots,L \)

Thus, its variance is given by:

\[ \text{Var}(\hat{\pi}_{s_{\text{sero}}}) = \sum_{h=1}^{L} W_h^2 \left[ \pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{P_h} \right]^2 \]

where:

\[ \hat{\pi}_h = \frac{n_{h1}}{n_h} \hat{\pi}_{h1} + \frac{n_{h2}}{n_h} \hat{\pi}_{h2} \]

\[ \text{Var}(\hat{\pi}_h) = \frac{\pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{n_h P_h}}{n_h} \]

The other computations are summarized below:

\[ \phi = \pi_h(1-\pi_h) + (1-P_h)(1-\pi_h) / P_h \]

\[ \hat{\pi}_{s_{\text{sero}}} = \sum_{h=1}^{L} W_h \hat{\pi}_h = 0.0874 \]

\[ \text{Var}(\hat{\pi}_{s_{\text{sero}}}) = \sum_{h=1}^{L} W_h^2 \left[ \pi_h(1-\pi_h) + \frac{(1-P_h)(1-\pi_h)}{P_h} \right]^2 = 0.00018 \]

\[ SE(\hat{\pi}_{s_{\text{sero}}}) = \sqrt{\text{Var}(\hat{\pi}_{s_{\text{sero}}})} = 0.0134 \]

The 95% confidence interval for HIV seroprevalence rate is given by:

\[ \hat{\pi}_{s_{\text{sero}}} \pm 1.96 \times SE(\hat{\pi}_{s_{\text{sero}}}) = 0.0874 \pm 1.96 \times 0.0134 = [0.061, 0.114] \]

DISCUSSION

This study has helped to avoid evasive answer on HIV
surveys. It was motivated by the fact that conventional data collection techniques usually cause evasive or untruthful responses when people are asked sensitive questions like their HIV serostatus. As a result, it is difficult to make accurate inferences from such unreliable data. This study has devised a mixed-stratified RRM using the work of Warner (1965), Arnab (2004), Quatember (2009), among others particularly for HIV seroprevalence surveys. The proposed model proved to be more efficient than a frontier similar model by Kim and Warde (2005).

### Conclusion

We have been able to develop a sensitive survey model for HIV seroprevalence. The model was used to estimate HIV seroprevalence rate in a small adult population using a sample size of 550 and a design parameter of 0.7. Using the survey data, the model estimated the HIV seroprevalence rate as 8.74% with a standard error of 0.0134 and 95% confidence bands of 6.1 and 11.4%, respectively. These estimates are for adults who are 18 years and above who attend a hospital. These results are consistent with that of Nigerian Sentinel Survey (2003) conducted by NACA, USAID and CDC which estimated the HIV seroprevalence in Kaduna State as 6.0%. Accordingly, the sentinel projected seroprevalence rate, using the EPP Package for the next ten years (2013) was 9.7%; very consistent with the 95% confidence interval. Hence, the RRTs herein can serve as new viable methods for HIV seroprevalence surveys.

### CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

### Abbreviations

AIDS, Acquired Immune Deficiency Syndrome; CDC, Centre for Disease Control; EPP, Epidemic Projection Package; HIV, Human Immunodeficiency Virus; NACA, National Agency for the Eradication of AIDS; RR, Randomized Response; RRM, Randomized Response Model; RRT, Randomized Response Technique; USAID, United States Agency for International Development.
ACKNOWLEDGEMENT

The authors wish to thank the anonymous reviewers for their time and constructive reviewing.

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